

Hierarchical Task Networks

- **Planning to perform tasks rather than to achieve goals**

Literature

- Malik Ghallab, Dana Nau, and Paolo Traverso. *Automated Planning – Theory and Practice*, chapter 11. Elsevier/Morgan Kaufmann, 2004.
- E. Sacerdoti. The nonlinear nature of plans. In: *Proc. IJCAI*, pages 206-214, 1975.
- A. Tate. Generating project networks. In: *Proc. IJCAI*, pages 888-893, 1977.

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HTN Planning

- HTN planning:
 - objective: perform a given set of tasks
- input includes:
 - set of operators
 - set of methods: recipes for decomposing a complex task into more primitive subtasks
- planning process:
 - decompose non-primitive tasks recursively until primitive tasks are reached

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HTN Planning

•HTN planning:

- world state represented by set of atoms and actions correspond to deterministic state transitions

•**objective: perform a given set of tasks**

- previously: achieve some goals

•input includes:

•**set of operators**

•**set of methods: recipes for decomposing a complex task into more primitive subtasks**

- methods: at a higher level of abstraction

- primitive task: can be performed directly by an operator instance

•planning process:

•**decompose non-primitive tasks recursively until primitive tasks are reached**

- HTN most widely used technique for real-world planning applications

- methods are a natural way to encode recipes (which should lead to solution plans only; reduces search significantly)

- methods reflect the way experts think about planning problems

Overview

- ◆ Simple Task Networks
 - HTN Planning
 - Extensions
 - State-Variable Representation

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Overview

◆ Simple Task Networks

◆ now: representation and planning algorithms for STNs

•HTN Planning

•Extensions

•State-Variable Representation

STN Planning

- STN: Simple Task Network
- what remains:
 - terms, literals, operators, actions, state transition function, plans
- what's new:
 - tasks to be performed
 - methods describing ways in which tasks can be performed
 - organized collections of tasks called task networks

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STN Planning

•STN: Simple Task Network

•STN: simplified version of the more general HTN case to be discussed later

•what remains:

•terms, literals, operators, actions, state transition function, plans

•what's new:

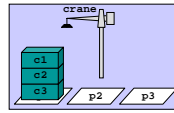
•tasks to be performed

•methods describing ways in which tasks can be performed

•organized collections of tasks called task networks

DWR Stack Moving Example

- task: move stack of containers from pallet p1 to pallet p3 in a way the preserves the order
- (informal) methods:
 - move via intermediate: move stack to intermediate pile (reversing order) and then to final destination (reversing order again)
 - move stack: repeatedly move the topmost container until the stack is empty
 - move topmost: take followed by put action



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DWR Stack Moving Example

•**task: move stack of containers from pallet p1 to pallet p3 in a way the preserves the order**

•**preserve order: each container should be on same container it is on originally**

•**(informal) methods:**

•**methods: possible subtasks and how they can be accomplished**

•**move via intermediate: move stack to intermediate pile (reversing order) and then to final destination (reversing order again)**

•**move stack: repeatedly move the topmost container until the stack is empty**

•**move topmost: take followed by put action**

•**action: no further decomposition required**

•**note: abstract concept: stack**

Tasks

- **task symbols:** $T_S = \{t_1, \dots, t_n\}$
 - operator names $\notin T_S$: primitive tasks
 - non-primitive task symbols: T_S - operator names
- **task:** $t_i(r_1, \dots, r_k)$
 - t_i : task symbol (primitive or non-primitive)
 - r_1, \dots, r_k : terms, objects manipulated by the task
 - ground task: are ground
- action a **accomplishes** ground primitive task $t_i(r_1, \dots, r_k)$ in state s iff
 - $\text{name}(a) = t_i$ and
 - a is applicable in s

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Tasks

• **task symbols:** $T_S = \{t_1, \dots, t_n\}$

- used for giving unique names to tasks
- **operator names $\notin T_S$: primitive tasks**
- **non-primitive task symbols: T_S - operator names**

• **task:** $t_i(r_1, \dots, r_k)$

- t_i : **task symbol (primitive or non-primitive)**
 - tasks: primitive iff task symbol is primitive
- r_1, \dots, r_k : **terms, objects manipulated by the task**
- **ground task: are ground**

• **action a accomplishes** ground primitive task $t_i(r_1, \dots, r_k)$ in state s iff

- action $a = (\text{name}(a), \text{precond}(a), \text{effects}(a))$
- **$\text{name}(a) = t_i$ and**
- **a is applicable in s**
 - applicability: s satisfies $\text{precond}(a)$

• note: unique operator names, hence primitive tasks can only be performed in one way – no search!

Simple Task Networks

- A simple task network w is an acyclic directed graph (U, E) in which
 - the node set $U = \{t_1, \dots, t_n\}$ is a set of tasks and
 - the edges in E define a partial ordering of the tasks in U .
- A task network w is ground/primitive if all tasks $t_u \in U$ are ground/primitive, otherwise it is unground/non-primitive.

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Simple Task Networks

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• simple task network: shortcut “task network”

Totally Ordered STNs

- ordering: $t_u < t_v$ in $w=(U,E)$ iff there is a path from t_u to t_v
- STN w is totally ordered iff E defines a total order on U
 - w is a sequence of tasks: $\langle t_1, \dots, t_n \rangle$
- Let $w = \langle t_1, \dots, t_n \rangle$ be a totally ordered, ground, primitive STN. Then the plan $\pi(w)$ is defined as:
 - $\pi(w) = \langle a_1, \dots, a_n \rangle$ where $a_i = t_i$; $1 \leq i \leq n$

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Totally Ordered STNs

- ordering: $t_u < t_v$ in $w=(U,E)$ iff there is a path from t_u to t_v
- STN w is totally ordered iff E defines a total order on U
 - w is a sequence of tasks: $\langle t_1, \dots, t_n \rangle$
 - sequence is special case of acyclic directed graph
 - t_1 : first task in U ; t_2 : second task in U ; ...; t_n : last task in U
- Let $w = \langle t_1, \dots, t_n \rangle$ be a totally ordered, ground, primitive STN. Then the plan $\pi(w)$ is defined as:
 - $\pi(w) = \langle a_1, \dots, a_n \rangle$ where $a_i = t_i$; $1 \leq i \leq n$

STNs: DWR Example

- tasks:
 - $t_1 = \text{take}(\text{crane}, \text{loc}, \text{c1}, \text{c2}, \text{p1})$: primitive, ground
 - $t_2 = \text{take}(\text{crane}, \text{loc}, \text{c2}, \text{c3}, \text{p1})$: primitive, ground
 - $t_3 = \text{move-stack}(\text{p1}, \text{q})$: non-primitive, unground
- task networks:
 - $w_1 = (\{t_1, t_2, t_3\}, \{(t_1, t_2), (t_1, t_3)\})$
 - partially ordered, non-primitive, unground
 - $w_2 = (\{t_1, t_2\}, \{(t_1, t_2)\})$
 - totally ordered: $w_2 = \langle t_1, t_2 \rangle$, ground, primitive
 - $\pi(w_2) = \langle \text{take}(\text{crane}, \text{loc}, \text{c1}, \text{c2}, \text{p1}), \text{take}(\text{crane}, \text{loc}, \text{c2}, \text{c3}, \text{p1}) \rangle$

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STNs: DWR Example

•tasks:

- $t_1 = \text{take}(\text{crane}, \text{loc}, \text{c1}, \text{c2}, \text{p1})$: primitive, ground
 - carne “crane” at location “loc” takes container “c1” of container “c2” in pile “p1”
- $t_2 = \text{take}(\text{crane}, \text{loc}, \text{c2}, \text{c3}, \text{p1})$: primitive, ground
- $t_3 = \text{move-stack}(\text{p1}, \text{q})$: non-primitive, unground
 - move the stack of containers on pallet “p2” to pallet “q” (variable)

•task networks:

- $w_1 = (\{t_1, t_2, t_3\}, \{(t_1, t_2), (t_1, t_3)\})$
 - partially ordered, non-primitive, unground
- $w_2 = (\{t_1, t_2\}, \{(t_1, t_2)\})$
 - totally ordered: $w_2 = \langle t_1, t_2 \rangle$, ground, primitive
 - $\pi(w_2) = \langle \text{take}(\text{crane}, \text{loc}, \text{c1}, \text{c2}, \text{p1}), \text{take}(\text{crane}, \text{loc}, \text{c2}, \text{c3}, \text{p1}) \rangle$

STN Methods

- Let M_S be a set of method symbols. An STN method is a 4-tuple $m=(name(m),task(m),precond(m),network(m))$ where:
 - $name(m)$:
 - the name of the method
 - syntactic expression of the form $n(x_1, \dots, x_k)$
 - $n \in M_S$: unique method symbol
 - x_1, \dots, x_k : all the variable symbols that occur in m ;
 - $task(m)$: a non-primitive task;
 - $precond(m)$: set of literals called the method's preconditions;
 - $network(m)$: task network (U, E) containing the set of subtasks U of m .

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STN Methods

• Let M_S be a set of method symbols. An STN method is a 4-tuple $m=(name(m),task(m),precond(m),network(m))$ where:

- method symbols: disjoint from other types of symbols
- STN method: also just called method
- **$name(m)$:**
 - **the name of the method**
 - unique name: no two methods can have the same name; gives an easy way to unambiguously refer to a method instances
 - **syntactic expression of the form $n(x_1, \dots, x_k)$**
 - **$n \in M_S$: unique method symbol**
 - **x_1, \dots, x_k : all the variable symbols that occur in m ;**
 - no “local” variables in method definition (may be relaxed in other formalisms)
- **$task(m)$: a non-primitive task;**
 - what task can be performed with this method
 - non-primitive: contains subtasks
- **$precond(m)$: set of literals called the method's preconditions;**
 - like operator preconditions: what must be true in state s for m to be applicable
 - no effects: not needed if problem is to refine/perform a task as opposed to achieving some effect

STN Methods: DWR Example (1)

- move topmost: take followed by put action
- **take-and-put($c, k, l, p_o, p_d, x_o, x_d$)**
 - task: **move-topmost(p_o, p_d)**
 - precondition: **top(c, p_o)**, **on(c, x_o)**, **attached(p_o, l)**, **belong(k, l)**, **attached(p_d, l)**, **top(x_d, p_d)**
 - subtasks: **⟨take(k, l, c, x_o, p_o), put(k, l, c, x_d, p_d)⟩**

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STN Methods: DWR Example (1)

•move topmost: take followed by put action

- simplest method from previous example

•take-and-put($c, k, l, p_o, p_d, x_o, x_d$)

- using crane k at location l , take container c from object x_o (container or pallet) in pile p_o and put it onto object x_d in pile p_d (o for origin, d for destination)

•task: move-topmost(p_o, p_d)

- move topmost container from pile p_o to pile p_d

•precond:

- top(c, p_o)**, **on(c, x_o)**: pile must be empty with container c on top

- attached(p_o, l)**, **belong(k, l)**, **attached(p_d, l)**: piles and crane must be at same location

- top(x_d, p_d)**: destination object must be top of its pile

•subtasks: **⟨take(k, l, c, x_o, p_o), put(k, l, c, x_d, p_d)⟩**

- simple macro operator combining two (primitive) operators (sequentially)

STN Methods: DWR Example (2)

- move stack: repeatedly move the topmost container until the stack is empty
- recursive-move(p_o, p_d, c, x_o)
 - task: move-stack(p_o, p_d)
 - precondition: top(c, p_o), on(c, x_o)
 - subtasks: \langle move-topmost(p_o, p_d), move-stack(p_o, p_d) \rangle
- no-move(p_o, p_d)
 - task: move-stack(p_o, p_d)
 - precondition: top(pallet, p_o)
 - subtasks: \langle

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STN Methods: DWR Example (2)

•**move stack: repeatedly move the topmost container until the stack is empty**

•**recursive-move(p_o, p_d, c, x_o)**

•move container c which must be on object x_o in pile p_o to the top of pile p_d

•**task: move-stack(p_o, p_d)**

•move the remainder of the stack from p_o to p_d : more abstract than method

•**precond: top(c, p_o), on(c, x_o)**

• p_o must be empty; c is the top container

•method is not applicable to empty piles!

•**subtasks: \langle move-topmost(p_o, p_d), move-stack(p_o, p_d) \rangle**

•recursive decomposition: move top container and then recursive invocation of method through task

•**no-move(p_o, p_d)**

•performs the task by doing nothing

•**task: move-stack(p_o, p_d)**

•as above

•**precond: top(pallet, p_o)**

•the pile must be empty (recursion ends here)

•**subtasks: \langle**

•do nothing does nothing

STN Methods: DWR Example (3)

- move via intermediate: move stack to intermediate pile (reversing order) and then to final destination (reversing order again)
- **move-stack-twice(p_o, p_i, p_d)**
 - task: **move-ordered-stack(p_o, p_d)**
 - precondition: -
 - subtasks: $\langle \text{move-stack}(p_o, p_i), \text{move-stack}(p_i, p_d) \rangle$

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STN Methods: DWR Example (3)

• **move via intermediate: move stack to intermediate pallet (reversing order) and then to final destination (reversing order again)**

• **move-stack-twice(p_o, p_i, p_d)**

• move the stack of containers in pile p_o first to intermediate pile p_i then to p_d , thus preserving the order

• **task: move-ordered-stack(p_o, p_d)**

• move the stack from p_o to p_d in an order-preserving way

• **precond: -**

• none; should mention that piles must be at same location and different

• **subtasks: $\langle \text{move-stack}(p_o, p_i), \text{move-stack}(p_i, p_d) \rangle$**

• the two stack moves

Applicability and Relevance

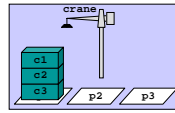
- A method instance m is applicable in a state s if
 - $\text{precond}^+(m) \subseteq s$ and
 - $\text{precond}^-(m) \cap s = \{ \}$.
- A method instance m is relevant for a task t if
 - there is a substitution σ such that $\sigma(t) = \text{task}(m)$.
- The decomposition of a task t by a relevant method m under σ is
 - $\delta(t, m, \sigma) = \sigma(\text{network}(m))$ or
 - $\delta(t, m, \sigma) = \sigma(\langle \text{subtasks}(m) \rangle)$ if m is totally ordered.

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Method Applicability and Relevance: DWR Example

- task $t = \text{move-stack}(p1, q)$
- state s (as shown)
- method instance $m_i = \text{recursive-move}(p1, p2, c1, c2)$
 - m_i is applicable in s
 - m_i is relevant for t under $\sigma = \{q \leftarrow p2\}$

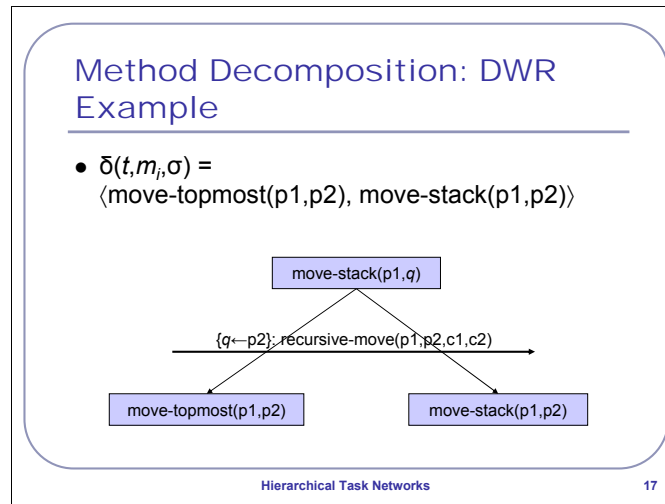


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Method Applicability and Relevance: DWR Example

- task $t = \text{move-stack}(p1, q)$
- state s (as shown)
- method instance $m_i = \text{recursive-move}(p1, p2, c1, c2)$
 - m_i is applicable in s
 - m_i is relevant for t under $\sigma = \{q \leftarrow p2\}$



Method Decomposition: DWR Example

• $\delta(t, m_i, \sigma) = \langle \text{move-topmost}(p1, p2), \text{move-stack}(p1, p2) \rangle$

• [figure]

• graphical representation (called a decomposition tree):

- view as AND/OR-graph: AND link – both subtasks need to be performed to perform super-task
- link is labelled with substitution and method instance used
- arrow under label indicates order in which subtasks need to be performed
- often leave out substitution (derivable) and sometimes method parameters (to save space)

Decomposition of Tasks in STNs

- Let
 - $w = (U, E)$ be a STN and
 - $t \in U$ be a task with no predecessors in w and
 - m a method that is relevant for t under some substitution σ with $\text{network}(m) = (U_m, E_m)$.
- The decomposition of t in w by m under σ is the STN $\delta(w, u, m, \sigma)$ where:
 - t is replaced in U by $\sigma(U_m)$ and
 - edges in E involving t are replaced by edges to appropriate nodes in $\sigma(U_m)$.

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Decomposition of Tasks in STNs

•idea: applying a method to a task in a network results in another network

•Let

• $w = (U, E)$ be a STN and

• $t \in U$ be a task with no predecessors in w and

• m a method that is relevant for t under some substitution σ with $\text{network}(m) = (U_m, E_m)$.

•The decomposition of t in w by m under σ is the STN $\delta(w, u, m, \sigma)$ where:

• t is replaced in U by $\sigma(U_m)$ and

•replacement with copy (method maybe used more than once)

•edges in E involving t are replaced by edges to appropriate nodes in $\sigma(U_m)$.

•every node in $\sigma(U_m)$ should come before nodes that came after t in E

• $\sigma(E_m)$ needs to be added to E to preserve internal method ordering

•ordering constraints must ensure that $\text{precond}(m)$ remains true even after subsequent decompositions

STN Planning Domains

- An STN planning domain is a pair $\mathcal{D}=(O,M)$ where:
 - O is a set of STRIPS planning operators and
 - M is a set of STN methods.
- \mathcal{D} is a total-order STN planning domain if every $m \in M$ is totally ordered.

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STN Planning Domains

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 - O is a set of STRIPS planning operators and
 - M is a set of STN methods.
- \mathcal{D} is a total-order STN planning domain if every $m \in M$ is totally ordered.

STN Planning Problems

- An STN planning problem is a 4-tuple $\mathcal{P}=(s_i, w_i, O, M)$ where:
 - s_i is the initial state (a set of ground atoms)
 - w_i is a task network called the initial task network and
 - $\mathcal{D}=(O, M)$ is an STN planning domain.
- \mathcal{P} is a total-order STN planning domain if w_i and \mathcal{D} are both totally ordered.

STN Planning Problems

- An STN planning problem is a 4-tuple $\mathcal{P}=(s_i, w_i, O, M)$ where:
 - s_i is the initial state (a set of ground atoms)
 - w_i is a task network called the initial task network and
 - $\mathcal{D}=(O, M)$ is an STN planning domain.
- \mathcal{P} is a total-order STN planning domain if w_i and \mathcal{D} are both totally ordered.

STN Solutions

- A plan $\pi = \langle a_1, \dots, a_n \rangle$ is a solution for an STN planning problem $\mathcal{P} = (s_i, w_i, O, M)$ if:
 - w_i is empty and π is empty;
 - or:
 - there is a primitive task $t \in w_i$ that has no predecessors in w_i and
 - $a_1 = t$ is applicable in s_i and
 - $\pi' = \langle a_2, \dots, a_n \rangle$ is a solution for $\mathcal{P}' = (\gamma(s_i, a_1), w_i - \{t\}, O, M)$
 - or:
 - there is a non-primitive task $t \in w_i$ that has no predecessors in w_i and
 - $m \in M$ is relevant for t , i.e. $\sigma(t) = \text{task}(m)$ and applicable in s_i and
 - π is a solution for $\mathcal{P}' = (s_i, \delta(w_i, t, m, \sigma), O, M)$.

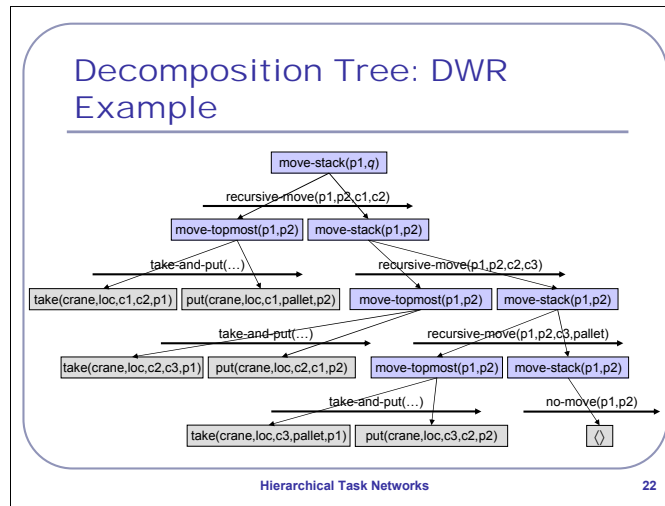
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STN Solutions

• A plan $\pi = \langle a_1, \dots, a_n \rangle$ is a solution for an STN planning problem $\mathcal{P} = (s_i, w_i, O, M)$ if:

- if π is a solution for \mathcal{P} , then we say that π accomplishes P
- intuition: there is a way to decompose w_i into π such that:
 - π is executable in s_i and
 - each decomposition is applicable in an appropriate state of the world
 - w_i is empty and π is empty;
- or:
 - there is a primitive task $t \in w_i$ that has no predecessors in w_i and
 - $a_1 = t$ is applicable in s_i and
 - $\pi' = \langle a_2, \dots, a_n \rangle$ is a solution for $\mathcal{P}' = (\gamma(s_i, a_1), w_i - \{t\}, O, M)$
- or:
 - there is a non-primitive task $t \in w_i$ that has no predecessors in w_i and
 - $m \in M$ is relevant for t , i.e. $\sigma(t) = \text{task}(m)$ and applicable in s_i and
 - π is a solution for $\mathcal{P}' = (s_i, \delta(w_i, t, m, \sigma), O, M)$.
- 2nd and 3rd case: recursive definition
 - if w_i is not totally ordered more than one node may have no predecessors and both cases may apply



Decomposition Tree: DWR Example

- choose method: recursive-move($p1,p2,c1,c2$) – binds variable q
- decompose into two sub-tasks
- choose method for first subtask: take-and-put: $c1$ from $c2$ onto pallet
- decompose into subtasks – primitive subtasks (grey) cannot be decomposed/correspond to actions
- choose method for second sub-task: recursive-move (recursive part)
- decompose (recursive)
- choose method and decompose (into primitive tasks): take-and-put: $c2$ from $c3$ onto $c1$
- choose method and decompose (recursive)
- choose method and decompose: take-and-put: $c3$ from pallet onto $c2$
- choose method (no-move) and decompose (empty plan)
- note:
 - (grey) leaf nodes of decomposition tree (primitive tasks) are actions of solution plan
 - (blue) inner nodes represent non-primitive task; decomposition results in sub-tree rooted at task according to decomposition function δ
 - no search required in this example

Ground-TFD: Pseudo Code

```
function Ground-TFD( $s, \langle t_1, \dots, t_k \rangle, O, M$ )
  if  $k=0$  return  $\diamond$ 
  if  $t_1$ .isPrimitive() then
    actions =  $\{(a, \sigma) \mid a = \sigma(t_1) \text{ and } a \text{ applicable in } s\}$ 
    if actions.isEmpty() then return failure
     $(a, \sigma) = \text{actions.chooseOne}()$ 
    plan  $\leftarrow$  Ground-TFD( $\gamma(s, a), \sigma(\langle t_2, \dots, t_k \rangle), O, M$ )
    if plan = failure then return failure
    else return  $\langle a \rangle \cdot \text{plan}$ 
  else
    methods =  $\{(m, \sigma) \mid m \text{ is relevant for } \sigma(t_1) \text{ and } m \text{ is applicable in } s\}$ 
    if methods.isEmpty() then return failure
     $(m, \sigma) = \text{methods.chooseOne}()$ 
    plan  $\leftarrow$  subtasks( $m$ )  $\cdot \sigma(\langle t_2, \dots, t_k \rangle)$ 
    return Ground-TFD( $s, \text{plan}, O, M$ )
```

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Ground-TFD: Pseudo Code

• TFD = Total-order Forward Decomposition; direct implementation of definition of STN solution

• **function Ground-TFD($s, \langle t_1, \dots, t_k \rangle, O, M$)**

• **if $k=0$ return \diamond**

• **if t_1 .isPrimitive() then**

• **$\text{actions} = \{(a, \sigma) \mid a = \sigma(t_1) \text{ and } a \text{ applicable in } s\}$**

• **if actions.isEmpty() then return failure**

• **$(a, \sigma) = \text{actions.chooseOne}()$**

• **plan \leftarrow Ground-TFD($\gamma(s, a), \sigma(\langle t_2, \dots, t_k \rangle), O, M$)**

• **if plan = failure then return failure**

• **else return $\langle a \rangle \cdot \text{plan}$**

• **else t_1 is non-primitive**

• **$\text{methods} = \{(m, \sigma) \mid m \text{ is relevant for } \sigma(t_1) \text{ and } m \text{ is applicable in } s\}$**

• **if methods.isEmpty() then return failure**

• **$(m, \sigma) = \text{methods.chooseOne}()$**

• **plan \leftarrow subtasks(m) $\cdot \sigma(\langle t_2, \dots, t_k \rangle)$**

• **return Ground-TFD(s, plan, O, M)**

TFD vs. Forward/Backward Search

- choosing actions:
 - TFD considers only applicable actions like forward search
 - TFD considers only relevant actions like backward search
- plan generation:
 - TFD generates actions execution order; current world state always known
- lifting:
 - Ground-TFD can be generalized to Lifted-TFD resulting in same advantages as lifted backward search

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TFD vs. Forward/Backward Search

•choosing actions:

- TFD considers only applicable actions like forward search

- TFD considers only relevant actions like backward search

- TFD combines advantages of both search directions – better efficiency

•plan generation:

- TFD generates actions execution order; current world state always known

- e.g. good for domain-specific heuristics

•lifting:

- Ground-TFD can be generalized to Lifted-TFD resulting in same advantages as lifted backward search

- avoids generating unnecessarily many actions (smaller branching factor)

- works for initial task list that is not ground

Ground-PFD: Pseudo Code

```
function Ground-PFD(s,w,O,M)
  if w.U={} return  $\langle \rangle$ 
  task  $\leftarrow \{t \in U \mid t \text{ has no predecessors in } w.E\}.chooseOne()$ 
  if task.isPrimitive() then
    actions =  $\{(a,\sigma) \mid a=\sigma(t_1) \text{ and } a \text{ applicable in } s\}$ 
    if actions.isEmpty() then return failure
    (a, $\sigma$ ) = actions.chooseOne()
    plan  $\leftarrow$  Ground-PFD( $\gamma(s,a),\sigma(w-\{task\}),O,M$ )
    if plan = failure then return failure
    else return (a) • plan
  else
    methods =  $\{(m,\sigma) \mid m \text{ is relevant for } \sigma(t_1) \text{ and } m \text{ is applicable in } s\}$ 
    if methods.isEmpty() then return failure
    (m, $\sigma$ ) = methods.chooseOne()
    return Ground-PFD(s,  $\delta(w,task,m,\sigma),O,M$ )
```

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Ground-PFD: Pseudo Code

- PFD = Partial-order Forward Decomposition; direct implementation of definition of STN solution
- function Ground-PFD(s,w,O,M)
- if w.U={} return $\langle \rangle$
- task $\leftarrow \{t \in U \mid t \text{ has no predecessors in } w.E\}.chooseOne()$
- if task.isPrimitive() then
- actions = $\{(a,\sigma) \mid a=\sigma(t_1) \text{ and } a \text{ applicable in } s\}$
- if actions.isEmpty() then return failure
- (a, σ) = actions.chooseOne()
- plan \leftarrow Ground-PFD($\gamma(s,a),\sigma(w-\{task\}),O,M$)
- if plan = failure then return failure
- else return (a) • plan
- else
- methods = $\{(m,\sigma) \mid m \text{ is relevant for } \sigma(t_1) \text{ and } m \text{ is applicable in } s\}$
- if methods.isEmpty() then return failure
- (m, σ) = methods.chooseOne()
- return Ground-PFD(s, $\delta(w,task,m,\sigma),O,M$)

Overview

- Simple Task Networks
- HTN Planning
- Extensions
- State-Variable Representation

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Overview

➤ Simple Task Networks

➤ just done: representation and planning algorithms for STNs

• HTN Planning

• now: generalizing the formalism and algorithm

• Extensions

• State-Variable Representation

Preconditions in STN Planning

- STN planning constraints:
 - ordering constraints: maintained in network
 - preconditions:
 - enforced by planning procedure
 - must know state to test for applicability
 - must perform forward search
- HTN Planning
 - additional bookkeeping maintains general constraints explicitly

Preconditions in STN Planning

- STN planning constraints:
 - ordering constraints: maintained in network
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 - must know state to test for applicability
 - must perform forward search
- HTN Planning
 - additional bookkeeping maintains general constraints explicitly

First and Last Network Nodes

- Let
 - $\pi = \langle a_1, \dots, a_n \rangle$ be a solution for w ,
 - $U' \subseteq U$ be a set of tasks in w , and
 - $A(U')$ the subset of actions in π such that each $a_i \in A(U')$ is a descendant of some $t \in U'$ in the decomposition tree.
- Then we define:
 - first(U', π) = the action $a_i \in A(U')$ that occurs first in π , and
 - last(U', π) = the action $a_i \in A(U')$ that occurs last in π .

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First and Last Network Nodes

•for defining the constraints in an HTN network

•Let

• $\pi = \langle a_1, \dots, a_n \rangle$ be a solution for w ,

•HTN solution will be defined later

• $U' \subseteq U$ be a set of tasks in w , and

• $A(U')$ the subset of actions in π such that each $a_i \in A(U')$ is a descendant of some $t \in U'$ in the decomposition tree.

•Then we define:

•first(U', π) = the action $a_i \in A(U')$ that occurs first in π ;
and

•last(U', π) = the action $a_i \in A(U')$ that occurs last in π .

•network is partially ordered; solution is totally ordered

•for a given set of subtasks, one action decomposing U' must occur first/last in the solution plan

Hierarchical Task Networks

- A (hierarchical) task network is a pair $w=(U,C)$, where:
 - U is a set of tasks and
 - C is a set of constraints of the following types:
 - $t_1 < t_2$: precedence constraint between tasks satisfied if in every solution π : $\text{last}(\{t_1\}, \pi) < \text{first}(\{t_2\}, \pi)$;
 - $\text{before}(U', l)$: satisfied if in every solution π : literal l holds in the state just before $\text{first}(U', \pi)$;
 - $\text{after}(U', l)$: satisfied if in every solution π : literal l holds in the state just after $\text{last}(U', \pi)$;
 - $\text{between}(U', U'', l)$: satisfied if in every solution π : literal l holds in every state after $\text{last}(U', \pi)$ and before $\text{first}(U'', \pi)$.

Hierarchical Task Networks

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Hierarchical Task Networks

• A (hierarchical) task network is a pair $w=(U,C)$, where:

• U is a set of tasks and

• C is a set of constraints of the following types:

• $t_1 < t_2$: precedence constraint between tasks satisfied if in every solution π : $\text{last}(\{t_1\}, \pi) < \text{first}(\{t_2\}, \pi)$;

• corresponds to edge in STN

• $\text{before}(U', l)$: satisfied if in every solution π : literal l holds in the state just before $\text{first}(U', \pi)$;

• $\text{after}(U', l)$: satisfied if in every solution π : literal l holds in the state just after $\text{last}(U', \pi)$;

• $\text{between}(U', U'', l)$: satisfied if in every solution π : literal l holds in every state after $\text{last}(U', \pi)$ and before $\text{first}(U'', \pi)$.

HTN Methods

- Let M_S be a set of method symbols. An HTN method is a 4-tuple $m=(name(m),task(m),subtasks(m),constr(m))$ where:
 - $name(m)$:
 - the name of the method
 - syntactic expression of the form $n(x_1,\dots,x_k)$
 - $n \in M_S$: unique method symbol
 - x_1,\dots,x_k : all the variable symbols that occur in m ;
 - $task(m)$: a non-primitive task;
 - $(subtasks(m),constr(m))$: a task network.

Hierarchical Task Networks

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HTN Methods

- extension of the definition of an STN method
- Let M_S be a set of method symbols. An HTN method is a 4-tuple $m=(name(m),task(m),subtasks(m),constr(m))$ where:
 - $name(m)$:
 - the name of the method
 - syntactic expression of the form $n(x_1,\dots,x_k)$
 - $n \in M_S$: unique method symbol
 - x_1,\dots,x_k : all the variable symbols that occur in m ;
 - $task(m)$: a non-primitive task;
 - $(subtasks(m),constr(m))$: a task network.

HTN Methods: DWR Example (1)

- move topmost: take followed by put action
- take-and-put($c, k, l, p_o, p_d, x_o, x_d$)
 - task: move-topmost(p_o, p_d)
 - network:
 - subtasks: $\{t_1 = \text{take}(k, l, c, x_o, p_o), t_2 = \text{put}(k, l, c, x_d, p_d)\}$
 - constraints: $\{t_1 < t_2, \text{before}(\{t_1\}, \text{top}(c, p_o)), \text{before}(\{t_1\}, \text{on}(c, x_o)), \text{before}(\{t_1\}, \text{attached}(p_o, l)), \text{before}(\{t_1\}, \text{belong}(k, l)), \text{before}(\{t_2\}, \text{attached}(p_d, l)), \text{before}(\{t_2\}, \text{top}(x_d, p_d))\}$

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HTN Methods: DWR Example (1)

- move topmost: take followed by put action
- take-and-put($c, k, l, p_o, p_d, x_o, x_d$)
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 - note: before-constraints refer to both tasks; more precise than STN representation of preconditions

HTN Methods: DWR Example (2)

- move stack: repeatedly move the topmost container until the stack is empty
- recursive-move(p_o, p_d, c, x_o)
 - task: move-stack(p_o, p_d)
 - network:
 - subtasks: $\{t_1 = \text{move-topmost}(p_o, p_d), t_2 = \text{move-stack}(p_o, p_d)\}$
 - constraints: $\{t_1 < t_2, \text{before}(\{t_1\}, \text{top}(c, p_o)), \text{before}(\{t_1\}, \text{on}(c, x_o))\}$
- move-one(p_o, p_d, c)
 - task: move-stack(p_o, p_d)
 - network:
 - subtasks: $\{t_1 = \text{move-topmost}(p_o, p_d)\}$
 - constraints: $\{\text{before}(\{t_1\}, \text{top}(c, p_o)), \text{before}(\{t_1\}, \text{on}(c, \text{pallet}))\}$

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HTN Methods: DWR Example (2)

•move stack: repeatedly move the topmost container until the stack is empty

•recursive-move(p_o, p_d, c, x_o)

•task: move-stack(p_o, p_d)

•network:

•subtasks: $\{t_1 = \text{move-topmost}(p_o, p_d), t_2 = \text{move-stack}(p_o, p_d)\}$

•constraints: $\{t_1 < t_2, \text{before}(\{t_1\}, \text{top}(c, p_o)), \text{before}(\{t_1\}, \text{on}(c, x_o))\}$

•move-one(p_o, p_d, c)

•task: move-stack(p_o, p_d)

•network:

•subtasks: $\{t_1 = \text{move-topmost}(p_o, p_d)\}$

•constraints: $\{\text{before}(\{t_1\}, \text{top}(c, p_o)), \text{before}(\{t_1\}, \text{on}(c, \text{pallet}))\}$

•note: problem with no-move: cannot add before-constraint when there are no tasks

•move-stack-twice(p_o, p_i, p_d) trivial; not shown again

HTN Decomposition

- Let $w=(U,C)$ be a task network, $t \in U$ a task, and m a method such that $\sigma(\text{task}(m))=t$. Then the decomposition of t in w using m under σ is defined as:

$$\delta(w,t,m,\sigma) = ((U-\{t\}) \cup \sigma(\text{subtasks}(m)), C' \cup \sigma(\text{constr}(m)))$$

where C' is modified from C as follows:

- for every precedence constraint in C that contains t , replace it with precedence constraints containing $\sigma(\text{subtasks}(m))$ instead of t ; and
- for every before-, after-, or between constraint over tasks U' containing t , replace U' with $(U'-\{t\}) \cup \sigma(\text{subtasks}(m))$.

HTN Decomposition

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$$\delta(w,t,m,\sigma) = ((U-\{t\}) \cup \sigma(\text{subtasks}(m)), C' \cup \sigma(\text{constr}(m)))$$

- new, additional constraints may introduce threats that need to be resolved

where C' is modified from C as follows:

• for every precedence constraint in C that contains t , replace it with precedence constraints containing $\sigma(\text{subtasks}(m))$ instead of t ; and

• example: let $\text{subtasks}(m)=\{t_1,t_2\}$ and $t \prec t' \in C$

• then replace $t \prec t'$ with $t_1 \prec t'$ and $t_2 \prec t'$

• cannot introduce inconsistencies (circles) since subtasks are new nodes

• for every before-, after-, or between constraint over tasks U' containing t , replace U' with $(U'-\{t\}) \cup \sigma(\text{subtasks}(m))$.

• example (other constraints): let $\text{subtasks}(m)=\{t_1,t_2\}$ and $\text{before}(\{t,t'\},l) \in C$

• then replace $\text{before}(\{t,t'\},l)$ with $\text{before}(\{t_1,t_2,t'\},l)$

• cannot introduce inconsistencies either

HTN Decomposition: Example

- network: $w = (\{t_1 = \text{move-stack}(p1, q)\}, \{\})$
- $\delta(w, t_1, \text{recursive-move}(p_o, p_d, c, x_o), \{p_o \leftarrow p1, p_d \leftarrow q\}) = w' =$
 - $(\{t_2 = \text{move-topmost}(p1, q), t_3 = \text{move-stack}(p1, q)\},$
 - $\{t_2 < t_3, \text{before}(\{t_2\}, \text{top}(c, p1)), \text{before}(\{t_2\}, \text{on}(c, x_o))\})$
- $\delta(w', t_2, \text{take-and-put}(c, k, l, p_o, p_d, x_o, x_d), \{p_o \leftarrow p1, p_d \leftarrow q\}) =$
 - $(\{t_3 = \text{move-stack}(p1, q), t_4 = \text{take}(k, l, c, x_o, p1), t_5 = \text{put}(k, l, c, x_d, q)\},$
 - $\{t_4 < t_3, t_5 < t_3, \text{before}(\{t_4, t_5\}, \text{top}(c, p1)), \text{before}(\{t_4, t_5\}, \text{on}(c, x_o))\} \cup$
 - $\{t_4 < t_5, \text{before}(\{t_4\}, \text{top}(c, p1)), \text{before}(\{t_4\}, \text{on}(c, x_o)), \text{before}(\{t_4\},$
 - $\text{attached}(p1, l)), \text{before}(\{t_4\}, \text{belong}(k, l)), \text{before}(\{t_5\},$
 - $\text{attached}(q, l)), \text{before}(\{t_5\}, \text{top}(x_d, q))\})$

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HTN Decomposition: Example

- **network: $w = (\{t_1 = \text{move-stack}(p1, q)\}, \{\})$**
 - initial, single task with no constraints
- **$\delta(w, t_1, \text{recursive-move}(p_o, p_d, c, x_o), \{p_o \leftarrow p1, p_d \leftarrow q\}) = w' =$**
 - **$(\{t_2 = \text{move-topmost}(p1, q), t_3 = \text{move-stack}(p1, q)\},$**
 - 2 instantiated subtasks from method
 - **$\{t_2 < t_3, \text{before}(\{t_2\}, \text{top}(c, p1)), \text{before}(\{t_2\}, \text{on}(c, x_o))\})$**
 - instantiated constraints from method
- **$\delta(w', t_2, \text{take-and-put}(c, k, l, p_o, p_d, x_o, x_d), \{p_o \leftarrow p1, p_d \leftarrow q\}) =$**
 - **$(\{t_3 = \text{move-stack}(p1, q), t_4 = \text{take}(k, l, c, x_o, p1),$**
 - **$t_5 = \text{put}(k, l, c, x_d, q)\},$**
 - t_3 : from input network w' ; t_4 and t_5 from method
 - **$\{t_4 < t_3, t_5 < t_3,$**
 - ordering did involve t_2 – replace with two constraints for new subtasks t_4 and t_5
 - **$\text{before}(\{t_4, t_5\}, \text{top}(c, p1)), \text{before}(\{t_4, t_5\}, \text{on}(c, x_o))\} \cup$**
 - replaced $\{t_2\}$ with $\{t_4, t_5\}$
 - **$\{t_4 < t_5, \text{before}(\{t_4\}, \text{top}(c, p1)), \text{before}(\{t_4\}, \text{on}(c, x_o)),$**
 - **$\text{before}(\{t_4\}, \text{attached}(p1, l)), \text{before}(\{t_4\}, \text{belong}(k, l)),$**
 - **$\text{before}(\{t_5\}, \text{attached}(q, l)), \text{before}(\{t_5\}, \text{top}(x_d, q))\})$**
 - instantiated constraints from new method

HTN Planning Domains and Problems

- An HTN planning domain is a pair $\mathcal{D}=(O,M)$ where:
 - O is a set of STRIPS planning operators and
 - M is a set of HTN methods.
- An HTN planning problem is a 4-tuple $\mathcal{P}=(s_i,w_i,O,M)$ where:
 - s_i is the initial state (a set of ground atoms)
 - w_i is a task network called the initial task network and
 - $\mathcal{D}=(O,M)$ is an HTN planning domain.

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HTN Planning Domains and Problems

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 - s_i is the initial state (a set of ground atoms)
 - w_i is a task network called the initial task network and
 - $\mathcal{D}=(O,M)$ is an HTN planning domain.

Solutions for Primitive HTNs

- Let (U,C) be a primitive HTN. A plan $\pi = \langle a_1, \dots, a_n \rangle$ is a solution for $\mathcal{P} = (s_i, (U,C), O, M)$ if there is a ground instance $(\sigma(U), \sigma(C))$ of (U,C) and a total ordering $\langle t_1, \dots, t_n \rangle$ of tasks in $\sigma(U)$ such that:
 - for $i=1 \dots n$: $\text{name}(a_i) = t_i$;
 - π is executable in s_i , i.e. $\gamma(s_i, \pi)$ is defined;
 - the ordering of $\langle t_1, \dots, t_n \rangle$ respects the ordering constraints in $\sigma(C)$;
 - for every constraint before (U', l) in $\sigma(C)$ where $t_k = \text{first}(U', \pi)$: l must hold in $\gamma(s_i, \langle a_1, \dots, a_{k-1} \rangle)$;
 - for every constraint after (U', l) in $\sigma(C)$ where $t_k = \text{last}(U', \pi)$: l must hold in $\gamma(s_i, \langle a_1, \dots, a_k \rangle)$;
 - for every constraint between (U', U'', l) in $\sigma(C)$ where $t_k = \text{first}(U', \pi)$ and $t_m = \text{last}(U'', \pi)$: l must hold in every state $\gamma(s_i, \langle a_1, \dots, a_j \rangle)$, $j \in \{k \dots m-1\}$.

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Solutions for Primitive HTNs

• Let (U,C) be a primitive HTN. A plan $\pi = \langle a_1, \dots, a_n \rangle$ is a solution for $\mathcal{P} = (s_i, (U,C), O, M)$ if there is a ground instance $(\sigma(U), \sigma(C))$ of (U,C) and a total ordering $\langle t_1, \dots, t_n \rangle$ of tasks in $\sigma(U)$ such that:

- for $i=1 \dots n$: $\text{name}(a_i) = t_i$;
- π is executable in s_i , i.e. $\gamma(s_i, \pi)$ is defined;
- the ordering of $\langle t_1, \dots, t_n \rangle$ respects the ordering constraints in $\sigma(C)$;
- for every constraint before (U', l) in $\sigma(C)$ where $t_k = \text{first}(U', \pi)$: l must hold in $\gamma(s_i, \langle a_1, \dots, a_{k-1} \rangle)$;
- for every constraint after (U', l) in $\sigma(C)$ where $t_k = \text{last}(U', \pi)$: l must hold in $\gamma(s_i, \langle a_1, \dots, a_k \rangle)$;
- for every constraint between (U', U'', l) in $\sigma(C)$ where $t_k = \text{first}(U', \pi)$ and $t_m = \text{last}(U'', \pi)$: l must hold in every state $\gamma(s_i, \langle a_1, \dots, a_j \rangle)$, $j \in \{k \dots m-1\}$.

Solutions for Non-Primitive HTNs

- Let $w = (U, C)$ be a non-primitive HTN. A plan $\pi = \langle a_1, \dots, a_n \rangle$ is a solution for $\mathcal{P} = (s_i, w, O, M)$ if there is a sequence of task decompositions that can be applied to w such that:
 - the result of the decompositions is a primitive HTN w' ; and
 - π is a solution for $\mathcal{P}' = (s_i, w', O, M)$.

Solutions for Non-Primitive HTNs

- Let $w = (U, C)$ be a non-primitive HTN. A plan $\pi = \langle a_1, \dots, a_n \rangle$ is a solution for $\mathcal{P} = (s_i, w, O, M)$ if there is a sequence of task decompositions that can be applied to w such that:
 - the result of the decompositions is a primitive HTN w' ;
 - and
 - π is a solution for $\mathcal{P}' = (s_i, w', O, M)$.

Abstract-HTN: Pseudo Code

```
function Abstract-HTN(s,U,C,O,M)  
  if (U,C).isInconsistent() then return failure  
  if U.isPrimitive() then  
    return extractSolution(s,U,C,O)  
  else  
    return decomposeTask(s,U,C,O,M)
```

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Abstract-HTN: Pseudo Code

- general schema for a function that implements HTN planning
- **function Abstract-HTN(*s,U,C,O,M*)**
- **if (*U,C*).isInconsistent() then return failure**
 - e.g. test for inconsistency of *C*, or apply other, domain-specific tests
- **if *U*.isPrimitive() then**
 - no further decompositions of tasks possible
- **return extractSolution(*s,U,C,O*)**
 - compute a total-order, grounded plan; may fail
- **else**
 - network still contains decomposable tasks
- **return decomposeTask(*s,U,C,O,M*)**
 - will recursively call Abstract-HTN function

extractSolution: Pseudo Code

```
function extractSolution(s,U,C,O)
   $\langle t_1, \dots, t_n \rangle \leftarrow U.\text{chooseSequence}(C)$ 
   $\langle a_1, \dots, a_n \rangle \leftarrow$ 
     $\langle t_1, \dots, t_n \rangle.\text{chooseGrounding}(s, C, O)$ 
  if  $\langle a_1, \dots, a_n \rangle.\text{satisfies}(C)$  then
    return  $\langle a_1, \dots, a_n \rangle$ 
  return failure
```

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extractSolution: Pseudo Code

• **function** extractSolution(*s,U,C,O*)

• $\langle t_1, \dots, t_n \rangle \leftarrow U.\text{chooseSequence}(C)$

• non-deterministically choose a serialization of the tasks in *U* that respects the ordering constraints in *C*

• $\langle a_1, \dots, a_n \rangle \leftarrow \langle t_1, \dots, t_n \rangle.\text{chooseGrounding}(s, C, O)$

• non-deterministically choose a grounding of the variables in t_1, \dots, t_n
• use *s* and *C* to ensure constraints hold, and *O* for type information if present

• **if** $\langle a_1, \dots, a_n \rangle.\text{satisfies}(C)$ **then**

• this test can be performed during the grounding

• **return** $\langle a_1, \dots, a_n \rangle$

• plan is a solution, return it

• **return** failure

decomposeTask: Pseudo Code

```
function decomposeTask(s,U,C,O,M)
  t ← U.nonPrimitives().selectOne()
  methods ← {(m,σ) | m∈M and σ(task(m))= σ(t)}
  if methods.isEmpty() then return failure
  (m,σ) ← methods.chooseOne()
  (U',C') ← δ((U,C),t,m,σ)
  (U',C') ← (U',C').applyCritic()
  return Abstract-HTN(s,U',C',O,M)
```

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decomposeTask: Pseudo Code

- **function decomposeTask(s,U,C,O,M)**
- **t ← U.nonPrimitives().selectOne()**
 - deterministically select a non-primitive task-node from the network
 - no backtracking required, all tasks must be decomposed eventually; selection important for efficiency
- **methods ← {(m,σ) | m∈M and σ(task(m))= σ(t)}**
 - substitution should be mgu for least commitment planner (generates smaller search space)
- **if methods.isEmpty() then return failure**
- **(m,σ) ← methods.chooseOne()**
 - non-deterministically choose a method that can be applied to decompose the task
- **(U',C') ← δ((U,C),t,m,σ)**
 - compute the decomposition
- **(U',C') ← (U',C').applyCritic()**
 - optional; may make arbitrary modifications, e.g. application-specific computations
 - soundness and completeness depends on this function
- **return Abstract-HTN(s,U',C',O,M)**

HTN vs. STRIPS Planning

- Since
 - HTN is generalization of STN Planning, and
 - STN problems can encode undecidable problems, but
 - STRIPS cannot encode such problems:
- **STN/HTN formalism is more expressive**
- non-recursive STN can be translated into equivalent STRIPS problem
 - but exponentially larger in worst case
- “regular” STN is equivalent to STRIPS

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HTN vs. STRIPS Planning

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 - **STN problems can encode undecidable problems, but**
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- **STN/HTN formalism is more expressive**
- **non-recursive STN can be translated into equivalent STRIPS problem**
 - **but exponentially larger in worst case**
- **“regular” STN is equivalent to STRIPS**
 - non-recursive
 - at most one non-primitive subtask per method
 - non-primitive sub-task must be last in sequence

Overview

- Simple Task Networks
- HTN Planning
- **Extensions**
- State-Variable Representation

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Overview

➡ Simple Task Networks

• HTN Planning

- just done: generalizing the formalism and algorithm

• Extensions

- now: approaches to extending the formalism and algorithm

• State-Variable Representation

Functions in Terms

- allow function terms in world state and method constraints
- ground versions of all planning algorithms may fail
 - potentially infinite number of ground instances of a given term
- lifted algorithms can be applied with most general unifier
 - least commitment approach instantiates only as far as necessary
 - plan-existence may not be decidable

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- ground versions of all planning algorithms may fail
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- lifted algorithms can be applied with most general unifier
 - least commitment approach instantiates only as far as necessary
 - plan-existence may not be decidable

Axiomatic Inference

- use theorem prover to infer derived knowledge within world states
 - undecidability of first-order logic in general
- idea: use restricted (decidable) subset of first-order logic: Horn clauses
 - only positive preconditions can be derived
 - precondition p is satisfied in state s iff p can be proved in s

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Axiomatic Inference

•use theorem prover to infer derived knowledge within world states

•undecidability of first-order logic in general

•idea: use restricted (decidable) subset of first-order logic: Horn clauses

•only positive preconditions can be derived

•precondition p is satisfied in state s iff p can be proved in s

•semantics of negative preconditions: closed world assumption?

Attached Procedures

- associate predicates with procedures
- modify planning algorithm
 - evaluate preconditions by
 - calling the procedure attached to the predicate symbol if there is such a procedure
 - test against world state (set-relation, theorem prover) otherwise
- soundness and completeness: depends on procedures

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Attached Procedures

- associate predicates with procedures
- modify planning algorithm
 - evaluate preconditions by
 - calling the procedure attached to the predicate symbol if there is such a procedure
 - test against world state (set-relation, theorem prover) otherwise
- applications:
 - perform numeric computations
 - query external data sources
- soundness and completeness: depends on procedures
- attached procedures to function symbols: critics

High-Level Effects

- allow user to declare effects for non-primitive methods
- aim:
 - establish preconditions
 - prune partial plans if high-level effects threaten preconditions
- increases efficiency
- problem: semantics

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High-Level Effects

- allow user to declare effects for non-primitive methods
- aim:
 - establish preconditions
 - prune partial plans if high-level effects threaten preconditions
- increases efficiency
- problem: semantics
 - can be defined in different ways

Other Extensions

- other constraints
 - time constraints
 - resource constraints
- extended goals
 - states to be avoided
 - required intermediate states
 - limited plan length
 - visit states multiple times

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Other Extensions

- other constraints
 - time constraints
 - resource constraints
- extended goals
 - states to be avoided
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Overview

- Simple Task Networks
- HTN Planning
- Extensions
- **State-Variable Representation**

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Overview

➔ Simple Task Networks

• HTN Planning

• Extensions

- just done: approaches to extending the formalism and algorithm

• State-Variable Representation

- now: different style of representation (used in O-Plan/I-Plan)

State Variables

- some relations are functions
 - example: $at(r1,loc1)$: relates robot $r1$ to location $loc1$ in some state
 - truth value changes from state to state
 - will only be true for exactly one location l in each state
- idea: represent such relations using state-variable functions mapping states into objects
 - example: functional representation:
 $rloc: robots \times S \rightarrow locations$

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State Variables

•some relations are functions

•**example: $at(r1,loc1)$: relates robot $r1$ to location $loc1$ in some state**

•**truth value changes from state to state**

•**will only be true for exactly one location l in each state**

•STRIPS state containing $at(r1,loc1)$ and $at(r1,loc2)$ usually inconsistent

•**idea: represent such relations using state-variable functions mapping states into objects**

•**advantage: reduces possibilities for inconsistent states, smaller state space**

•**example: functional representation:
 $rloc: robots \times S \rightarrow locations$**

•in general: maps objects and state into object

• $rloc$ is state-variable symbol that denotes state-variable function

States in the State-Variable Representation

- Let X be a set of state-variable functions. A k -ary state variable is an expression of the form $x(v_1, \dots, v_k)$ where:
 - $x \in X$ is a state-variable function and
 - v_i is either an object constant or an object variable.
- A state-variable state description is a set of expressions of the form $x_s = c$ where:
 - x_s is a ground state variable $x(v_1, \dots, v_k)$ and
 - c is an object constant.

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States in the State-Variable Representation

• **Let X be a set of state-variable functions. A k -ary state variable is an expression of the form $x(v_1, \dots, v_k)$ where:**

- **$x \in X$ is a state-variable function and**
- **v_i is either an object constant or an object variable.**
 - object variables as opposed to state variables
 - ground if all v_i are object constants
 - additionally: v_i may be typed
- state variable is a characteristic attribute of a state

• **A state-variable state description is a set of expressions of the form $x_s = c$ where:**

- **x_s is a ground state variable $x(v_1, \dots, v_k)$ and**
- **c is an object constant.**
- as for ground atoms in STRIPS states, state is implicit
- state description will usually give all values of ground state variables
- values of state variables are not independent

DWR Example: State-Variable State Descriptions

- simplified: no cranes, no piles
- state-variable functions:
 - rloc: robots \times S \rightarrow locations
 - rolad: robots \times S \rightarrow containers \cup {nil}
 - cpos: containers \times S \rightarrow locations \cup robots
- sample state-variable state descriptions:
 - {rloc(r1)=loc1, rload(r1)=nil, cpos(c1)=loc1, cpos(c2)=loc2, cpos(c3)=loc2}
 - {rloc(r1)=loc1, rload(r1)=c1, cpos(c1)=r1, cpos(c2)=loc2, cpos(c3)=loc2}

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DWR Example: State-Variable State Descriptions

•simplified: no cranes, no piles

- robots can load and unload containers autonomously

•state-variable functions:

•rloc: robots \times S \rightarrow locations

- location of a robot in a state

•rolad: robots \times S \rightarrow containers \cup {nil}

- what a robot has loaded in a state; nil for nothing loaded

•cpos: containers \times S \rightarrow locations \cup robots

- where a container is in a state; at a location or on some robot

•sample state-variable state descriptions:

•{rloc(r1)=loc1, rload(r1)=nil, cpos(c1)=loc1, cpos(c2)=loc2, cpos(c3)=loc2}

•{rloc(r1)=loc1, rload(r1)=c1, cpos(c1)=r1, cpos(c2)=loc2, cpos(c3)=loc2}

Operators in the State-Variable Representation

- A state-variable planning operator is a triple (name(o), precondition(o), effects(o)) where:
 - name(o) is a syntactic expression of the form $n(x_1, \dots, x_k)$ where n is a (unique) symbol and x_1, \dots, x_k are all the object variables that appear in o ,
 - precondition(o) are the unions of a state-variable state description and some rigid relations, and
 - effects(o) are sets of expressions of the form $x_s \leftarrow v_{k+1}$ where:
 - x_s is a ground state variable $x(v_1, \dots, v_k)$ and
 - v_{k+1} is an object constant or an object variable.

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Operators in the State-Variable Representation

• A state-variable planning operator is a triple (name(o), precondition(o), effects(o)) where:

• name(o) is a syntactic expression of the form $n(x_1, \dots, x_k)$ where n is a (unique) symbol and x_1, \dots, x_k are all the object variables that appear in o ,

• looks like name of a STRIPS planning operator

• precondition(o) are the unions of a state-variable state description and some rigid relations, and

• set of state variable equals value expressions and some rigid relations (as in STRIPS operators)

• values of state variables refer to state before the operator is applied

• effects(o) are sets of expressions of the form $x_s \leftarrow v_{k+1}$ where:

• x_s is a ground state variable $x(v_1, \dots, v_k)$ and

• v_{k+1} is an object constant or an object variable.

• similar to state but assignment operator instead of equals sign

• updates in effects refer to state after operator is applied

• as for STRIPS operators, actions are ground instances of operators

DWR Example: State-Variable Operators

- $\text{move}(r,l,m)$
 - precondition: $\text{rloc}(r)=l, \text{adjacent}(l,m)$
 - effects: $\text{rloc}(r)\leftarrow m$
- $\text{load}(r,c,l)$
 - precondition: $\text{rloc}(r)=l, \text{cpos}(c)=l, \text{rload}(r)=\text{nil}$
 - effects: $\text{cpos}(c)\leftarrow r, \text{rload}(r)\leftarrow c$
- $\text{unload}(r,c,l)$
 - precondition: $\text{rloc}(r)=l, \text{rload}(r)=c$
 - effects: $\text{rload}(r)\leftarrow \text{nil}, \text{cpos}(c)\leftarrow l$

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DWR Example: Operators

•simplified domain: no piles, no cranes – only three operators:

• $\text{move}(r,l,m)$

•move robot r from location l to adjacent location m

•**precond: $\text{rloc}(r)=l, \text{adjacent}(l,m)$**

•adjacent: rigid relation

•**effects: $\text{rloc}(r)\leftarrow m$**

• $\text{load}(r,c,l)$

•robot r loads container c at location l

•**precond: $\text{rloc}(r)=l, \text{cpos}(c)=l, \text{rload}(r)=\text{nil}$**

•**effects: $\text{cpos}(c)\leftarrow r, \text{rload}(r)\leftarrow c$**

• $\text{unload}(r,c,l)$

•robot r unloads container c at location l

•**precond: $\text{rloc}(r)=l, \text{rload}(r)=c$**

•**effects: $\text{rload}(r)\leftarrow \text{nil}, \text{cpos}(c)\leftarrow l$**

Applicability and State Transitions

- Let a be an action and s a state. Then a is applicable in s iff:
 - all rigid relations mentioned in $\text{precond}(a)$ hold, and
 - if $x_s=c \in \text{precond}(a)$ then $x_s=c \in s$.
- The state transition function γ for an action a in state s is defined as $\gamma(s,a) = \{x_s=c \mid x \in X\}$ where:
 - $x_s \leftarrow c \in \text{effects}(a)$ or
 - $x_s=c \in s$ otherwise.

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Applicability and State Transitions

- Let a be an action and s a state. Then a is applicable in s iff:
 - all rigid relations mentioned in $\text{precond}(a)$ hold, and
 - as in STRIPS representation
 - if $x_s=c \in \text{precond}(a)$ then $x_s=c \in s$.
 - if values of state variables in preconditions agree with same values in state
- The state transition function γ for an action a in state s is defined as $\gamma(s,a) = \{x_s=c \mid x \in X\}$ where:
 - $x_s \leftarrow c \in \text{effects}(a)$ or
 - update the values of state variables in the effects
 - $x_s=c \in s$ otherwise.
 - keep other values from previous state

State-Variable Planning Domains

- Let X be a set of state-variable functions. A state-variable planning domain on X is a restricted state-transition system $\Sigma=(S,A,\gamma)$ such that:
 - S is a set of state-variable state descriptions,
 - A is a set of ground instances of some state-variable planning operators O ,
 - $\gamma:S\times A\rightarrow S$ where
 - $\gamma(s,a)=\{x_s=c \mid x\in X \text{ and } x_s\leftarrow c \in \text{effects}(a) \text{ or } x_s=c \in s \text{ otherwise}\}$ if a is applicable in s
 - $\gamma(s,a)=\text{undefined}$ otherwise,
 - S is closed under γ

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State-Variable Planning Domains

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 - $\gamma(s,a)=\text{undefined}$ otherwise,
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State-Variable Planning Problems

- A state-variable planning problem is a triple $\mathcal{P}=(\Sigma, s_i, g)$ where:
 - $\Sigma=(S, A, \gamma)$ is a state-variable planning domain on some set of state-variable functions X
 - $s_i \in S$ is the initial state
 - g is a set of expressions of the form $x_s=c$ describing the goal such that the set of goal states is: $S_g=\{s \in S \mid x_s=c \in s\}$

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State-Variable Planning Problems

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• g is a set of expressions of the form $x_s=c$ describing the goal such that the set of goal states is: $S_g=\{s \in S \mid x_s=c \in s\}$

• a goal is a specification of the values of some ground state variables

• goals are like preconditions without rigid relations

• definitions for plan, reachable states, and solutions as for propositional case

Relevance and Regression Sets

- Let $\mathcal{P}=(\Sigma, s_i, g)$ be a state-variable planning problem. An action $a \in A$ is relevant for g if
 - $g \cap \text{effects}(a) \neq \{\}$ and
 - for every $x_s=c \in g$, there is no $x_s \leftarrow d \in \text{effects}(a)$ such that $c \neq d$.
- The regression set of g for a relevant action $a \in A$ is:
 - $\gamma^{-1}(g, a) = (g - \vartheta(a)) \cup \text{precond}(a)$ where
 - $\vartheta(a) = \{x_s=c \mid x_s \leftarrow c \in \text{effects}(a)\}$
- definition for all regression sets $\Gamma^<(g)$ exactly as for propositional case

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Relevance and Regression Sets

• Let $\mathcal{P}=(\Sigma, s_i, g)$ be a state-variable planning problem. An action $a \in A$ is relevant for g if

• $g \cap \text{effects}(a) \neq \{\}$ and

• a has an effect that contributes to g

• for every $x_s=c \in g$, there is no $x_s \leftarrow d \in \text{effects}(a)$ such that $c \neq d$.

• effects of a do not change any of the state variables in g

• The regression set of g for a relevant action $a \in A$ is:

• $\gamma^{-1}(g, a) = (g - \vartheta(a)) \cup \text{precond}(a)$ where

• $\vartheta(a) = \{x_s=c \mid x_s \leftarrow c \in \text{effects}(a)\}$

• necessary to change syntax: replace left arrow with equals sign

• otherwise definition is as before

• definition for all regression sets $\Gamma^<(g)$ exactly as for propositional case

Statement of a State-Variable Planning Problem

- A statement of a state-variable planning problem is a triple $P=(O,s_i,g)$ where:
 - O is a set of planning operators in an appropriate state-variable planning domain $\Sigma=(S,A,\gamma)$ on X
 - s_i is the initial state in an appropriate state-variable planning problem $\mathcal{P}=(\Sigma,s_i,g)$
 - g is a goal in the same state-variable planning problem \mathcal{P}

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- g is a goal in the same state-variable planning problem \mathcal{P}

Translation: STRIPS to State-Variable Representation

- Let $P=(O,s_i,g)$ be a statement of a classical planning problem. In the operators O , in the initial state s_i , and in the goal g :
 - replace every positive literal $p(t_1,\dots,t_n)$ with a state-variable expression $p(t_1,\dots,t_n)=1$ or $p(t_1,\dots,t_n)\leftarrow 1$ in the operators' effects, and
 - replace every negative literal $\neg p(t_1,\dots,t_n)$ with a state-variable expression $p(t_1,\dots,t_n)=0$ or $p(t_1,\dots,t_n)\leftarrow 0$ in the operators' effects.

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Translation: STRIPS to State-Variable Representation

• Let $P=(O,s_i,g)$ be a statement of a classical planning problem. In the operators O , in the initial state s_i , and in the goal g :

• replace every positive literal $p(t_1,\dots,t_n)$ with a state-variable expression $p(t_1,\dots,t_n)=1$ or $p(t_1,\dots,t_n)\leftarrow 1$ in the operators' effects, and

• replace every negative literal $\neg p(t_1,\dots,t_n)$ with a state-variable expression $p(t_1,\dots,t_n)=0$ or $p(t_1,\dots,t_n)\leftarrow 0$ in the operators' effects.

• result is a statement of a state-variable planning problem

Translation: State-Variable to STRIPS Representation

- Let $P=(O,s_i,g)$ be a statement of a state-variable planning problem. In the operators' preconditions, in the initial state s_i , and in the goal g :
 - replace every state-variable expression $p(t_1,\dots,t_n)=v$ with an atom $p(t_1,\dots,t_n,v)$, and
- in the operators' effects:
 - replace every state-variable assignment $p(t_1,\dots,t_n)\leftarrow v$ with a pair of literals $p(t_1,\dots,t_n,v)$, $\neg p(t_1,\dots,t_n,w)$, and add $p(t_1,\dots,t_n,w)$ to the respective operators preconditions.

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Translation: State-Variable to STRIPS Representation

•Let $P=(O,s_i,g)$ be a statement of a state-variable planning problem. In the operators' preconditions, in the initial state s_i , and in the goal g :

•replace every state-variable expression $p(t_1,\dots,t_n)=v$ with an atom $p(t_1,\dots,t_n,v)$, and

•in the operators' effects:

•replace every state-variable assignment $p(t_1,\dots,t_n)\leftarrow v$ with a pair of literals $p(t_1,\dots,t_n,v)$, $\neg p(t_1,\dots,t_n,w)$, and add $p(t_1,\dots,t_n,w)$ to the respective operators preconditions.

•result is a statement of a STRIPS planning problem

Overview

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Overview

➔ Simple Task Networks

• HTN Planning

• Extensions

• State-Variable Representation

- just done: different style of representation (used in O-Plan/I-Plan)