Some basic network definitions and Random Graphs

Social and Technological Networks

Rik Sarkar

2015

Some properties of Networks

• Diameter

- How far are nodes from each other ?
- How small is small?
- Expansion
 - Look at friends, their friends, their friends...
 - How fast does that set grow?
 - How fast can something spread?
- Clustering coefficient
 - How clustered is the network?
 - Are people just individually friends, or do they have groups/cliques?

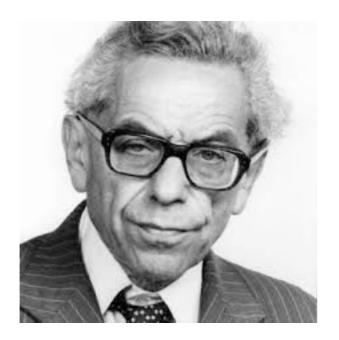
Properties of networks

- We want to define them in a measurable way
- And show what they imply

Random graphs

- The most basic, most unstructured graphs
- Forms a baseline to see what we can expect in absence of any force (social, technological...)
- Many real network have a random component
 - Many things occur without plan or reason

Erdos-Renyi Random Graphs





Erdos-Renyi Random Graphs $\mathcal{G}(n, p)$

- n: Number of vertices
- p: Probability that any particular edge exists

Expectations

- Expected total number of edges: $\binom{n}{2}p$
- Expected number of edges at any vertex: (n-1)p

Isolated vertices

- For what values of p
 - Random graph likely to have isolated vertices

Isolated vertices

- What happens to the number of isolated vertices
 - As p increases ?

Isolated vertices

• Likely when p

$$0 < \frac{\ln n}{n}$$

• Unlikely when $p > \frac{\ln n}{n}$

Useful inequalities

$$\left(1+\frac{1}{x}\right)^x \le e$$
$$\left(1-\frac{1}{x}\right)^x \le \frac{1}{e}$$

Isolated vertices

- Show threshold behavior
- Tipping point
- Percolation threshold
- •

Configuration model of random graphs

- Suppose we have to construct a random graph
 - With given degrees for vertices

 $d_1, d_2, d_3, \ldots d_n$

- At each vertex *i*, we create *di* out-edges
 - Then we pair up the out edges randomly
 - produces a random graph with given degrees
- If all degree = d
 - graph is d-regular

Expansion

- The smallest surface to volume ratio
 - Compare the number of edges sticking out to number of vertices

Expanders

- Every graph has some value of alpha
- A class of graphs can be called expanders if they all have
 - small degrees
 - α above some constant

Examples of non-expanders

Examples of non-expanders

- Tree
- Complete graph
- Grid

Random graphs

- A random d-regular graph is an expander
 - with

$$\alpha = f(d)$$

- An Erdos renyi graph is an expander
- Both, with high probability (w.h.p)

Expanders have short diameters

 $O(\frac{d}{\alpha} \lg n)$