Probabilistic Modelling and Reasoning
Exercises 2 — Notes

Spring 2022
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These notes are intended to give a summary of relevant concepts from the lectures which are helpful to complete the exercises. It is not intended to cover the lectures thoroughly. Learning this content is not a replacement for working through the lecture material and the exercises.

Factorisation and independence— For two non-negative functions ϕ_A and ϕ_B:

$$x \perp\!\!\!\perp y \mid z \Leftrightarrow p(x,y,z) = \frac{1}{Z}\phi_A(x,z)\phi_B(y,z) \qquad Z = \int_{x,y,z} \phi_A(x,z)\phi_B(y,z)dxdydz \tag{1}$$

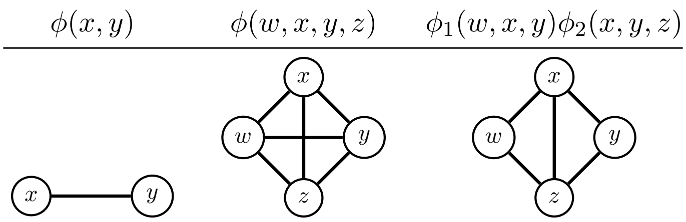
Gibbs distribution — A class of pdfs/pmfs that factorise into factors of sets of variables.

$$p(x_1,\dots,x_d) = \frac{1}{Z}\prod_c \phi_c(\mathcal{X}_c) \qquad \mathcal{X}_c \subseteq \{x_1,\dots,x_d\} \tag{2}$$

Energy based model — A model where energy functions are used in place of factors, this is useful as we can work with sums of energies which are in log-space.

$$p(x_1,\dots,x_d) = \frac{1}{Z}\exp\left[-\sum_c E_c(\mathcal{X}_c)\right] \qquad E_c(\mathcal{X}_c) = -\log\left(\phi_c(\mathcal{X}_c)\right) \tag{3}$$

Undirected graphical model — All variables x_i are associated with one node, each set of variables $\mathcal{X}_c$ for a factor ϕ_c are maximally connected with edges.



Independence and separation in undirected graphical models — Two sets of variables X and Y are separated by Z if, after removing the Z-nodes, there is no path between any variable $x \in X$ and $y \in Y$. Implies conditional independence for all distributions p that factorise over the graph. They satisfy the global Markov property relative to the undirected graph.

Local Markov property — A distribution $p(\mathbf{x})$ satisfies the local Markov property relative to an undirected graph if

$$x \perp\!\!\!\perp X \setminus (x \cup \text{ne}(x)) \mid \text{ne}(x) \quad \forall x \in X \tag{4}$$

holds for p.

Pairwise Markov property — A distribution $p(\mathbf{x})$ satisfies the pairwise Markov property relative to an undirected graph if

$$x_i \perp\!\!\!\perp x_j \mid X \setminus (x_i, x_j) \quad \forall x_i, x_j \in X \quad \textbf{s.t. } x_i \notin \text{ne}(x_j) \tag{5}$$

holds for p.