THE UNIVERSITY of EDINBURGH **informatics**

Probabilistic Modelling and Reasoning
Exercises 1 — Notes

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These notes are intended to give a summary of relevant concepts from the lectures which are helpful to complete the exercises. It is not intended to cover the lectures thoroughly. Learning this content is not a replacement for working through the lecture material and the exercises.

Topological ordering $(x_1, , \dots, x_d)$ — For all x_i, x_j connected by a directed edge $x_i \to x_j$, x_i should appear before x_j in the ordering

Ordered Markov property — A distribution $p(x_1, \dots, x_d)$ satisfies the ordered Markov property if $\forall x_i \, \exists \pi_i$ **s.t.** $x_i \perp\!\!\!\perp \text{pre}_i \setminus \pi_i \mid \pi_i$, i.e. $p(x_i|\text{pre}_i) = p(x_i|\pi_i)$, where:

- pre_i is the set of nodes before x_i in a topological ordering
- π_i is a minimal subset of pre_i

For example, in graphs $\pi_i = \text{pa}_i$ (parents of x_i)

DAG connections

Connection	Serial	Diverging	Converging
Graph	$x \rightarrow z \rightarrow y$	$x \leftarrow z \rightarrow y$	$x \rightarrow z \leftarrow y$
$p(x, y)$	$x \not\perp\!\!\!\perp y$ – trail active	$x \not\perp\!\!\!\perp y$ – trail active	$x \perp\!\!\!\perp y$ – trail blocked
$p(x, y\|z)$	$x \perp\!\!\!\perp y \mid z$ – trail blocked	$x \perp\!\!\!\perp y \mid z$ – trail blocked	$x \not\perp\!\!\!\perp y \mid z$ – trail active
			$x \not\perp\!\!\!\perp y \mid desc(z)$ – trail active

D-separation — $X \perp\!\!\!\perp Y \mid Z$ if every trail from $\forall x \in X$ to $\forall y \in Y$ is blocked by Z

Note, d-separation is not complete – it may not capture all independencies

Global directed Markov property — A distribution $p(x_1, \dots, x_d)$ satisfies the global directed Markov property if all independencies asserted by d-separation hold for $p(x_1, \dots, x_d)$.

Local directed Markov property — A distribution $p(x_1, \dots, x_d)$ satisfies the local directed Markov property if $x_i \perp\!\!\!\perp \text{nondesc}(x_i) \setminus \text{pa}_i \mid \text{pa}_i$ holds for all i, i.e. $p(x_i|\text{nondesc}(x_i)) = p(x_i|\text{pa}_i)$ for all i.

Markov blanket MB(x_i) — The minimal set of variables $\text{MB}(x_i)$ that makes x_i independent from all other variables.

$$x_i \perp\!\!\!\perp X \setminus \{x_i \cup \text{MB}(x_i)\} \mid \text{MB}(x_i) \tag{1}$$

$$\text{MB}(x_i) = parents(x_i) \cup children(x_i) \cup \{parents(children(x_i)) \setminus x_i\} \tag{2}$$