

*These notes are intended to give a summary of relevant concepts from the lectures which are helpful to complete the exercises. It is not intended to cover the lectures thoroughly. Learning this content is not a replacement for working through the lecture material and the exercises.*

**Factorisation and independence** — For two non-negative functions  $\phi_A$  and  $\phi_B$ :

$$x \perp\!\!\!\perp y \mid z \Leftrightarrow p(x, y, z) = \frac{1}{Z} \phi_A(x, z) \phi_B(y, z) \quad Z = \int_{x, y, z} \phi_A(x, z) \phi_B(y, z) dx dy dz \quad (1)$$

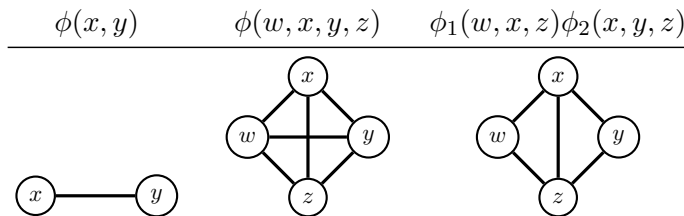
**Gibbs distribution** — A class of pdfs/pmfs that factorise into factors of sets of variables.

$$p(x_1, \dots, x_d) = \frac{1}{Z} \prod_c \phi_c(\mathcal{X}_c) \quad \mathcal{X}_c \subseteq \{x_1, \dots, x_d\} \quad (2)$$

**Energy based model** — A model where energy functions are used in place of factors, this is useful as we can work with sums of energies which are in log-space.

$$p(x_1, \dots, x_d) = \frac{1}{Z} \exp \left[ - \sum_c E_c(\mathcal{X}_c) \right] \quad E_c(\mathcal{X}_c) = - \log(\phi_c(\mathcal{X}_c)) \quad (3)$$

**Undirected graphical model** — All variables  $x_i$  are associated with one node, each set of variables  $\mathcal{X}_c$  for a factor  $\phi_c$  are maximally connected with edges.



**Independence and separation in undirected graphical models** — Two sets of variables  $X$  and  $Y$  are separated by  $Z$  if, after removing the  $Z$ -nodes, there is no path between any variable  $x \in X$  and  $y \in Y$ . Implies conditional independence for all distributions  $p$  that factorise over the graph. They satisfy the global Markov property relative to the undirected graph.

**Local Markov property** — A distribution  $p(\mathbf{x})$  satisfies the local Markov property relative to an undirected graph if

$$x \perp\!\!\!\perp X \setminus (x \cup \text{ne}(x)) \mid \text{ne}(x) \quad \forall x \in X \quad (4)$$

holds for  $p$ .

**Pairwise Markov property** — A distribution  $p(\mathbf{x})$  satisfies the pairwise Markov property relative to an undirected graph if

$$x_i \perp\!\!\!\perp x_j \mid X \setminus (x_i, x_j) \quad \forall x_i, x_j \in X \quad \text{s.t. } x_i \notin \text{ne}(x_j) \quad (5)$$

holds for  $p$ .