Random Graphs continued

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Recap

- Erdos-Renyi model of Random graphs
  - Edges exist with Pr = p
- Configuration model of random graphs
  - Degrees given for vertices, edges are random connections
- ER show Threshold phenomenon at p = (ln n)/(n-1)
  - tipping point, phase transition, percolation threshold

Recap

- Expansion α
  - In any set S smaller than half V
    - There are at least α |S| out-going edges

Recap

- Polynomials n^β are large: they grow fast with n
  - Even when β is a small constant
- Logs are small: they grow slowly
  - E.g. in n node network, the id of each node is at least lg n bits in size

Expanders have short diameters

$$O(\frac{d}{\alpha} \lg n)$$

- Where d is the degree of vertices (we assume fixed constant degree) of all vertices

Clustering

- In social networks, people with mutual friends are often friends
- Two people A, C and a mutual neighbor B form a triad
  - Open triad: Only edges AB and BC exist
  - Closed triad: All edges AB, BC, CA exist

Clustering coefficient

- Probability of having closed triads
- CC of a node A
    - Fraction of pairs of A's neighbors that are neighbors
- Average CC : average of CC of all nodes
- Global CC is ratio : #closed triads / #all triads

CC in Erdos Renyi graphs

- Is small compared to n
- Grows slower than n

CC in ER graphs

Absence of short cycles

- ER graphs are unlikely to have short cycles

Absence of short cycles

- Implies local neighborhoods $B_r(v)$
  - have a tree structure

Community structure in random graphs

- Random graphs do not have “communities”
  - Since neighbors of a node are usually not neighbors of each-other

Course materials

- Usually, I will upload materials for a class as follows:
- Lecture X
    - Slides
    - lecture notes (includes exercises)
        - Solutions will be uploaded few days later
- Reading (chapter of references), parts of papers etc [exam material]
- ipython notebook code from class.
- Additional reading lists
    - relevant papers etc
    - Not for direct exam, but helpful to your general understanding
- Other materials, links to relevant stuff etc.

Study suggestions

- Main references:
    - David Easley, Jon Kleinberg - Networks, Crowds and Markets
    - David Kempe - Structure and dynamics of information in networks
- Please study the lecture notes.
- Do the exercises. Check your answer with solutions.
    - Keep up with the class. Otherwise it will get harder to understand later things
- Compare your writing of the answer with the solution
    - technical writing needs to be precise and correct
- In some cases, I may ask you to read something beforehand.