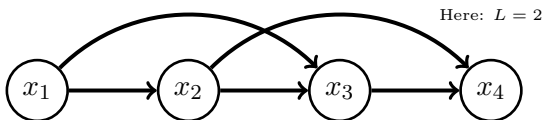


These notes are intended to give a summary of relevant concepts from the lectures which are helpful to complete the exercises. It is not intended to cover the lectures thoroughly. Learning this content is not a replacement for working through the lecture material and the exercises.

Markov chains — A distribution factorised such that each variable x_i depends on L previous (contiguous) nodes $\{x_{i-L}, \dots, x_{i-1}\}$

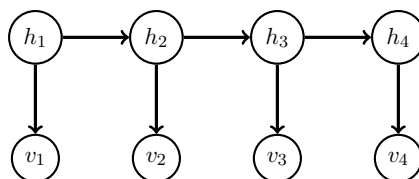
$$p(x_1, \dots, x_d) = \prod_{i=1}^d p(x_i | x_{i-L}, \dots, x_{i-1})$$


For $L = 1$ we have a 1st-order Markov chain, $p(x_1, \dots, x_d) = \prod_{i=1}^d p(x_i | x_{i-1})$

The transition distribution $p(x_i, | x_{i-1})$ gives the probability of transitioning to different states. However, if this does not depend on i , then the Markov chain is said to be homogeneous.

Hidden Markov model (HMM) — A 1st-order Markov chain on latent variables h_i (hid-dens), with an additional set of visible variables v_i that represent observations. An emission distribution $p(v_i | h_i)$ gives the probabilities of the observations v_i (visibles) taking different values, if the observations are real-valued then $p(v_i | h_i)$ will be a probability density function.

$$p(h_{1:d}, v_{1:d}) = p(v_1 | h_1)p(h_1) \prod_{i=2}^d p(v_i | h_i)p(h_i | h_{i-1})$$



An HMM is said to be stationary if its transition and emission distributions don't depend on i .

Alpha-recursion A recursive process that propagates information forwards, from h_{s-1} to h_s

$$\alpha(h_s) = p(v_s | h_s) \sum_{h_{s-1}} p(h_s | h_{s-1})\alpha(h_{s-1}) = p(h_s, v_{1:s}) \propto p(h_s | v_{1:s}) \quad (1)$$

$$\alpha(h_1) = p(h_1)p(v_1 | h_1) = p(h_1, v_1) \propto p(h_1 | v_1) \quad (2)$$

Used to compute $p(h_t | v_{1:t})$ (filtering).

Beta-recursion A recursive process that propagates information backwards, from h_s to h_{s-1}

$$\beta(h_{s-1}) = \sum_{h_s} p(v_s | h_s)p(h_s | h_{s-1})\beta(h_s) = p(v_{s:u} | h_{s-1}) \quad (3)$$

$$\beta(h_u) = 1 \quad (4)$$

Together with the alpha-recursion, used to compute $p(h_t | v_{1:u})$ (smoothing).