Some basic network definitions and Random Graphs

Social and Technological Networks

Rik Sarkar

2015

Some properties of Networks

- Diameter
  - How far are nodes from each other ?
  - How small is small?
- Expansion
  - Look at friends, their friends, their friends…
  - How fast does that set grow?
  - How fast can something spread?
- Clustering coefficient
  - How clustered is the network?
  - Are people just individually friends, or do they have groups/cliques?

Properties of networks

- We want to define them in a measurable way
- And show what they imply

Random graphs

- The most basic, most unstructured graphs
- Forms a baseline to see what we can expect in absence of any force (social, technological…)
- Many real network have a random component
    - Many things occur without plan or reason

Erdos-Renyi Random Graphs

Erdos-Renyi Random Graphs

$$\mathcal{G}(n, p)$$

- n: Number of vertices
- p: Probability that any particular edge exists

Expectations

- Expected total number of edges: $\binom{n}{2}p$
- Expected number of edges at any vertex: $(n-1)p$

Isolated vertices

- For what values of p
  - Random graph likely to have isolated vertices

Isolated vertices

- What happens to the number of isolated vertices
    - As p increases ?

Isolated vertices

- Likely when $p < \frac{\ln n}{n}$
- Unlikely when $p > \frac{\ln n}{n}$

Useful inequalities

$$\left(1+\frac{1}{x}\right)^x \leq e$$

$$\left(1-\frac{1}{x}\right)^x \leq \frac{1}{e}$$

Isolated vertices

- Show threshold behavior
- *Tipping point*
- *Percolation threshold*
- *…*

Configuration model of random graphs

- Suppose we have to construct a random graph
  - With given degrees for vertices

$$d_1, d_2, d_3, \ldots d_n$$

- At each vertex *i*, we create *di* out-edges
  - Then we pair up the out edges randomly
  - produces a random graph with given degrees
- If all degree = d
  - graph is d-regular

Expansion

- The smallest surface to volume ratio
  - Compare the number of edges sticking out to number of vertices

Expanders

- Every graph has some value of alpha
- A class of graphs can be called expanders if they all have
  - small degrees
  - α above some constant

Examples of non-expanders

Examples of non-expanders

- Tree
- Complete graph
- Grid

Random graphs

- A random d-regular graph is an expander
    - with

$$\alpha = f(d)$$

- An Erdos renyi graph is an expander
- Both, with high probability (w.h.p)

Expanders have short diameters

$$O(\frac{d}{\alpha} \lg n)$$