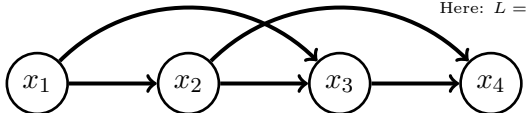


These notes are intended to give a summary of relevant concepts from the lectures which are helpful to complete the exercises. It is not intended to cover the lectures thoroughly. Learning this content is not a replacement for working through the lecture material and the exercises.

**Markov chains** — A distribution factorised such that each variable  $x_i$  depends on  $L$  previous (contiguous) nodes  $\{x_{i-L}, \dots, x_{i-1}\}$

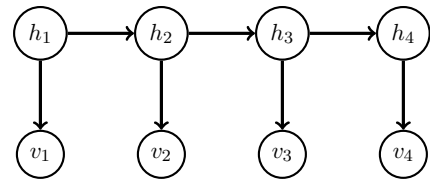
$$p(x_1, \dots, x_d) = \prod_{i=1}^d p(x_i | x_{i-L}, \dots, x_{i-1})$$


For  $L = 1$  we have a 1<sup>st</sup>-order Markov chain,  $p(x_1, \dots, x_d) = \prod_{i=1}^d p(x_i | x_{i-1})$

The transition distribution  $p(x_i, | x_{i-1})$  gives the probability of transitioning to different states. However, if this does not depend on  $i$ , then the Markov chain is said to be homogeneous.

**Hidden Markov model (HMM)** — A 1<sup>st</sup>-order Markov chain on latent variables  $h_i$  (hid-dens), with an additional set of visible variables  $v_i$  that represent observations. An emission distribution  $p(v_i | h_i)$  gives the probabilities of the observations  $v_i$  (visibles) taking different values, if the observations are real-valued then  $p(v_i | h_i)$  will be a probability density function.

$$p(h_{1:d}, v_{1:d}) = p(v_1 | h_1)p(h_1) \prod_{i=2}^d p(v_i | h_i)p(h_i | h_{i-1})$$



An HMM is said to be stationary if its transition and emission distributions don't depend on  $i$ .

**Alpha-recursion** A recursive process that propagates information forwards, from  $h_{s-1}$  to  $h_s$

$$\alpha(h_s) = p(v_s | h_s) \sum_{h_{s-1}} p(h_s | h_{s-1})\alpha(h_{s-1}) = p(h_s, v_{1:s}) \propto p(h_s | v_{1:s}) \quad (1)$$

$$\alpha(h_1) = p(h_1)p(v_1 | h_1) = p(h_1, v_1) \propto p(h_1 | v_1) \quad (2)$$

Used to compute  $p(h_t | v_{1:t})$  (filtering).

**Beta-recursion** A recursive process that propagates information backwards, from  $h_s$  to  $h_{s-1}$

$$\beta(h_{s-1}) = \sum_{h_s} p(v_s | h_s)p(h_s | h_{s-1})\beta(h_s) = p(v_{s:u} | h_{s-1}) \quad (3)$$

$$\beta(h_u) = 1 \quad (4)$$

Together with the alpha-recursion, used to compute  $p(h_t | v_{1:u})$  (smoothing).