Social and Technological Networks Edinburgh, 2018

Notes 4. Growth, expansion and doubling dimension of metrics.

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Q 1. *Consider the unweighted grid graph with the intrinsic metric. Is this metric space the same as the two dimensional plane $\mathbb{R}^2$ with the L_1 metric?*

Q 2. *What do balls in the plane $\mathbb{R}^2$ look like, when considered in L_1, L_2 and L_∞ metrics? [Hint: Take a grid, and draw l_1 and L_∞ balls.]*

Q 3. *Check that in a balanced binary tree, a ball of radius r cannot be covered by a constant number of balls of radius $r/2$.*

Q 4. *What are the Growth, Doubling dimension and Expansion of following types of graphs:*

- *The $1-D$ chain.*
- *The $2-D$ grid.*
- *The balanced binary tree*

Q 5. *Suppose we have a set of wireless nodes arranged in a finite grid with sides $1/2$ and we consider the unit disk graph. Now suppose that we go an adding nodes at random and updating the UDG, how do the growth and doubling dimension change? In particular, suppose we have added nodes such that any disk of radius $1/2$ has $\Theta(k)$ nodes. What can you say about the growth and doubling dimension?*

Q 6. *A graph is said to have bounded growth (sometimes called polynomial bounded growth), if the growth of metric balls is bounded by $O(r^d)$ for some constant d. Prove that a graph with bounded growth has bounded doubling dimension. [Also prove that bounded doubling dimension does not imply bounded growth.]*

Q 7. *Suppose we generalize the definition of doubling dimension to c-multiplicative dimension. That is, a metric space has bounded c-multiplicative dimension if any ball of radius r can be covered by a constant number of balls of radius r/c. Show that bounded doubling dimension implies bounded c-multiplicative dimension, and vice versa.*