Social and Technological Networks	Lecture 1. Introduction
Basics and sample problems	
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Lecture Summary

1. *Networks is a graph*
   The underlying structure of many systems
2. *Basic Introduction*
   See Slides.
3. *In this document*
   Some basic definitions below.
4. *Exercises*

A network is a graph.

Definition 1 *A network or a graph is denoted $G = (V, E)$. Where V is a set of nodes and E is a set of edges. An edge $e = (a, b)$ is a pair, where $a, b \in V$.*

Note that (a, b) denotes an ordered pair, that is, a pair where the order of elements matter. This is definitely true for some types of edges such as links which go *from* page a *to* page b. In other cases, such as friendship, the direction may not be relevant, although the notation remains the same. When the direction is relevant, we call it a *directed* graph.

A *weighted* network comes associated with function $w : E \to \mathbb{R}$, assigning a weight to each edge. Unweighted network usually implies all weights equal 1.

Definition 2 *A path P is a sequence of distinct vertices $s = v_0, v_1, v_2, \ldots v_k = t$, where each pair $(v_i, v_{i+1}) \in E$ is an edge. Vertices s and t at start and end of the path are usually called the source and the destination.*

Usually, the word *path* implies that all vertices are distinct. If this is not the case, that should be mentioned explicitly. Sometimes such paths with non-distinct vertices are called a *walk*.

The number k is the *length* of the path in an unweighted graph. In weighted graphs, the *length* is usually taken as sum of the edges: $\sum_{(a,b)\in P} w((a, b))$.

The *graph distance or hop distance* between two nodes (another name for vertices) is usually measured as the length of the shortest path connecting them. Let us represent the hop distance between a and b as $d_G(a, b)$. In cases, where the graph is understood, we can drop the subscript G.

The graph in Fig. 1 is called a grid graph, shown embedded in the plane. If we assume that each edge has length 1, we call it a unit-grid graph. For vertices p_1 and p_2, we can define the distance in two ways:

- The usual distance in the plane, usually called the Euclidean distance or L_2 distance: $|p_1 - p_2|_2 = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

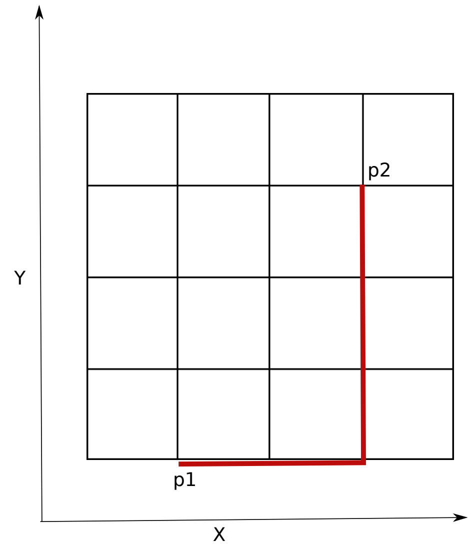

Figure 1: Grid graph

- The graph distance, which in this case is $|x_1 - x_2| + |y_1 - y_2|$ (shown by the red path in the Figure.)

The formula $|p_1 - p_2|_1 = |x_1 - x_2| + |y_1 - y_2|$ is called the L_1 distance. Note that in general graph distance does not equal the L_1 distance. However, in case of the unit grid, they are equal.

The L_1 distance is sometimes also called the Manhattan distance, because the streets in manhattan are laid out like a grid, and a path between two points is rectilinear, like the red curve in the Fig 1. In fact, any shortest path between the two points will have same length in the grid. Try them out!

Definition 3 (Ball of radius r.) *A ball of radius r centered at vertex v, written as $B_r(v)$, is the set of vertices u such that $d(v, u) \le r$.*

The number of vertices in this set is written as $|B_r(v)|$.

Exercise 1 *How many edges can a graph have? (assuming there is at most one edge between any two vertices.) If each possible edge exists with a probability p, what should be the value of p such that the expected number of edges at each vertex is 1?*

Exercise 2 *Suppose every year Mr. X makes double the number of friends he made last year (starting with making 1 friend in first year). In how many years will he make n friends? (asymptotic notation is fine.)*

Exercise 3 *Suppose we throw k balls into n bins randomly, what is the probability that bin 1 remains empty?*

Exercise 4 *Show that for a unit grid in a plane (as above), $|v_1 - v_2|_1 = \Theta(|v_1 - v_2|_2)$, for any v_1, v_2 in the graph.*

Exercise 5 *Show that for a unit grid, $|B_r(v)| = O(r^2)$.*