
Event Calculus Reasoning Through Satisfiability

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Abstract

We present an implemented method for encoding reasoning problems of a discrete version of the classical logic event calculus in propositional conjunctive normal form, enabling the problems to be solved efficiently by off-the-shelf complete satisfiability (SAT) solvers. We build on the previous encoding method of Shanahan and Witkowski, extending it to support causal constraints, concurrent events, determining fluents, effect axioms with conditions, events triggered by conditions, gradual change, incompletely specified initial situations, state constraints, and release from the commonsense law of inertia. We present an alternative classical logic axiomatization of the event calculus and prove its equivalence to a standard axiomatization for integer time. We describe our encoding method based on the alternative axiomatization and prove its correctness. We evaluate the method on 14 benchmark reasoning problems for the event calculus and compare performance with the causal calculator on eight problems in the zoo world domain.

Keywords: Commonsense reasoning, reasoning about action and change, event calculus, satisfiability, automated deduction.

1 Introduction

The classical logic event calculus [37, 39, 26], which is based on the original event calculus of Kowalski and Sergot [18], is a well-developed formalism for reasoning about action and change. It has been used to represent such things as beliefs and car crashes [20], egg cracking [27, 43], robot mail delivery [42], and robot sensors [38]. It is able to cope with representational problems such as the representation of conditional effects of events, triggered events, events with nondeterministic effects, events with indirect effects, gradual change, and the commonsense law of inertia.

To date, most reasoning with the classical logic event calculus has been carried out using one of two methods: (1) manual theorem proving by humans [27, 43] or (2) automated theorem proving through logic programming [42]. Citing research demonstrating the efficiency of planning using satisfiability (SAT) [16, 17], Shanahan and Witkowski [44] proposed that event calculus planning be carried out using satisfiability and presented a method for encoding event calculus planning problems as satisfiability problems. They demonstrated the efficiency of satisfiability over abductive logic programming for solving event calculus planning problems. However, the method of Shanahan and Witkowski applies only to a highly restricted subset of the event calculus.

The goals of this paper are:

1. to describe and prove the correctness of a method for encoding event calculus reasoning problems as satisfiability problems for a larger subset of the event calculus,
2. to describe a method that enables event calculus reasoning problems to be solved efficiently,
3. to present an alternative classical logic axiomatization of the event calculus useful for satisfiability encoding and prove its equivalence to a standard axiomatization for integer time, and
4. to evaluate the method on benchmark reasoning problems.

We build on Shanahan and Witkowski's method, extending it to support causal constraints, concurrent events, determining fluents, effect axioms with conditions, events triggered by conditions, gradual change, incompletely specified initial situations, state constraints, and release from the commonsense law of inertia.

The efficiency of our method hinges on the use of two techniques: (1) reformulation of the classical logic axiomatization of the event calculus in order to eliminate triply quantified time from most axioms, and (2) elimination from the reasoning problem of a large number of ground atoms stemming from effect axioms and gradual change axioms.

This paper is organized as follows. In Section 2 we present a standard classical logic axiomatization of the event calculus that serves as a point of reference. In Section 3 we present an alternative axiomatization and prove its equivalence to the axiomatization of Section 2 for integer time. In Section 4 we present and prove the correctness of our satisfiability encoding method for model finding, deduction, and abduction. In Section 5 we evaluate the method. Finally we present conclusions.

2 The event calculus

The classical logic event calculus [39] is based on many-sorted predicate calculus with equality. There are sorts for events, fluents, timepoints, and domain objects. A classical logic axiomatization of the event calculus has been described in a paper by Miller and Shanahan [26]. In that paper, several alternative axiomatizations are provided that subtract or add various features of the event calculus. In this section, we fix one set of axioms to serve as a point of reference. We combine axioms from the following sections of the paper:

- Section 2, which provides the basic classical logic axiomatization of the event calculus,
- Section 3.2, which revises the axioms of Section 2 for a version of the event calculus in which initiating and terminating a fluent at the same time produces inconsistency,
- Section 3.5, which adds axioms to those of Section 2 to support gradual change, and
- Section 3.7, which revises the axioms of Section 2 for a version of the event calculus in which fluents may be released from the commonsense law of inertia.

2.1 Event calculus predicates

The basic predicates of the event calculus axiomatization are as follows:

1. *Happens*(a, t): Event a occurs at timepoint t .
2. *HoldsAt*(f, t): Fluent f is true at timepoint t .
3. *ReleasedAt*(f, t): Fluent f is released from the commonsense law of inertia at timepoint t .
4. *Initiates*(a, f, t): If event a occurs at timepoint t , then fluent f becomes true after t and is no longer released from the commonsense law of inertia after t .
5. *Terminates*(a, f, t): If event a occurs at timepoint t , then fluent f becomes false after t and is no longer released from the commonsense law of inertia after t .
6. *Releases*(a, f, t): If event a occurs at timepoint t , then fluent f becomes released from the commonsense law of inertia after t .
7. *Trajectory*(f_1, t_1, f_2, t_2): If fluent f_1 is initiated by an event that occurs at timepoint t_1 and $t_2 > 0$, then fluent f_2 is true at timepoint $t_1 + t_2$.

8. *AntiTrajectory*(f_1, t_1, f_2, t_2): If fluent f_1 is terminated by an event that occurs at timepoint t_1 and $t_2 > 0$, then fluent f_2 is true at timepoint $t_1 + t_2$.

2.2 Event calculus (EC) axiomatization

The axiomatization of the event calculus is as follows. The following definitional axioms are from Miller and Shanahan Section 2 [26]:

AXIOM EC1

$$\text{Clipped}(t_1, f, t_2) \stackrel{\text{def}}{=} \exists a, t [\text{Happens}(a, t) \wedge t_1 \leq t < t_2 \wedge \text{Terminates}(a, f, t)].$$

AXIOM EC2

$$\text{Declipped}(t_1, f, t_2) \stackrel{\text{def}}{=} \exists a, t [\text{Happens}(a, t) \wedge t_1 \leq t < t_2 \wedge \text{Initiates}(a, f, t)].$$

A definitional axiom $\Gamma_1 \stackrel{\text{def}}{=} \Gamma_2$ indicates that Γ_1 is a notational shorthand for Γ_2 . That is, all occurrences of the compact notation Γ_1 are to be replaced by the more complex formula Γ_2 . In this paper, free occurrences of variables in formulas are assumed to be universally quantified.

The following definitional axioms are from Miller and Shanahan Section 3.2 [26]:

AXIOM EC9B

$$\text{StoppedIn}(t_1, f, t_2) \stackrel{\text{def}}{=} \exists a, t [\text{Happens}(a, t) \wedge t_1 < t < t_2 \wedge \text{Terminates}(a, f, t)].$$

AXIOM EC10B

$$\text{StartedIn}(t_1, f, t_2) \stackrel{\text{def}}{=} \exists a, t [\text{Happens}(a, t) \wedge t_1 < t < t_2 \wedge \text{Initiates}(a, f, t)].$$

The following axioms for gradual change are obtained from EC11F and EC12F of Miller and Shanahan Section 3.5 [26] by removing $\neg \text{Frame}(f_2)$, which is not needed:

AXIOM EC11F'

$$[\text{Happens}(a, t_1) \wedge \text{Initiates}(a, f_1, t_1) \wedge 0 < t_2 \wedge \text{Trajectory}(f_1, t_1, f_2, t_2) \wedge \neg \text{Clipped}(t_1, f_1, t_1 + t_2)] \Rightarrow \text{HoldsAt}(f_2, t_1 + t_2).$$

AXIOM EC12F'

$$[\text{Happens}(a, t_1) \wedge \text{Terminates}(a, f_1, t_1) \wedge 0 < t_2 \wedge \text{AntiTrajectory}(f_1, t_1, f_2, t_2) \wedge \neg \text{Declipped}(t_1, f_1, t_1 + t_2)] \Rightarrow \text{HoldsAt}(f_2, t_1 + t_2).$$

The following definitional axiom is obtained from EC17H of Miller and Shanahan Section 3.7 [26] by changing $t_1 \leq t \leq t_2$ to $t_1 < t \leq t_2$ in order to conform to the version of the event calculus described in Miller and Shanahan Section 3.2 [26]:

AXIOM EC17H'

$$\text{PersistsBetween}(t_1, f, t_2) \stackrel{\text{def}}{=} \neg \exists t [\text{ReleasedAt}(f, t) \wedge t_1 < t \leq t_2].$$

The following definitional axiom is from Miller and Shanahan Section 3.7 [26]:

AXIOM EC14H

$$\text{ReleasedBetween}(t_1, f, t_2) \stackrel{\text{def}}{=} \exists a, t [\text{Happens}(a, t) \wedge t_1 \leq t < t_2 \wedge \text{Releases}(a, f, t)].$$

The following axioms, which deal with the frame problem [24, 9], are from Miller and Shanahan Section 3.7 [26]:

AXIOM EC5H

$$[\neg \text{HoldsAt}(f, t_1) \wedge t_1 < t_2 \wedge \text{PersistsBetween}(t_1, f, t_2) \wedge \neg \text{Clipped}(t_1, f, t_2)] \Rightarrow \text{HoldsAt}(f, t_2).$$

AXIOM EC6H

$$[\neg \text{HoldsAt}(f, t_1) \wedge t_1 < t_2 \wedge \text{PersistsBetween}(t_1, f, t_2) \wedge \neg \text{Declipped}(t_1, f, t_2)] \Rightarrow \neg \text{HoldsAt}(f, t_2).$$

AXIOM EC18H

$$[\text{ReleasedAt}(f, t_1) \wedge t_1 < t_2 \wedge \neg \text{Clipped}(t_1, f, t_2) \wedge \neg \text{Declipped}(t_1, f, t_2)] \Rightarrow \text{ReleasedAt}(f, t_2).$$

AXIOM EC19H

$$[\neg \text{ReleasedAt}(f, t_1) \wedge t_1 < t_2 \wedge \neg \text{ReleasedBetween}(t_1, f, t_2)] \Rightarrow \neg \text{ReleasedAt}(f, t_2).$$

Axioms EC5H and EC6H are frame axioms for *HoldsAt*, and EC18H and EC19H are frame axioms for *ReleasedAt*.

The following definitional axiom is obtained from EC14H of Miller and Shanahan Section 3.7 [26] by changing *ReleasedBetween* to *ReleasedIn*, which will be used below, and $t_1 \leq t < t_2$ to $t_1 < t < t_2$ in order to conform to the version of the event calculus described in Miller and Shanahan Section 3.2 [26]:

AXIOM EC14H'

$$\text{ReleasedIn}(t_1, f, t_2) \stackrel{\text{def}}{=} \exists a, t [\text{Happens}(a, t) \wedge t_1 < t < t_2 \wedge \text{Releases}(a, f, t)].$$

A fluent can be in one of four states at a given timepoint: true and released, true and not released, false and released, or false and not released. The remaining axioms describe how the occurrence of an event affects the states of fluents. They are obtained from EC3H, EC4H, EC15H, and EC16H of Miller and Shanahan Section 3.7 [26] by changing *Clipped* to *StoppedIn*, *Declipped* to *StartedIn*, and *ReleasedBetween* to *ReleasedIn*, in order to conform to the version of the event calculus described in Miller and Shanahan Section 3.2 [26]:

AXIOM EC3H'

$$[\text{Happens}(a, t_1) \wedge \text{Initiates}(a, f, t_1) \wedge t_1 < t_2 \wedge \neg \text{StoppedIn}(t_1, f, t_2) \wedge \neg \text{ReleasedIn}(t_1, f, t_2)] \Rightarrow \text{HoldsAt}(f, t_2).$$

AXIOM EC4H'

$$[\text{Happens}(a, t_1) \wedge \text{Terminates}(a, f, t_1) \wedge t_1 < t_2 \wedge \neg \text{StartedIn}(t_1, f, t_2) \wedge \neg \text{ReleasedIn}(t_1, f, t_2)] \Rightarrow$$

$\neg \text{HoldsAt}(f, t_2).$

AXIOM EC15H'

$[\text{Happens}(a, t_1) \wedge \text{Releases}(a, f, t_1) \wedge t_1 < t_2 \wedge \neg \text{StoppedIn}(t_1, f, t_2) \wedge \neg \text{StartedIn}(t_1, f, t_2)] \Rightarrow \text{ReleasedAt}(f, t_2).$

AXIOM EC16H'

$[\text{Happens}(a, t_1) \wedge [\text{Initiates}(a, f, t_1) \vee \text{Terminates}(a, f, t_1)] \wedge t_1 < t_2 \wedge \neg \text{ReleasedIn}(t_1, f, t_2)] \Rightarrow \neg \text{ReleasedAt}(f, t_2).$

2.3 Domain descriptions

Let EC be the conjunction of EC11F', EC12F', EC5H, EC6H, EC18H, EC19H, EC3H', EC4H', EC15H', and EC16H'. Define $\text{CIRC}[\Phi; \rho_1, \dots, \rho_n]$ as the circumscription [23, 21] of the predicate symbols $\rho_1, \dots, \rho_n$ in the formula Φ .

DEFINITION 2.1

An event calculus *domain description* is given by

$$\text{CIRC}[\Sigma; \text{Initiates}, \text{Terminates}, \text{Releases}] \wedge \text{CIRC}[\Delta; \text{Happens}] \wedge \Omega \wedge \Psi \wedge \Pi \wedge \Gamma \wedge \text{EC}$$

where

- Σ is a conjunction of *Initiates*, *Terminates*, and *Releases* formulas,
- Δ is a conjunction of *Happens* and temporal ordering formulas,
- Ω is the uniqueness-of-names axioms for all the event symbols conjoined with the uniqueness-of-names axioms for all the fluent symbols,
- Ψ is a conjunction of state constraints,
- Π is a conjunction of *Trajectory* and *AntiTrajectory* formulas, and
- Γ is a conjunction of *HoldsAt* and *ReleasedAt* formulas.

EXAMPLE 2.2 (Domain description)

First we axiomatize some simple knowledge about falling objects. We use a state constraint that says that an object has a unique height:

$$\text{HoldsAt}(\text{Height}(o, h_1), t) \wedge \text{HoldsAt}(\text{Height}(o, h_2), t) \Rightarrow h_1 = h_2. \quad (2.1)$$

We add an effect axiom that states that if an object starts falling, then it will be falling:

$$\text{Initiates}(\text{StartFalling}(o), \text{Falling}(o), t). \quad (2.2)$$

We add a second effect axiom that states that if an object starts falling, then its height will be released from the commonsense law of inertia:

$$\text{Releases}(\text{StartFalling}(o), \text{Height}(o, h), t). \quad (2.3)$$

Next we use a gradual change axiom that states that if an object starts falling at time t_1 when its height is h_1 , then its height at time t_2 will be $\text{Max}(0, h_1 - t_2^2)$:

$$\begin{aligned} \text{HoldsAt}(\text{Height}(o, h_1), t_1) \wedge h_2 = \text{Max}(0, h_1 - t_2^2) \Rightarrow \\ \text{Trajectory}(\text{Falling}(o), t_1, \text{Height}(o, h_2), t_2). \end{aligned} \quad (2.4)$$

Next we use a trigger axiom that states that if an object is falling and its height is zero, then it will hit the ground:

$$\begin{aligned} \text{HoldsAt}(\text{Falling}(o), t) \wedge \text{HoldsAt}(\text{Height}(o, 0), t) \Rightarrow \\ \text{Happens}(\text{HitsGround}(o), t). \end{aligned} \quad (2.5)$$

We add another effect axiom that states that if the height of an object is h and the object hits the ground, then its height will no longer be released from the commonsense law of inertia and its height will be h :

$$\begin{aligned} \text{HoldsAt}(\text{Height}(o, h), t) \Rightarrow \\ \text{Initiates}(\text{HitsGround}(o), \text{Height}(o, h), t). \end{aligned} \quad (2.6)$$

Finally we add an effect axiom that states that if an object hits the ground, then the object will no longer be falling:

$$\text{Terminates}(\text{HitsGround}(o), \text{Falling}(o), t). \quad (2.7)$$

Now we add formulas describing an initial situation and an event occurrence:

$$\neg \text{HoldsAt}(\text{Falling}(\text{Leaf}), 0) \quad (2.8)$$

$$\neg \text{ReleasedAt}(\text{Falling}(\text{Leaf}), 0) \quad (2.9)$$

$$\text{HoldsAt}(\text{Height}(\text{Leaf}, 9), 0) \quad (2.10)$$

$$\neg \text{ReleasedAt}(\text{Height}(\text{Leaf}, h), 0) \quad (2.11)$$

$$\text{Happens}(\text{StartFalling}(\text{Leaf}), 0). \quad (2.12)$$

We may now reason using this domain description as follows. Let Σ be the conjunction of (2.2), (2.3), (2.6), and (2.7). Let Δ be the conjunction of (2.5) and (2.12). Let Ω be the uniqueness-of-names axioms for event and fluent symbols. Let Ψ be (2.1). Let Π be (2.4). Let Γ be the conjunction of (2.8), (2.9), (2.10), and (2.11). We can then show the following:

$$\begin{aligned} & \text{CIRC}[\Sigma; \text{Initiates}, \text{Terminates}, \text{Releases}] \wedge \\ & \text{CIRC}[\Delta; \text{Happens}] \wedge \Omega \wedge \Psi \wedge \Pi \wedge \Gamma \wedge \text{EC} \models \\ & \quad \text{HoldsAt}(\text{Falling}(\text{Leaf}), 1) \wedge \\ & \quad \text{HoldsAt}(\text{Height}(\text{Leaf}, 8), 1) \wedge \\ & \quad \text{HoldsAt}(\text{Height}(\text{Leaf}, 5), 2) \wedge \\ & \quad \text{HoldsAt}(\text{Height}(\text{Leaf}, 0), 3) \wedge \\ & \quad \text{Happens}(\text{HitsGround}(\text{Leaf}), 3) \wedge \\ & \quad \neg \text{HoldsAt}(\text{Falling}(\text{Leaf}), 4) \wedge \\ & \quad \text{HoldsAt}(\text{Height}(\text{Leaf}, 0), 4). \end{aligned}$$

3 The discrete event calculus

We desire an efficient satisfiability encoding for the event calculus. Observe that the standard event calculus axiomatization of the previous section involves triple quantification over timepoints. This is not ideal since it leads to an encoding size proportional to the cube of the number of timepoints.

In this section, we show that if one restricts the timepoint sort to the integers, then it becomes possible to eliminate triply quantified timepoints from each of the event calculus axioms and replace them with only singly quantified timepoints except for the two axioms that deal with gradual change. We present an alternative classical logic axiomatization of the event calculus, which we call the *discrete event calculus*, and prove that it is logically equivalent to the standard axiomatization if the timepoint sort is restricted to the integers.

It is worth pointing out that this equivalence does not require restriction to a finite universe, as in our propositional encoding of Section 4. It merely requires restriction of timepoints to the (infinite) set of integers.

3.1 Discrete event calculus (DEC) axiomatization

The axioms of the discrete event calculus are as follows. We start with two definitional axioms identical to EC1 and EC2:

AXIOM DEC1

$$\text{Clipped}(t_1, f, t_2) \stackrel{\text{def}}{=} \exists a, t [\text{Happens}(a, t) \wedge t_1 \leq t < t_2 \wedge \text{Terminates}(a, f, t)].$$

AXIOM DEC2

$$\text{Declipped}(t_1, f, t_2) \stackrel{\text{def}}{=} \exists a, t [\text{Happens}(a, t) \wedge t_1 \leq t < t_2 \wedge \text{Initiates}(a, f, t)].$$

We then have axioms for gradual change identical to EC11F' and EC12F':

AXIOM DEC3

$$[\text{Happens}(a, t_1) \wedge \text{Initiates}(a, f_1, t_1) \wedge 0 < t_2 \wedge \text{Trajectory}(f_1, t_1, f_2, t_2) \wedge \neg \text{Clipped}(t_1, f_1, t_1 + t_2)] \Rightarrow \text{HoldsAt}(f_2, t_1 + t_2).$$

AXIOM DEC4

$$[\text{Happens}(a, t_1) \wedge \text{Terminates}(a, f_1, t_1) \wedge 0 < t_2 \wedge \text{AntiTrajectory}(f_1, t_1, f_2, t_2) \wedge \neg \text{Declipped}(t_1, f_1, t_1 + t_2)] \Rightarrow \text{HoldsAt}(f_2, t_1 + t_2).$$

We have new frame axioms that resemble explanation closure frame axioms [12, 35, 3, 33, 34], extended to allow fluents to be released from the commonsense law of inertia:

AXIOM DEC5

$$[\text{HoldsAt}(f, t) \wedge \neg \text{ReleasedAt}(f, t + 1) \wedge \neg \exists a [\text{Happens}(a, t) \wedge \text{Terminates}(a, f, t)]] \Rightarrow \text{HoldsAt}(f, t + 1).$$

710 Event Calculus Reasoning Through Satisfiability

AXIOM DEC6

$$[\neg \text{HoldsAt}(f, t) \wedge \neg \text{ReleasedAt}(f, t + 1) \wedge \neg \exists a [\text{Happens}(a, t) \wedge \text{Initiates}(a, f, t)]] \Rightarrow \neg \text{HoldsAt}(f, t + 1).$$

AXIOM DEC7

$$[\text{ReleasedAt}(f, t) \wedge \neg \exists a [\text{Happens}(a, t) \wedge [\text{Initiates}(a, f, t) \vee \text{Terminates}(a, f, t)]]] \Rightarrow \text{ReleasedAt}(f, t + 1).$$

AXIOM DEC8

$$[\neg \text{ReleasedAt}(f, t) \wedge \neg \exists a [\text{Happens}(a, t) \wedge \text{Releases}(a, f, t)]] \Rightarrow \neg \text{ReleasedAt}(f, t + 1).$$

We have axioms that describe how the occurrence of an event affects the states of fluents:

AXIOM DEC9

$$[\text{Happens}(a, t) \wedge \text{Initiates}(a, f, t)] \Rightarrow \text{HoldsAt}(f, t + 1).$$

AXIOM DEC10

$$[\text{Happens}(a, t) \wedge \text{Terminates}(a, f, t)] \Rightarrow \neg \text{HoldsAt}(f, t + 1).$$

AXIOM DEC11

$$[\text{Happens}(a, t) \wedge \text{Releases}(a, f, t)] \Rightarrow \text{ReleasedAt}(f, t + 1).$$

AXIOM DEC12

$$[\text{Happens}(a, t) \wedge [\text{Initiates}(a, f, t) \vee \text{Terminates}(a, f, t)]] \Rightarrow \neg \text{ReleasedAt}(f, t + 1).$$

Let DEC be the conjunction of DEC3 through DEC12.

3.2 Equivalence of EC and DEC

We now prove the equivalence of EC and DEC if the timepoint sort is restricted to the integers. Before doing so, a number of lemmas are required.

LEMMA 3.1

If the timepoint sort is restricted to the integers, then

$$\text{DEC} \Rightarrow \text{EC5H}.$$

PROOF. Suppose DEC. Let τ_1 and τ_2 be arbitrary integer timepoints and β be an arbitrary fluent. We must show

$$[\text{HoldsAt}(\beta, \tau_1) \wedge \tau_1 < \tau_2 \wedge \text{PersistsBetween}(\tau_1, \beta, \tau_2) \wedge \neg \text{Clipped}(\tau_1, \beta, \tau_2)] \Rightarrow \text{HoldsAt}(\beta, \tau_2). \quad (3.1)$$

Case 1: $\tau_1 \geq \tau_2$. (3.1) is trivially satisfied.

Case 2: $\tau_1 < \tau_2$. We proceed by mathematical induction.

Base case: We show that (3.1) is true for $\tau_2 = \tau_1 + 1$. Suppose

$$\text{HoldsAt}(\beta, \tau_1) \wedge \text{PersistsBetween}(\tau_1, \beta, \tau_1 + 1) \wedge \neg \text{Clipped}(\tau_1, \beta, \tau_1 + 1).$$

From $PersistsBetween(\tau_1, \beta, \tau_1 + 1)$ and the definitional axiom EC17H', we have $\neg ReleasedAt(\beta, \tau_1 + 1)$. From $\neg Clipped(\tau_1, \beta, \tau_1 + 1)$ and the definitional axiom EC1, we have $\neg \exists a[Happens(a, \tau_1) \wedge Terminates(a, \beta, \tau_1)]$. From this, $HoldsAt(\beta, \tau_1)$, $\neg ReleasedAt(\beta, \tau_1 + 1)$, and DEC5, we have $HoldsAt(\beta, \tau_1 + 1)$ as required.

Induction step: Suppose (3.1) is true for $\tau_2 = k$, $k > \tau_1$ (induction hypothesis):

$$[HoldsAt(\beta, \tau_1) \wedge \tau_1 < k \wedge PersistsBetween(\tau_1, \beta, k) \wedge \neg Clipped(\tau_1, \beta, k)] \Rightarrow HoldsAt(\beta, k). \quad (3.2)$$

We must show that (3.1) is true for $\tau_2 = k + 1$. Suppose

$$HoldsAt(\beta, \tau_1) \wedge \tau_1 < k + 1 \wedge PersistsBetween(\tau_1, \beta, k + 1) \wedge \neg Clipped(\tau_1, \beta, k + 1).$$

From $PersistsBetween(\tau_1, \beta, k + 1)$ and EC17H', we have $PersistsBetween(\tau_1, \beta, k)$. From $\neg Clipped(\tau_1, \beta, k + 1)$ and EC1, we have $\neg Clipped(\tau_1, \beta, k)$. From this, $HoldsAt(\beta, \tau_1)$, $\tau_1 < k$, $PersistsBetween(\tau_1, \beta, k)$, and the induction hypothesis (3.2), we have $HoldsAt(\beta, k)$. From $PersistsBetween(\tau_1, \beta, k + 1)$ and EC17H', we have $\neg ReleasedAt(\beta, k + 1)$. From $\neg Clipped(\tau_1, \beta, k + 1)$ and EC1, we have $\neg \exists a[Happens(a, k) \wedge Terminates(a, \beta, k)]$. From this, $HoldsAt(\beta, k)$, $\neg ReleasedAt(\beta, k + 1)$, and DEC5, we have $HoldsAt(\beta, k + 1)$ as required. ■

LEMMA 3.2

If the timepoint sort is restricted to the integers, then

$$DEC \Rightarrow EC6H.$$

PROOF. The proof is identical to that of Lemma 3.1, except that $\neg HoldsAt$ is substituted for $HoldsAt$, $Initiates$ is substituted for $Terminates$, DEC6 is substituted for DEC5, $Declipped$ is substituted for $Clipped$, EC2 is substituted for EC1, and EC6H is substituted for EC5H. ■

LEMMA 3.3

If the timepoint sort is restricted to the integers, then

$$DEC \Rightarrow EC18H.$$

PROOF. Suppose DEC. Let τ_1 and τ_2 be arbitrary integer timepoints and β be an arbitrary fluent. We must show

$$[ReleasedAt(\beta, \tau_1) \wedge \tau_1 < \tau_2 \wedge \neg Clipped(\tau_1, \beta, \tau_2) \wedge \neg Declipped(\tau_1, \beta, \tau_2)] \Rightarrow ReleasedAt(\beta, \tau_2). \quad (3.3)$$

Case 1: $\tau_1 \geq \tau_2$. (3.3) is trivially satisfied.

Case 2: $\tau_1 < \tau_2$. We proceed by mathematical induction.

Base case: We show that (3.3) is true for $\tau_2 = \tau_1 + 1$. Suppose

$$ReleasedAt(\beta, \tau_1) \wedge \tau_1 < \tau_1 + 1 \wedge \neg Clipped(\tau_1, \beta, \tau_1 + 1) \wedge \neg Declipped(\tau_1, \beta, \tau_1 + 1).$$

From $\neg Declipped(\tau_1, \beta, \tau_1 + 1)$ and EC2, we have $\neg \exists a[Happens(a, \tau_1) \wedge Initiates(a, \beta, \tau_1)]$. From $\neg Clipped(\tau_1, \beta, \tau_1 + 1)$ and EC1, we have $\neg \exists a[Happens(a, \tau_1) \wedge Terminates(a, \beta, \tau_1)]$. From $ReleasedAt(\beta, \tau_1)$, $\neg \exists a[Happens(a, \tau_1) \wedge Initiates(a, \beta, \tau_1)]$, $\neg \exists a[Happens(a, \tau_1) \wedge Terminates(a, \beta, \tau_1)]$, and DEC7, we have $ReleasedAt(\beta, \tau_1 + 1)$ as required.

Induction step: Suppose (3.3) is true for $\tau_2 = k, k > \tau_1$ (induction hypothesis):

$$[ReleasedAt(\beta, \tau_1) \wedge \tau_1 < k \wedge \neg Clipped(\tau_1, \beta, k) \wedge \neg Declipped(\tau_1, \beta, k)] \Rightarrow ReleasedAt(\beta, k). \quad (3.4)$$

We must show that (3.3) is true for $\tau_2 = k + 1$. Suppose

$$ReleasedAt(\beta, \tau_1) \wedge \tau_1 < k + 1 \wedge \neg Clipped(\tau_1, \beta, k + 1) \wedge \neg Declipped(\tau_1, \beta, k + 1).$$

From $\neg Clipped(\tau_1, \beta, k + 1)$ and EC1, we have $\neg Clipped(\tau_1, \beta, k)$. From $\neg Declipped(\tau_1, \beta, k + 1)$ and EC2, we have $\neg Declipped(\tau_1, \beta, k)$. From this, $ReleasedAt(\beta, \tau_1), \tau_1 < k, \neg Clipped(\tau_1, \beta, k)$, and the induction hypothesis (3.4), we have $ReleasedAt(\beta, k)$. From $\neg Declipped(\tau_1, \beta, k + 1)$ and EC2, we have $\neg \exists a[Happens(a, k) \wedge Initiates(a, \beta, k)]$. From $\neg Clipped(\tau_1, \beta, k + 1)$ and EC1, we have $\neg \exists a[Happens(a, k) \wedge Terminates(a, \beta, k)]$. From $ReleasedAt(\beta, k), \neg \exists a[Happens(a, k) \wedge Initiates(a, \beta, k)], \neg \exists a[Happens(a, k) \wedge Terminates(a, \beta, k)]$, and DEC7, we have $ReleasedAt(\beta, k + 1)$ as required. ■

The following lemma is used in Theorem 3.9 as well as Lemmas 3.5 and 3.6:

LEMMA 3.4

If the timepoint sort is restricted to the integers, then

$$DEC \Rightarrow EC19H.$$

PROOF. Suppose DEC. Let τ_1 and τ_2 be arbitrary integer timepoints and β be an arbitrary fluent. We must show

$$[\neg ReleasedAt(\beta, \tau_1) \wedge \tau_1 < \tau_2 \wedge \neg ReleasedBetween(\tau_1, \beta, \tau_2)] \Rightarrow \neg ReleasedAt(\beta, \tau_2). \quad (3.5)$$

Case 1: $\tau_1 \geq \tau_2$. (3.5) is trivially satisfied.

Case 2: $\tau_1 < \tau_2$. We proceed by mathematical induction.

Base case: We show that (3.5) is true for $\tau_2 = \tau_1 + 1$. Suppose

$$\neg ReleasedAt(\beta, \tau_1) \wedge \tau_1 < \tau_1 + 1 \wedge \neg ReleasedBetween(\tau_1, \beta, \tau_1 + 1).$$

From $\neg ReleasedBetween(\tau_1, \beta, \tau_1 + 1)$ and EC14H, we have $\neg \exists a[Happens(a, \tau_1) \wedge Releases(a, \beta, \tau_1)]$. From this, $\neg ReleasedAt(\beta, \tau_1)$, and DEC8, we have $\neg ReleasedAt(\beta, \tau_1 + 1)$ as required.

Induction step: Suppose (3.5) is true for $\tau_2 = k, k > \tau_1$ (induction hypothesis):

$$[\neg ReleasedAt(\beta, \tau_1) \wedge \tau_1 < k \wedge \neg ReleasedBetween(\tau_1, \beta, k)] \Rightarrow \neg ReleasedAt(\beta, k). \quad (3.6)$$

We must show that (3.5) is true for $\tau_2 = k + 1$. Suppose

$$\neg ReleasedAt(\beta, \tau_1) \wedge \tau_1 < k + 1 \wedge \neg ReleasedBetween(\tau_1, \beta, k + 1).$$

From $\neg ReleasedBetween(\tau_1, \beta, k + 1)$ and EC14H, we have $\neg ReleasedBetween(\tau_1, \beta, k)$. From this, $\neg ReleasedAt(\beta, \tau_1), \tau_1 < k$, and the induction hypothesis (3.6), we have $\neg ReleasedAt(\beta, k)$. From $\neg ReleasedBetween(\tau_1, \beta, k + 1)$ and EC14H, we have $\neg \exists a[Happens(a, k) \wedge Releases(a, \beta, k)]$. From this, $\neg ReleasedAt(\beta, k)$, and DEC8, we have $\neg ReleasedAt(\beta, k + 1)$ as required. ■

LEMMA 3.5

If the timepoint sort is restricted to the integers, then

$$\text{DEC} \Rightarrow \text{EC3H}'.$$

PROOF. Suppose DEC. Let τ_1 and τ_2 be arbitrary integer timepoints, α be an arbitrary event, and β be an arbitrary fluent. We must show

$$\begin{aligned} & [\text{Happens}(\alpha, \tau_1) \wedge \text{Initiates}(\alpha, \beta, \tau_1) \wedge \tau_1 < \tau_2 \wedge \\ & \neg \text{StoppedIn}(\tau_1, \beta, \tau_2) \wedge \neg \text{ReleasedIn}(\tau_1, \beta, \tau_2)] \Rightarrow \\ & \text{HoldsAt}(\beta, \tau_2). \end{aligned} \quad (3.7)$$

Case 1: $\tau_1 \geq \tau_2$. (3.7) is trivially satisfied.

Case 2: $\tau_1 < \tau_2$. We proceed by mathematical induction.

Base case: We show that (3.7) is true for $\tau_2 = \tau_1 + 1$. Suppose

$$\begin{aligned} & \text{Happens}(\alpha, \tau_1) \wedge \text{Initiates}(\alpha, \beta, \tau_1) \wedge \tau_1 < \tau_1 + 1 \wedge \\ & \neg \text{StoppedIn}(\tau_1, \beta, \tau_1 + 1) \wedge \neg \text{ReleasedIn}(\tau_1, \beta, \tau_1 + 1). \end{aligned}$$

From this and DEC9, we have $\text{HoldsAt}(\beta, \tau_1 + 1)$, as required.

Induction step: Suppose (3.7) is true for $\tau_2 = k$, $k > \tau_1$ (induction hypothesis):

$$\begin{aligned} & [\text{Happens}(\alpha, \tau_1) \wedge \text{Initiates}(\alpha, \beta, \tau_1) \wedge \tau_1 < k \wedge \\ & \neg \text{StoppedIn}(\tau_1, \beta, k) \wedge \neg \text{ReleasedIn}(\tau_1, \beta, k)] \Rightarrow \\ & \text{HoldsAt}(\beta, k). \end{aligned} \quad (3.8)$$

We must show that (3.7) is true for $\tau_2 = k + 1$. Suppose

$$\begin{aligned} & \text{Happens}(\alpha, \tau_1) \wedge \text{Initiates}(\alpha, \beta, \tau_1) \wedge \tau_1 < k + 1 \wedge \\ & \neg \text{StoppedIn}(\tau_1, \beta, k + 1) \wedge \neg \text{ReleasedIn}(\tau_1, \beta, k + 1). \end{aligned}$$

From $\text{Happens}(\alpha, \tau_1)$, $\text{Initiates}(\alpha, \beta, \tau_1)$, and DEC9, we have $\text{HoldsAt}(\beta, \tau_1 + 1)$. From $\neg \text{StoppedIn}(\tau_1, \beta, k + 1)$ and EC9B, we have $\neg \text{StoppedIn}(\tau_1, \beta, k)$. From $\neg \text{ReleasedIn}(\tau_1, \beta, k + 1)$ and EC14H', we have $\neg \text{ReleasedIn}(\tau_1, \beta, k)$. From $\text{Happens}(\alpha, \tau_1)$, $\text{Initiates}(\alpha, \beta, \tau_1)$, $\tau_1 < k$, $\neg \text{StoppedIn}(\tau_1, \beta, k)$, $\neg \text{ReleasedIn}(\tau_1, \beta, k)$, and the induction hypothesis (3.8), we have $\text{HoldsAt}(\beta, k)$. From $\text{Happens}(\alpha, \tau_1)$, $\text{Initiates}(\alpha, \beta, \tau_1)$, and DEC12, we have $\neg \text{ReleasedAt}(\beta, \tau_1 + 1)$. From $\neg \text{ReleasedIn}(\tau_1, \beta, k + 1)$, EC14H', and EC14H, we have $\neg \text{ReleasedBetween}(\tau_1 + 1, \beta, k + 1)$. From Lemma 3.4 and DEC, we have EC19H. From $\neg \text{ReleasedAt}(\beta, \tau_1 + 1)$, $\tau_1 + 1 < k + 1$, $\neg \text{ReleasedBetween}(\tau_1 + 1, \beta, k + 1)$, and EC19H, we have $\neg \text{ReleasedAt}(\beta, k + 1)$. From $\neg \text{StoppedIn}(\tau_1, \beta, k + 1)$ and EC9B, we have $\neg \exists a [\text{Happens}(a, k) \wedge \text{Terminates}(a, \beta, k)]$. From this, $\text{HoldsAt}(\beta, k)$, $\neg \text{ReleasedAt}(\beta, k + 1)$, and DEC5, we have $\text{HoldsAt}(\beta, k + 1)$ as required. ■

LEMMA 3.6

If the timepoint sort is restricted to the integers, then

$$\text{DEC} \Rightarrow \text{EC4H}'.$$

PROOF. The proof is identical to that of Lemma 3.5, except that $\neg \text{HoldsAt}$ is substituted for HoldsAt , Terminates is substituted for Initiates , DEC10 is substituted for DEC9, StartedIn is substituted for StoppedIn , EC10B is substituted for EC9B, and DEC6 is substituted for DEC5. ■

LEMMA 3.7

If the timepoint sort is restricted to the integers, then

$$\text{DEC} \Rightarrow \text{EC15H}'.$$

PROOF. Suppose DEC. Let τ_1 and τ_2 be arbitrary integer timepoints, α be an arbitrary event, and β be an arbitrary fluent. We must show

$$\begin{aligned} & [\text{Happens}(\alpha, \tau_1) \wedge \text{Releases}(\alpha, \beta, \tau_1) \wedge \tau_1 < \tau_2 \wedge \\ & \neg \text{StoppedIn}(\tau_1, \beta, \tau_2) \wedge \neg \text{StartedIn}(\tau_1, \beta, \tau_2)] \Rightarrow \\ & \text{ReleasedAt}(\beta, \tau_2). \end{aligned} \quad (3.9)$$

Case 1: $\tau_1 \geq \tau_2$. (3.9) is trivially satisfied.

Case 2: $\tau_1 < \tau_2$. We proceed by mathematical induction.

Base case: We show that (3.9) is true for $\tau_2 = \tau_1 + 1$. Suppose

$$\begin{aligned} & \text{Happens}(\alpha, \tau_1) \wedge \text{Releases}(\alpha, \beta, \tau_1) \wedge \tau_1 < \tau_1 + 1 \wedge \\ & \neg \text{StoppedIn}(\tau_1, \beta, \tau_1 + 1) \wedge \neg \text{StartedIn}(\tau_1, \beta, \tau_1 + 1). \end{aligned}$$

From $\text{Happens}(\alpha, \tau_1)$, $\text{Releases}(\alpha, \beta, \tau_1)$, and DEC11, we have $\text{ReleasedAt}(\beta, \tau_1 + 1)$, as required.

Induction step: Suppose (3.9) is true for $\tau_2 = k$, $k > \tau_1$ (induction hypothesis):

$$\begin{aligned} & [\text{Happens}(\alpha, \tau_1) \wedge \text{Releases}(\alpha, \beta, \tau_1) \wedge \tau_1 < k \wedge \\ & \neg \text{StoppedIn}(\tau_1, \beta, k) \wedge \neg \text{StartedIn}(\tau_1, \beta, k)] \Rightarrow \\ & \text{ReleasedAt}(\beta, k). \end{aligned} \quad (3.10)$$

We must show that (3.9) is true for $\tau_2 = k + 1$. Suppose

$$\begin{aligned} & \text{Happens}(\alpha, \tau_1) \wedge \text{Releases}(\alpha, \beta, \tau_1) \wedge \tau_1 < k + 1 \wedge \\ & \neg \text{StoppedIn}(\tau_1, \beta, k + 1) \wedge \neg \text{StartedIn}(\tau_1, \beta, k + 1). \end{aligned}$$

From $\text{Happens}(\alpha, \tau_1)$, $\text{Releases}(\alpha, \beta, \tau_1)$, and DEC11, we have $\text{ReleasedAt}(\beta, \tau_1 + 1)$. From $\neg \text{StoppedIn}(\tau_1, \beta, k + 1)$ and EC9B, we have $\neg \text{StoppedIn}(\tau_1, \beta, k)$. From $\neg \text{StartedIn}(\tau_1, \beta, k + 1)$ and EC10B, we have $\neg \text{StartedIn}(\tau_1, \beta, k)$. From $\text{Happens}(\alpha, \tau_1)$, $\text{Releases}(\alpha, \beta, \tau_1)$, $\tau_1 < k$, $\neg \text{StoppedIn}(\tau_1, \beta, k)$, $\neg \text{StartedIn}(\tau_1, \beta, k)$, and the induction hypothesis (3.10), we have $\text{ReleasedAt}(\beta, k)$. From $\neg \text{StoppedIn}(\tau_1, \beta, k + 1)$ and EC9B, we have $\neg \exists a [\text{Happens}(a, k) \wedge \text{Terminates}(a, \beta, k)]$. From $\neg \text{StartedIn}(\tau_1, \beta, k + 1)$ and EC10B, we have $\neg \exists a [\text{Happens}(a, k) \wedge \text{Initiates}(a, \beta, k)]$. From this, $\text{ReleasedAt}(\beta, k)$, $\neg \exists a [\text{Happens}(a, k) \wedge \text{Terminates}(a, \beta, k)]$, and DEC7, we have $\text{ReleasedAt}(\beta, k + 1)$ as required. ■

LEMMA 3.8

If the timepoint sort is restricted to the integers, then

$$\text{DEC} \Rightarrow \text{EC16H}'.$$

PROOF. Suppose DEC. Let τ_1 and τ_2 be arbitrary integer timepoints, α be an arbitrary event, and β be an arbitrary fluent. We must show

$$\begin{aligned} & [\text{Happens}(\alpha, \tau_1) \wedge [\text{Initiates}(\alpha, \beta, \tau_1) \vee \text{Terminates}(\alpha, \beta, \tau_1)] \wedge \\ & \tau_1 < \tau_2 \wedge \neg \text{ReleasedIn}(\tau_1, \beta, \tau_2)] \Rightarrow \\ & \neg \text{ReleasedAt}(\beta, \tau_2). \end{aligned} \quad (3.11)$$

Case 1: $\tau_1 \geq \tau_2$. (3.11) is trivially satisfied.

Case 2: $\tau_1 < \tau_2$. We proceed by mathematical induction.

Base case: We show that (3.11) is true for $\tau_2 = \tau_1 + 1$. Suppose

$$\begin{aligned} & \text{Happens}(\alpha, \tau_1) \wedge [\text{Initiates}(\alpha, \beta, \tau_1) \vee \text{Terminates}(\alpha, \beta, \tau_1)] \wedge \\ & \tau_1 < \tau_1 + 1 \wedge \neg \text{ReleasedIn}(\tau_1, \beta, \tau_1 + 1). \end{aligned}$$

From $\text{Happens}(\alpha, \tau_1)$, $[\text{Initiates}(\alpha, \beta, \tau_1) \vee \text{Terminates}(\alpha, \beta, \tau_1)]$, and DEC12, we have $\neg \text{ReleasedAt}(\beta, \tau_1 + 1)$, as required.

Induction step: Suppose (3.11) is true for $\tau_2 = k$, $k > \tau_1$ (induction hypothesis):

$$\begin{aligned} & [\text{Happens}(\alpha, \tau_1) \wedge [\text{Initiates}(\alpha, \beta, \tau_1) \vee \text{Terminates}(\alpha, \beta, \tau_1)] \wedge \\ & \tau_1 < k \wedge \neg \text{ReleasedIn}(\tau_1, \beta, k)] \Rightarrow \\ & \neg \text{ReleasedAt}(\beta, k). \end{aligned} \tag{3.12}$$

We must show that (3.11) is true for $\tau_2 = k + 1$. Suppose

$$\begin{aligned} & \text{Happens}(\alpha, \tau_1) \wedge [\text{Initiates}(\alpha, \beta, \tau_1) \vee \text{Terminates}(\alpha, \beta, \tau_1)] \wedge \\ & \tau_1 < k + 1 \wedge \neg \text{ReleasedIn}(\tau_1, \beta, k + 1). \end{aligned}$$

From $\text{Happens}(\alpha, \tau_1)$, $\text{Initiates}(\alpha, \beta, \tau_1) \vee \text{Terminates}(\alpha, \beta, \tau_1)$, and DEC12, we have $\neg \text{ReleasedAt}(\beta, \tau_1 + 1)$. From $\neg \text{ReleasedIn}(\tau_1, \beta, k + 1)$ and EC14H', we have $\neg \text{ReleasedIn}(\tau_1, \beta, k)$. From this, $\text{Happens}(\alpha, \tau_1)$, $\text{Initiates}(\alpha, \beta, \tau_1) \vee \text{Terminates}(\alpha, \beta, \tau_1)$, $\tau_1 < k$, and the induction hypothesis (3.12), we have $\neg \text{ReleasedAt}(\beta, k)$. From $\neg \text{ReleasedIn}(\tau_1, \beta, k + 1)$ and EC14H', we have $\neg \exists a [\text{Happens}(a, k) \wedge \text{Releases}(a, \beta, k)]$. From this, $\neg \text{ReleasedAt}(\beta, k)$, and DEC8, we have $\neg \text{ReleasedAt}(\beta, k + 1)$ as required. ■

Now we proceed to the equivalence theorem:

THEOREM 3.9

If the timepoint sort is restricted to the integers, then

$$EC \Leftrightarrow DEC.$$

PROOF. We prove the two directions separately.

($EC \Rightarrow DEC$)

Suppose EC. DEC3 is identical to EC11F' and DEC4 is identical to EC12F'. DEC5 follows from EC5H by universal instantiation, substituting $t_1 + 1$ for t_2 . Similarly, DEC6, DEC7, DEC8, DEC9, DEC10, DEC11, and DEC12 follow from EC6H, EC18H, EC19H, EC3H', EC4H', EC15H', and EC16H', respectively, by universal instantiation, substituting $t_1 + 1$ for t_2 .

($EC \Leftarrow DEC$)

Suppose DEC. EC11F' is identical to DEC3 and EC12F' is identical to DEC4. EC5H, EC6H, EC18H, EC19H, EC3H', EC4H', EC15H', and EC16H' follow from Lemmas 3.1, 3.2, 3.3, 3.4, 3.5, 3.6, 3.7, and 3.8, respectively. ■

4 The encoding method

In order to perform event calculus reasoning through satisfiability, we must construct an efficient satisfiability encoding of a domain description

$$\text{CIRC}[\Sigma; \text{Initiates}, \text{Terminates}, \text{Releases}] \wedge \text{CIRC}[\Delta; \text{Happens}] \wedge \Omega \wedge \Psi \wedge \Pi \wedge \Gamma \wedge EC.$$

In this section, we describe our method for constructing such an encoding and prove a form of equivalence between a domain description and its encoding. Our basic method performs model finding. After describing the method in the context of model finding, we discuss how the method is also used to solve deduction and abduction problems.

4.1 *Restriction to a finite universe*

In order to use a satisfiability solver [8], we must transform event calculus problems into the propositional calculus. A satisfiability solver takes as input a set of Boolean variables and a propositional formula over those variables and produces as output zero or more models or satisfying truth assignments—truth assignments for the variables such that the formula is true. Satisfiability solvers take a propositional formula in conjunctive normal form: a conjunction of clauses, where each clause is a disjunction of literals, where each literal is a variable or a negated variable. A complete satisfiability solver produces all satisfying truth assignments.

Following Shanahan and Witkowski [44], we restrict the predicate calculus to a finite universe [16, 13]. We restrict the event calculus to finite sets of variables, constants, function symbols, predicate symbols, sorts, events, fluents, timepoints, and domain objects. We restrict the timepoint sort to a finite set of integers $\{0, 1, 2, \dots, n\}$ for some $n \geq 0$.

Note that we may now ignore Ω since the propositional calculus already incorporates the unique names assumption.

4.2 *Definitions*

We start with some definitions.

DEFINITION 4.1

A *comparison* is a formula of the form $t_1 < t_2$, $t_1 \leq t_2$, $t_1 = t_2$, $t_1 \geq t_2$, $t_1 > t_2$, or $t_1 \neq t_2$, where t_1 and t_2 are terms.

DEFINITION 4.2

If t is a variable, then a *condition over t* is defined as follows: (1) A comparison is a condition over t . (2) If f is a term, then $HoldsAt(f, t)$ and $\neg HoldsAt(f, t)$ are conditions over t . (3) If c_1 and c_2 are conditions over t , then $c_1 \wedge c_2$ and $c_1 \vee c_2$ are conditions over t . (4) If v is a variable and c is a condition over t , then $\exists v c$ is a condition over t .

DEFINITION 4.3

If π is the predicate symbol *Initiates*, *Terminates*, or *Releases*, then a π *effect axiom* is a formula of the form $\forall a, f, t [\Theta(a, f, t) \Rightarrow \pi(a, f, t)]$, where $\Theta(a, f, t)$ is a condition over t with only a, f , and t free.

DEFINITION 4.4

A π *effect description* is a collection of π effect axioms written as a single, logically equivalent π effect axiom of the form $\forall a, f, t [\Theta(a, f, t) \Rightarrow \pi(a, f, t)]$, where $\Theta(a, f, t)$ is a condition over t with only a, f , and t free.

Let Σ_{init} be the *Initiates* effect description $\forall a, f, t [\Theta_{init}(a, f, t) \Rightarrow Initiates(a, f, t)]$.

Let Σ_{term} be the *Terminates* effect description $\forall a, f, t [\Theta_{term}(a, f, t) \Rightarrow Terminates(a, f, t)]$.

Let Σ_{rel} be the *Releases* effect description $\forall a, f, t [\Theta_{rel}(a, f, t) \Rightarrow Releases(a, f, t)]$.

DEFINITION 4.5

A *trigger axiom* is a formula of the form $\forall a, t [\Upsilon(a, t) \Rightarrow Happens(a, t)]$, where $\Upsilon(a, t)$ is a condition over t with only a and t free.

DEFINITION 4.6

A *trigger description* is a collection of trigger axioms written as a single, logically equivalent trigger axiom of the form $\forall a, t [\Upsilon(a, t) \Rightarrow \text{Happens}(a, t)]$, where $\Upsilon(a, t)$ is a condition over t with only a and t free.

DEFINITION 4.7

An *event occurrence* is a formula of the form $\text{Happens}(a, t)$, where a is an event ground term and t is a timepoint ground term.

DEFINITION 4.8

An *event occurrence description* is a collection of event occurrences written as a single, logically equivalent trigger axiom of the form $\forall a, t [\Upsilon(a, t) \Rightarrow \text{Happens}(a, t)]$, where $\Upsilon(a, t)$ is a condition over t with only a and t free.

DEFINITION 4.9

An *event description* is a trigger description and an event occurrence description written as a single, logically equivalent trigger axiom of the form $\forall a, t [\Upsilon(a, t) \Rightarrow \text{Happens}(a, t)]$, where $\Upsilon(a, t)$ is a condition over t with only a and t free.

Let Δ be an event description.

DEFINITION 4.10

A *state constraint* is a formula of the form (1) $c_1 \Rightarrow c_2$ or (2) $c_1 \Leftrightarrow c_2$, where c_1 and c_2 are conditions over some variable t .

Let Ψ be a conjunction of state constraints.

DEFINITION 4.11

If π is the predicate symbol *Trajectory* or *AntiTrajectory*, then a π *gradual change axiom* is a formula of the form $\forall f_1, t_1, f_2, t_2 [\Xi(f_1, t_1, f_2, t_2) \Rightarrow \pi(f_1, t_1, f_2, t_2)]$, where $\Xi(f_1, t_1, f_2, t_2)$ is a condition over t_1 with only f_1, t_1, f_2 , and t_2 free.

DEFINITION 4.12

A π *gradual change description* is a collection of π gradual change axioms written as a single, logically equivalent trajectory axiom of the form $\forall f_1, t_1, f_2, t_2 [\Xi(f_1, t_1, f_2, t_2) \Rightarrow \pi(f_1, t_1, f_2, t_2)]$, where $\Xi(f_1, t_1, f_2, t_2)$ is a condition over t_1 with only f_1, t_1, f_2 , and t_2 free.

Let Π_{traj} be the *Trajectory* gradual change description $\forall f_1, t_1, f_2, t_2 [\Xi_{traj}(f_1, t_1, f_2, t_2) \Rightarrow \text{Trajectory}(f_1, t_1, f_2, t_2)]$.

Let Π_{anti} be the *AntiTrajectory* gradual change description $\forall f_1, t_1, f_2, t_2 [\Xi_{anti}(f_1, t_1, f_2, t_2) \Rightarrow \text{AntiTrajectory}(f_1, t_1, f_2, t_2)]$.

DEFINITION 4.13

A *state description* is a conjunction of formulas of the form $\text{HoldsAt}(f, t)$, $\neg \text{HoldsAt}(f, t)$, $\text{ReleasedAt}(f, t)$, or $\neg \text{ReleasedAt}(f, t)$, where f is a fluent ground term and t is a timepoint ground term.

Let Γ be a state description.

4.3 Computing circumscription

Our encoding method requires computation of circumscription of effect and event descriptions. We perform these computations using two theorems of Lifschitz [21]. The first theorem provides a rule for computing circumscription using predicate completion:

THEOREM 4.14

Let ρ be an n -ary predicate symbol and $\Gamma(x_1, \dots, x_n)$ be a formula with only $x_1, \dots, x_n$ free. If $\Gamma(x_1, \dots, x_n)$ does not mention ρ , then the circumscription $CIRC[\forall x_1, \dots, x_n [\Gamma(x_1, \dots, x_n) \Rightarrow \rho(x_1, \dots, x_n)]; \rho]$ is equivalent to $\forall x_1, \dots, x_n [\Gamma(x_1, \dots, x_n) \Leftrightarrow \rho(x_1, \dots, x_n)]$.

PROOF. See the proof of Proposition 2 by Lifschitz [21]. ■

The second theorem provides a rule for computing circumscription of several predicates.

DEFINITION 4.15

A formula Γ is *positive relative to a predicate symbol* ρ if all mentions of ρ in Γ are in the range of an even number of negations in an equivalent formula obtained by eliminating $\Rightarrow$ and $\Leftrightarrow$ from Γ .

THEOREM 4.16

Let $\rho_1, \dots, \rho_n$ be predicate symbols and Γ be a formula. If Γ is positive relative to every ρ_i , then $CIRC[\Gamma; \rho_1, \dots, \rho_n]$ is equivalent to $\bigwedge_{i=1}^n CIRC[\Gamma; \rho_i]$.

PROOF. See the proof of Proposition 14 by Lifschitz [21]. ■

4.4 Description of the encoding method

We now describe our method for encoding a problem given by Σ_{init} , Σ_{term} , Σ_{rel} , Δ , Ψ , Π_{traj} , Π_{anti} , and Γ .

First, we use the axiomatization DEC of Section 3 instead of the axiomatization EC of Section 2 in order to reduce triply quantified time to singly quantified time in most axioms.

Second, observe that EC and DEC contain atoms involving *Initiates*, *Terminates*, *Releases*, *Trajectory* and *AntiTrajectory*, which may lead to a large number of ground atoms. For example, $Initiates(a, f, t)$ gives rise to $A \cdot F \cdot T$ ground atoms, where A is the number of events, F is the number of fluents, and T is the number of timepoints. Therefore, in order to eliminate such atoms, we expand DEC by performing the following substitutions:

$$\begin{aligned} Initiates(a, f, t) &\Rightarrow \Theta_{init}(a, f, t) \\ Terminates(a, f, t) &\Rightarrow \Theta_{term}(a, f, t) \\ Releases(a, f, t) &\Rightarrow \Theta_{rel}(a, f, t) \\ Trajectory(f_1, t_1, f_2, t_2) &\Rightarrow \Xi_{traj}(f_1, t_1, f_2, t_2) \\ AntiTrajectory(f_1, t_1, f_2, t_2) &\Rightarrow \Xi_{anti}(f_1, t_1, f_2, t_2). \end{aligned}$$

For example, if Σ_{init} is

$$[a = Hold(p, o) \wedge f = Holding(p, o)] \Rightarrow Initiates(a, f, t)$$

then we replace DEC9 with

$$[Happens(a, t) \wedge [a = Hold(p, o) \wedge f = Holding(p, o)]] \Rightarrow HoldsAt(f, t + 1).$$

Third, we compute $CIRC[\Delta; Happens]$ using Theorems 4.14 and 4.16.

Fourth, we conjoin Ψ , Γ , the expanded DEC, and $CIRC[\Delta; Happens]$.

Fifth, we instantiate quantifiers by replacing $\forall x \Phi(x)$ with $\bigwedge_i \Phi(x_i)$ and $\exists x \Phi(x)$ with $\bigvee_i \Phi(x_i)$, where x_i are the constants of the sort of x . This gives a propositional calculus formula.

Sixth, we simplify the formula using standard techniques [31, pp. 35–36].

Seventh, we convert the formula to conjunctive normal form using standard techniques [6, pp. 17–18].

Finally, we construct a one-to-one and onto map B that maps the ground atoms of the formula to Boolean variables. We construct an inverse map B^{-1} from B . We construct a formula to pass to the satisfiability solver by replacing each ground atom u in the formula with $B(u)$.

In order to perform model finding, we feed the formula to a satisfiability solver. We decode satisfying truth assignments produced by the solver by applying B^{-1} . Model finding is useful in many applications such as determining what can possibly occur given a formalization [1, 10].

Our implementation employs two additional optimizations in order to reduce the size of the encoding further. First, note that converting to conjunctive normal form using standard techniques may result in a combinatorial explosion. We convert to a compact conjunctive normal form using the technique of renaming subformulas [32, 11]. Second, instead of using a single sort for all domain objects, we allow the use of a number of domain-specific sorts.

In order to reduce the encoding time further, our implementation simplifies the expanded DEC in order to eliminate the quantification over events and fluents. For example, we simplify

$$[Happens(a, t) \wedge [a = Hold(p, o) \wedge f = Holding(p, o)]] \Rightarrow HoldsAt(f, t + 1)$$

to

$$Happens(Hold(p, o), t) \Rightarrow HoldsAt(Holding(p, o), t + 1).$$

EXAMPLE 4.17 (Encoding of a domain description)

Consider the following domain description. We have an *Initiates* effect description that states that if a person holds an object, then the person will be holding the object:

$$[a = Hold(p, o) \wedge f = Holding(p, o)] \Rightarrow Initiates(a, f, t). \quad (4.1)$$

We have a state description that says that at timepoint 0, person $P1$ is not holding object $O1$ and $Holding(P1, O1)$ is not released from the commonsense law of inertia:

$$\neg HoldsAt(Holding(P1, O1), 0) \wedge \neg ReleasedAt(Holding(P1, O1), 0). \quad (4.2)$$

We have an event description that states that at timepoint 0, person $P1$ holds object $O1$:

$$[a = Hold(P1, O1) \wedge t = 0] \Rightarrow Happens(a, t). \quad (4.3)$$

Suppose that 0 and 1 are the only constants of the timepoint sort, $P1$ is the only constant of the person sort, and $O1$ is the only constant of the object sort. The conjunctive normal form encoding of this domain description then consists of 10 clauses. We have the clauses for (4.2):

$$\begin{aligned} &\neg HoldsAt(Holding(P1, O1), 0) \\ &\neg ReleasedAt(Holding(P1, O1), 0). \end{aligned}$$

We have the following clauses, which result from the expansion of DEC5, DEC6, DEC7, DEC8, DEC9, and DEC12, respectively, given (4.1):

$$\begin{aligned} &ReleasedAt(Holding(P1, O1), 1) \vee HoldsAt(Holding(P1, O1), 1) \vee \\ &\quad \neg HoldsAt(Holding(P1, O1), 0) \end{aligned}$$

$$\begin{aligned} & \text{Happens}(\text{Hold}(P1, O1), 0) \vee \text{HoldsAt}(\text{Holding}(P1, O1), 0) \vee \\ & \text{ReleasedAt}(\text{Holding}(P1, O1), 1) \vee \neg \text{HoldsAt}(\text{Holding}(P1, O1), 1) \end{aligned}$$

$$\begin{aligned} & \text{Happens}(\text{Hold}(P1, O1), 0) \vee \text{ReleasedAt}(\text{Holding}(P1, O1), 1) \vee \\ & \neg \text{ReleasedAt}(\text{Holding}(P1, O1), 0) \end{aligned}$$

$$\text{ReleasedAt}(\text{Holding}(P1, O1), 0) \vee \neg \text{ReleasedAt}(\text{Holding}(P1, O1), 1)$$

$$\text{HoldsAt}(\text{Holding}(P1, O1), 1) \vee \neg \text{Happens}(\text{Hold}(P1, O1), 0)$$

$$\neg \text{Happens}(\text{Hold}(P1, O1), 0) \vee \neg \text{ReleasedAt}(\text{Holding}(P1, O1), 1).$$

Note that axioms DEC10 and DEC11 are trivially satisfied since no *Terminates* or *Releases* formulas are in the domain description. We have the following clauses, which are equivalent to the circumscription of *Happens* in (4.3):

$$\begin{aligned} & \neg \text{Happens}(\text{Hold}(P1, O1), 1) \\ & \text{Happens}(\text{Hold}(P1, O1), 0). \end{aligned}$$

We construct a map from ground atoms to Boolean variables:

$$\begin{aligned} \text{Happens}(\text{Hold}(P1, O1), 0) & \rightarrow 1 \\ \text{HoldsAt}(\text{Holding}(P1, O1), 0) & \rightarrow 2 \\ \text{ReleasedAt}(\text{Holding}(P1, O1), 0) & \rightarrow 3 \\ \text{Happens}(\text{Hold}(P1, O1), 1) & \rightarrow 4 \\ \text{ReleasedAt}(\text{Holding}(P1, O1), 1) & \rightarrow 5 \\ \text{HoldsAt}(\text{Holding}(P1, O1), 1) & \rightarrow 6. \end{aligned}$$

We convert the clauses into the standard DIMACS format for satisfiability problems [5]:

```
p cnf 6 10
-2 0
-3 0
5 6 -2 0
1 2 5 -6 0
1 5 -3 0
3 -5 0
6 -1 0
-1 -5 0
-4 0
1 0
```

The first line specifies the number of variables and clauses. The remaining lines are the clauses. Each line consists of a sequence of numbers. A negated variable v is represented by $-v$; a non-negated variable v is represented by v . Each line is terminated with the number 0.

We invoke a satisfiability solver on the problem, which produces one model as output:

1 -2 -3 -4 -5 6

By applying the inverse of the above map, we get:

$$\begin{aligned} & \text{Happens}(\text{Hold}(P1, O1), 0) \\ & \neg \text{HoldsAt}(\text{Holding}(P1, O1), 0) \\ & \neg \text{ReleasedAt}(\text{Holding}(P1, O1), 0) \\ & \neg \text{Happens}(\text{Hold}(P1, O1), 1) \\ & \neg \text{ReleasedAt}(\text{Holding}(P1, O1), 1) \\ & \text{HoldsAt}(\text{Holding}(P1, O1), 1). \end{aligned}$$

4.5 Equivalence

We now prove a form of equivalence between a domain description and the encoding of the domain description produced by our method. We start with some definitions and a lemma.

DEFINITION 4.18

An *intermediate atom* is an atom of the form $\text{Initiates}(\alpha, \beta, \tau)$, $\text{Terminates}(\alpha, \beta, \tau)$, $\text{Releases}(\alpha, \beta, \tau)$, $\text{Trajectory}(\beta_1, \tau_1, \beta_2, \tau_2)$, or $\text{AntiTrajectory}(\beta_1, \tau_1, \beta_2, \tau_2)$, where α is an event, β , β_1 , and β_2 are fluents, and τ , τ_1 , and τ_2 are timepoints.

DEFINITION 4.19

Let Θ_{init} , Θ_{term} , Θ_{rel} , Ξ_{traj} , and Ξ_{anti} be as defined above. If Λ is a formula, then $\text{Expand}[\Lambda; \Theta_{init}; \Theta_{term}; \Theta_{rel}; \Xi_{traj}; \Xi_{anti}]$ is defined as follows:

1. $\Theta_{init}(\alpha, \beta, \tau)$, if Λ is of the form $\text{Initiates}(\alpha, \beta, \tau)$,
2. $\Theta_{term}(\alpha, \beta, \tau)$, if Λ is of the form $\text{Terminates}(\alpha, \beta, \tau)$,
3. $\Theta_{rel}(\alpha, \beta, \tau)$, if Λ is of the form $\text{Releases}(\alpha, \beta, \tau)$,
4. $\Xi_{traj}(\beta_1, \tau_1, \beta_2, \tau_2)$, if Λ is of the form $\text{Trajectory}(\beta_1, \tau_1, \beta_2, \tau_2)$,
5. $\Xi_{anti}(\beta_1, \tau_1, \beta_2, \tau_2)$, if Λ is of the form $\text{AntiTrajectory}(\beta_1, \tau_1, \beta_2, \tau_2)$, and
6. Λ with each intermediate atom λ replaced with $\text{Expand}[\lambda; \Theta_{init}; \Theta_{term}; \Theta_{rel}; \Xi_{traj}; \Xi_{anti}]$, otherwise.

DEFINITION 4.20

If Σ_{init} , Σ_{term} , Σ_{rel} , Θ_{init} , Θ_{term} , Θ_{rel} , Δ , Ψ , Π_{traj} , Π_{anti} , Ξ_{traj} , Ξ_{anti} , and Γ are as defined above, then Encode is defined as follows:

$$\begin{aligned} & \text{Encode}[\text{DEC}; \Sigma_{init}; \Sigma_{term}; \Sigma_{rel}; \Delta; \Psi; \Pi_{traj}; \Pi_{anti}; \Gamma] = \\ & \text{Expand}[\text{DEC}; \Theta_{init}; \Theta_{term}; \Theta_{rel}; \Xi_{traj}; \Xi_{anti}] \wedge \text{CIRC}[\Delta; \text{Happens}] \wedge \Psi \wedge \Gamma. \end{aligned}$$

LEMMA 4.21

Let $x_1, \dots, x_n, y_1, \dots, y_k$ be distinct atoms. Let $X_1, \dots, X_n, Y_1, \dots, Y_k$ be propositional formulas not mentioning any of the atoms $x_1, \dots, x_n, y_1, \dots, y_k$. Let Z be a propositional formula in conjunctive normal form mentioning all of the atoms $x_1, \dots, x_n, y_1, \dots, y_k$. Let $F = [\bigwedge_{i=1}^n X_i \Leftrightarrow x_i] \wedge [\bigwedge_{i=1}^k Y_i \Rightarrow y_i] \wedge Z$. Let G be Z with each occurrence of x_i replaced by X_i and each occurrence of y_i replaced by Y_i . Let A_G be the set of atoms mentioned in G . Let $\mathcal{T}_G$ be a truth assignment $\mathcal{T}_G : A_G \rightarrow \{T, F\}$. Let $\overline{\mathcal{T}}_G$ be the extension of $\mathcal{T}_G$ to propositional formulas mentioning the atoms A_G . Let $A_F = A_G \cup \{x_1, \dots, x_n, y_1, \dots, y_k\}$. Let $\mathcal{T}_F$ be the truth assignment $\mathcal{T}_F : A_F \rightarrow \{T, F\}$ defined as follows:

$$\mathcal{T}_F(u) = \begin{cases} \overline{\mathcal{T}}_G(X_i) & \text{if } u = x_i \text{ for some } i \\ \overline{\mathcal{T}}_G(Y_i) & \text{if } u = y_i \text{ for some } i \\ \overline{\mathcal{T}}_G(u) & \text{otherwise.} \end{cases}$$

Let $\overline{\mathcal{T}}_F$ be the extension of $\mathcal{T}_F$ to propositional formulas mentioning the atoms A_F . Then it is the case that

$$\overline{\mathcal{T}}_F(F) = T \text{ if and only if } \overline{\mathcal{T}}_G(G) = T.$$

PROOF. Let $Z_1, \dots, Z_m$ be the conjuncts of Z and $G_1, \dots, G_m$ be the corresponding conjuncts of G . We prove each direction separately.

($\Rightarrow$) Suppose $\overline{\mathcal{T}}_F(F) = T$. We must show that for every $i \in \{1, \dots, m\}$, $\overline{\mathcal{T}}_G(G_i) = T$. Let i be an arbitrary element of $\{1, \dots, m\}$. From $\overline{\mathcal{T}}_F(Z) = T$, which follows from $\overline{\mathcal{T}}_F(F) = T$, it follows that $\overline{\mathcal{T}}_F(Z_i) = T$. Let $L_1, \dots, L_p$ be the disjuncts of Z_i and $M_1, \dots, M_p$ be the corresponding disjuncts of G_i . From the definition of $\overline{\mathcal{T}}_F$, it follows that $\bigwedge_{j=1}^p \overline{\mathcal{T}}_F(L_j) = \overline{\mathcal{T}}_G(M_j)$. From this and $\overline{\mathcal{T}}_F(\bigvee_{j=1}^p L_j) = T$, it follows that $\overline{\mathcal{T}}_G(\bigvee_{j=1}^p M_j) = T$, as required.

($\Leftarrow$) Suppose $\overline{\mathcal{T}}_G(G) = T$. We must show that (1) for every $i \in \{1, \dots, n\}$, $\overline{\mathcal{T}}_F(X_i \Leftrightarrow x_i) = T$, (2) for every $i \in \{1, \dots, k\}$, $\overline{\mathcal{T}}_F(Y_i \Rightarrow y_i) = T$, and (3) for every $i \in \{1, \dots, m\}$, $\overline{\mathcal{T}}_F(Z_i) = T$.

(1) Let i be an arbitrary element of $\{1, \dots, n\}$. From the definition of $\overline{\mathcal{T}}_F$, it follows that $\overline{\mathcal{T}}_F(X_i)$ and $\overline{\mathcal{T}}_F(x_i)$ have the same truth value.

(2) Let i be an arbitrary element of $\{1, \dots, k\}$. From the definition of $\overline{\mathcal{T}}_F$, it follows that $\overline{\mathcal{T}}_F(Y_i) = F$ or $\overline{\mathcal{T}}_F(y_i) = T$.

(3) Let i be an arbitrary element of $\{1, \dots, m\}$. From $\overline{\mathcal{T}}_G(G) = T$, it follows that $\overline{\mathcal{T}}_G(G_i) = T$. Let $M_1, \dots, M_p$ be the disjuncts of G_i and $L_1, \dots, L_p$ be the corresponding disjuncts of Z_i . From the definition of $\overline{\mathcal{T}}_F$, it follows that $\bigwedge_{j=1}^p \overline{\mathcal{T}}_G(M_j) = \overline{\mathcal{T}}_F(L_j)$. From this and $\overline{\mathcal{T}}_G(\bigvee_{j=1}^p M_j) = T$, it follows that $\overline{\mathcal{T}}_F(\bigvee_{j=1}^p L_j) = T$, as required. ■

DEFINITION 4.22

A truth assignment $\mathcal{T} : U \rightarrow \{T, F\}$ with a set V removed is defined as a truth assignment $\mathcal{T}' : [U - V] \rightarrow \{T, F\}$ such that for every $\lambda \in U - V$, $\mathcal{T}'(\lambda) = \mathcal{T}(\lambda)$.

DEFINITION 4.23

The *grounding* of a formula Λ is a formula obtained from Λ by instantiating quantifiers and simplifying.

We now proceed to the equivalence theorem:

THEOREM 4.24

Restrict the logic to a finite universe as specified above. If $\Sigma_{init}, \Sigma_{term}, \Sigma_{rel}, \Theta_{init}, \Theta_{term}, \Theta_{rel}, \Delta, \Psi, \Pi_{traj}, \Pi_{anti}, \Xi_{traj}, \Xi_{anti}$, and Γ are as defined above, then the satisfying truth assignments with intermediate atoms removed of:

$$E = CIRC[\Sigma_{init} \wedge \Sigma_{term} \wedge \Sigma_{rel}; Initiates, Terminates, Releases] \wedge CIRC[\Delta; Happens] \wedge \Psi \wedge \Pi_{traj} \wedge \Pi_{anti} \wedge \Gamma \wedge EC$$

are the same as the satisfying truth assignments with intermediate atoms removed of:

$$D = Encode[DEC; \Sigma_{init}; \Sigma_{term}; \Sigma_{rel}; \Delta; \Psi; \Pi_{traj}; \Pi_{anti}; \Gamma].$$

PROOF. Let $x_1, \dots, x_n$ be all the atoms in the grounding of E of the form *Initiates*(α, β, τ), *Terminates*(α, β, τ), and *Releases*(α, β, τ). Let $y_1, \dots, y_k$ be all the atoms in the grounding of E of

the form $Trajectory(\beta_1, \tau_1, \beta_2, \tau_2)$ and $AntiTrajectory(\beta_1, \tau_1, \beta_2, \tau_2)$. Let $X_i = Expand[x_i; \Theta_{init}; \Theta_{term}; \Theta_{rel}; \Xi_{traj}; \Xi_{anti}]$. Let $Y_i = Expand[y_i; \Theta_{init}; \Theta_{term}; \Theta_{rel}; \Xi_{traj}; \Xi_{anti}]$. Let Z be the grounding of $CIRC[\Delta; Happens] \wedge \Psi \wedge \Gamma \wedge DEC$ written in conjunctive normal form. Let $F = [\bigwedge_{i=1}^n X_i \Leftrightarrow x_i] \wedge [\bigwedge_{i=1}^k Y_i \Rightarrow y_i] \wedge Z$. Let $G = Expand[Z; \Theta_{init}; \Theta_{term}; \Theta_{rel}; \Xi_{traj}; \Xi_{anti}]$. Let A_G be the set of atoms mentioned in G . Let $\mathcal{T}_G$ be a truth assignment $\mathcal{T}_G : A_G \rightarrow \{T, F\}$. Let $\overline{\mathcal{T}}_G$ be the extension of $\mathcal{T}_G$ to propositional formulas mentioning the atoms A_G . Let $A_F = A_G \cup \{x_1, \dots, x_n, y_1, \dots, y_k\}$. Let $\mathcal{T}_F$ be the truth assignment $\mathcal{T}_F : A_F \rightarrow \{T, F\}$ defined as follows:

$$\mathcal{T}_F(u) = \begin{cases} \overline{\mathcal{T}}_G(X_i) & \text{if } u = x_i \text{ for some } i \\ \overline{\mathcal{T}}_G(Y_i) & \text{if } u = y_i \text{ for some } i \\ \overline{\mathcal{T}}_G(u) & \text{otherwise.} \end{cases}$$

Let $\overline{\mathcal{T}}_F$ be the extension of $\mathcal{T}_F$ to propositional formulas mentioning the atoms A_F . From Theorems 4.14 and 4.16, and the definition of $Expand$, we have

$$CIRC[\Sigma_{init} \wedge \Sigma_{term} \wedge \Sigma_{rel}; Initiates, Terminates, Releases] \Leftrightarrow [\bigwedge_{i=1}^n X_i \Leftrightarrow x_i]. \quad (4.4)$$

From the definition of $Expand$, we have

$$[\Pi_{traj} \wedge \Pi_{anti}] \Leftrightarrow [\bigwedge_{i=1}^k Y_i \Rightarrow y_i]. \quad (4.5)$$

From Theorem 3.9 we have $EC \Leftrightarrow DEC$. From (4.4), (4.5), $Z \Leftrightarrow CIRC[\Delta; Happens] \wedge \Psi \wedge \Gamma \wedge DEC$, and $EC \Leftrightarrow DEC$, we have $E \Leftrightarrow F$. From the definition of $Encode$, we have $D \Leftrightarrow Expand[DEC; \Theta_{init}; \Theta_{term}; \Theta_{rel}; \Xi_{traj}; \Xi_{anti}] \wedge CIRC[\Delta; Happens] \wedge \Psi \wedge \Gamma$. From the definition of $Expand$ and since $CIRC[\Delta; Happens] \wedge \Psi \wedge \Gamma$ does not mention any intermediate atoms, we have $D \Leftrightarrow Expand[DEC \wedge CIRC[\Delta; Happens] \wedge \Psi \wedge \Gamma; \Theta_{init}; \Theta_{term}; \Theta_{rel}; \Xi_{traj}; \Xi_{anti}]$. From $Z \Leftrightarrow CIRC[\Delta; Happens] \wedge \Psi \wedge \Gamma \wedge DEC$ we have $D \Leftrightarrow Expand[Z; \Theta_{init}; \Theta_{term}; \Theta_{rel}; \Xi_{traj}; \Xi_{anti}]$. From this, we have $D \Leftrightarrow G$. From Lemma 4.21, $E \Leftrightarrow F$, and $D \Leftrightarrow G$, we have $\overline{\mathcal{T}}_F(E) = T$ if and only if $\overline{\mathcal{T}}_G(D) = T$. From this, it follows that the satisfying truth assignments with intermediate atoms removed of E are the same as the satisfying truth assignments with intermediate atoms removed of D , as required. ■

4.6 Deduction

We may use our encoding method to perform deduction:

THEOREM 4.25

Restrict the logic to a finite universe as specified above. Let $\Sigma_{init}, \Sigma_{term}, \Sigma_{rel}, \Delta, \Psi, \Pi_{traj}, \Pi_{anti}$, and Γ be as defined above. If Γ' is a state description, then

$$CIRC[\Sigma_{init} \wedge \Sigma_{term} \wedge \Sigma_{rel}; Initiates, Terminates, Releases] \wedge CIRC[\Delta; Happens] \wedge \Psi \wedge \Pi_{traj} \wedge \Pi_{anti} \wedge \Gamma \wedge EC \models \Gamma'$$

if and only if

$$Encode[DEC; \Sigma_{init}; \Sigma_{term}; \Sigma_{rel}; \Delta; \Psi; \Pi_{traj}; \Pi_{anti}; \Gamma] \models \Gamma'.$$

PROOF. This follows from Theorem 4.24, since none of the atoms of Γ' are intermediate. ■

There are two ways to determine whether the encoding entails Γ' . (1) We may run a complete satisfiability solver on our encoding and the negation of $B(\Gamma')$. The encoding entails Γ' iff the solver does not find any satisfying truth assignments. (2) We may run a complete satisfiability solver on our encoding, producing a set of satisfying truth assignments. The encoding entails Γ' iff for every satisfying truth assignment, for every conjunct λ of Γ' , $B(\lambda)$ is assigned to T . This method has the benefit of filling in additional information (model finding).

4.7 Abduction

We may also use our encoding method to perform event calculus abduction [36, 4, 42]. We start with some definitions:

DEFINITION 4.26

A *goal* is a state description.

DEFINITION 4.27

Let Σ_{init} , Σ_{term} , Σ_{rel} , Ψ , Π_{traj} , Π_{anti} , and Γ be as defined above. Let Γ' be a goal, Δ_{occ} be an event occurrence description, and Δ_{trig} be a trigger description. Δ_{occ} is a *plan* for Γ' if and only if

$$\begin{aligned} &CIRC[\Sigma_{init} \wedge \Sigma_{term} \wedge \Sigma_{rel}; \textit{Initiates}, \textit{Terminates}, \textit{Releases}] \wedge \\ &CIRC[\Delta_{trig} \wedge \Delta_{occ}; \textit{Happens}] \wedge \Psi \wedge \Pi_{traj} \wedge \Pi_{anti} \wedge \Gamma \wedge EC \models \Gamma'. \end{aligned}$$

THEOREM 4.28

Restrict the logic to a finite universe as specified above. Let Σ_{init} , Σ_{term} , Σ_{rel} , Ψ , Π_{traj} , Π_{anti} , and Γ be as defined above. Let Γ' be a goal and Δ_{trig} be a trigger description. The following algorithm finds all plans for Γ' :

1. Create an empty list of plans.
2. For each satisfying truth assignment $\mathcal{T}$ of $Encode[DEC; \Sigma_{init}; \Sigma_{term}; \Sigma_{rel}; \Delta_{trig}; \Psi; \Pi_{traj}; \Pi_{anti}; \Gamma \wedge \Gamma']$.
 - (a) Let Δ_{occ} be an event occurrence description constructed from the set of all ground atoms λ of the form $Happens(\alpha, \tau)$ such that $\mathcal{T}(\lambda) = T$.
 - (b) If $Encode[DEC; \Sigma_{init}; \Sigma_{term}; \Sigma_{rel}; \Delta_{trig} \wedge \Delta_{occ}; \Psi; \Pi_{traj}; \Pi_{anti}; \Gamma] \models \Gamma'$, then add Δ_{occ} to the list of plans.

PROOF. This follows from Theorem 4.24, since none of the atoms of Γ' are intermediate and for every Δ_{occ} none of the atoms of Δ_{occ} are intermediate. ■

Thus we find all plans for Γ' as follows. We first run a complete satisfiability solver on our encoding augmented with Γ' . For each satisfying truth assignment, we form a candidate plan Δ_{occ} consisting of a set of *Happens* atoms. We then run the complete solver on the encoding augmented with Δ_{occ} and the negation of Γ' . Δ_{occ} is a plan for Γ' iff the solver does not find any satisfying truth assignments.

5 Evaluation

In this section, we evaluate our method. First, we compare the method to that of Shanahan and Witkowski [44]. Second, we evaluate our method and that of Shanahan and Witkowski on a set of

14 event calculus problems. Third, we compare the performance of our method to that of the causal calculator [10], a tool for reasoning about action and change using the language of causal theories [22].

5.1 Comparison with Shanahan and Witkowski's method

Table 1 compares the coverage of our method for satisfiability-based event calculus reasoning and that of Shanahan and Witkowski [44]. Causal constraints deal with the instantaneous propagation of interacting indirect effects, as in idealized electronic circuits [45]. Our method handles problems involving causal constraints provided that four new predicates and four new axioms are added to the formulation of the event calculus, as described by Shanahan [41].

Our method handles problems involving concurrent events with cumulative or canceling effects [39, pp. 301–304] provided that the problems are formulated in the style of Miller and Shanahan [26, pp. 460–461]. Since our method supports effect axioms with conditions, fluents that are released from the commonsense law of inertia, and incompletely specified initial situations, our method supports the use of determining fluents to enable events with nondeterministic effects [40].

Neither method supports disjunctive event axioms [39, pp. 342–345] and neither method supports compound event axioms [40]. The circumscription of *Happens* in such axioms cannot be computed using Theorem 4.14 since *Happens* is mentioned in $\Gamma(x_1, \dots, x_n)$.

Event precondition axioms are formulas of the form

$$Happens(a, t) \Rightarrow \text{condition over } t.$$

Event precondition axioms may be used in our method by incorporating them into Ψ , with the caveat that if the initial situation is not completely specified, then *Happens*(*a*, *t*) becomes a plan for the condition over *t* (see the discussion of Miller and Shanahan [26, p. 465]). Fluent precondition axioms are the same as effect axioms with conditions.

5.2 Evaluation on event calculus benchmark problems

We have implemented our method within a tool for satisfiability-based reasoning in the event calculus [30]. The entire implementation consists of about 10,000 lines of code, with the critical portions (about 4,000 lines) written in the C language for maximum runtime efficiency. The tool invokes the Relsat 2.0 complete satisfiability solver [2].

We conducted an evaluation of our encoding method and the method of Shanahan and Witkowski [44] on a set of 14 benchmark reasoning problems that have been described for the event calculus by Shanahan [39, 40]. Table 2 provides the results of the evaluation. For each problem, the presence of a mark indicates that the method is able to handle, and in the case of our encoding method was successfully able to solve, the problem. Our encoding method was able to solve 11 of the 14 problems; the previous method handles only one due to its limited coverage of the event calculus. Our encoding method was not able to handle the problems involving disjunctive event axioms, compound events, and effect constraints because it does not support those features of the event calculus. In order to run using our method, SUPERMARKETTROLLEY was reformulated using the method of Miller and Shanahan [26, pp. 460–461]. The problems were solved in less than one second.

TABLE 1. Coverage of event calculus satisfiability encoding methods (S&W = Shanahan and Witkowski [44])

Feature of the event calculus	S&W	Us
causal constraints		✓
compound event axioms		
concurrent events		✓
continuous time		
determining fluents for nondeterminism		✓
discrete time	✓	✓
disjunctive event axioms		
effect axioms without conditions	✓	✓
effect axioms with conditions		✓
effect constraints		
event precondition axioms	✓	✓
fluent precondition axioms		✓
gradual change axioms		✓
incompletely specified initial situations		✓
release from the commonsense law of inertia		✓
state constraints		✓
three-argument <i>Happens</i>		
trigger axioms		✓

5.3 Evaluation on zoo world problems

We conducted a performance comparison of our tool and the causal calculator (CCALC) [10]. We performed the comparison using a collection of zoo world problems proposed by Erik Sandewall and formalized in the language of CCALC [1]. We translated the CCALC formalization of the zoo world into the event calculus, and used our tool to solve the same set of zoo world test problems solved by CCALC. The CCALC formalization consists of 62 causal laws and our event calculus translation consists of 78 axioms.

Table 3 provides the results of the comparison. The performance of our tool on the test problems is comparable to that of CCALC. The columns of this table are: (1) the number of variables in the satisfiability problem, (2) the number of clauses in the problem, (3) the time taken to encode the problem, and (4) the time taken by the Relsat 2.0 satisfiability solver to solve the problem. Encoding and solution times are elapsed wall-clock time in seconds on a machine with a 1.8 GHz Intel Pentium 4 processor and 512 megabytes of RAM. The CCALC encoding time is the sum of the grounding and completion times. The CCALC runs were performed with CCALC 2.0 beta 8.3 and SWI-Prolog 5.0.10.

6 Conclusion

We have described a method for encoding reasoning problems of a discrete version of the classical logic event calculus in propositional conjunctive normal form, enabling the problems to be solved efficiently by off-the-shelf complete satisfiability solvers. The method has been implemented as a

TABLE 2. Event calculus benchmark problems solved by satisfiability encoding methods (S&W = Shanahan and Witkowski [44])

Problem	Reasoning type	Notable features	S&W	Us
BUSRIDE [39, pp. 359–361]	abduction	abduction disjunctive event axiom state constraint		
CHESSBOARD [40]	model finding	determining fluent state constraint		✓
COINTOSS [40]	model finding	determining fluent		✓
COMMUTER [40]	deduction	compound event		
DEADORALIVE [39, p. 324]	deduction	state constraint		✓
HAPPY [40]	deduction	state constraint		✓
KITCHENSINK [39, pp. 326–329]	deduction	release from inertia gradual change trigger axiom		✓
RUSSIANTURKEY [40]	model finding	release from inertia		✓
STOLENCAR [39, p. 359]	abduction	abduction	✓	✓
STUFFYROOM [39, pp. 288–289]	deduction	state constraint		✓
SUPERMARKETTROLLEY [39, pp. 302–304]	deduction	concurrent event		✓
THIELSCHERCIRCUIT [40]	deduction	causal constraint		✓
WALKINGTURKEY [40]	deduction	effect constraint		
YALE [39, pp. 322–323]	deduction	effect axiom with condition		✓

tool for event calculus reasoning about action and change. The tool successfully solves 11 of 14 benchmark commonsense reasoning problems described for the event calculus and has performance comparable to the causal calculator in the zoo world domain.

Several tools now exist for reasoning about action and change. The most similar ones to ours are the causal calculator, VITAL [7, 19], and $\mathcal{E}$ -RES [14, 15]. $\mathcal{E}$ -RES is inspired by the event calculus and a mapping between the event calculus and $\mathcal{E}$ has been described [26].

The advantages of our tool are its efficiency and ease of use due to the familiarity of the classical logic event calculus, which is a straightforward extension of first-order logic. The disadvantages of our tool are that it does not support compound events, continuous time, disjunctive event axioms, and effect constraints.

TABLE 3. Comparison with CCalc (c) on zoo problems (wall times in seconds)

Problem	Variables	Clauses	Encode	Solve
ZOO TEST1	3,609	23,355	29.43	0.92
c	2,693	31,881	14.83	12.24
ZOO TEST2	1,072	5,587	2.44	0.14
c	1,116	8,870	3.67	0.17
ZOO TEST3	1,989	12,370	10.70	0.44
c	1,726	17,895	15.08	0.61
ZOO TEST4.1	3,609	23,352	29.38	1.06
c	2,770	32,193	14.98	4.14
ZOO TEST4.2	4,443	28,993	43.81	1.67
c	3,292	39,354	14.90	11.54
ZOO TEST5.1	1,812	12,692	23.97	0.77
c	1,483	18,120	44.69	5.04
ZOO TEST5.2	1,812	12,694	23.90	0.68
c	1,483	18,122	44.69	5.06
ZOO TEST6	1,179	6,877	4.88	0.21
c	1,127	10,428	14.85	1.10

We have begun to use the tool to develop applications. We are developing a commonsense knowledge base, or library of reusable event calculus representations of commonsense knowledge, for use with the tool. We are applying the tool and the commonsense knowledge base to the problem of making inferences and filling in missing information in story understanding [25, 28, 29]. Our approach consists of (1) using a semantic parser to build a semantic parse of a story text, (2) feeding the semantic parse to the tool, which produces one or more models of the story, and (3) using the models to answer questions about the story.

In addition to building applications and extending the method to support more features of the event calculus, another area for future work is to parallelize the method to run on a computing grid so that much larger problems can be solved even more quickly.

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Received 17 June 2003