Probabilistic Modelling and Reasoning
— Course Recap —

Michael U. Gutmann

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School of Informatics, The University of Edinburgh

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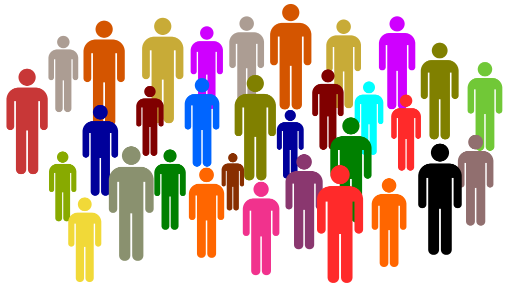
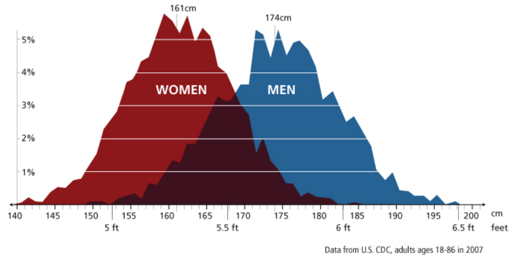
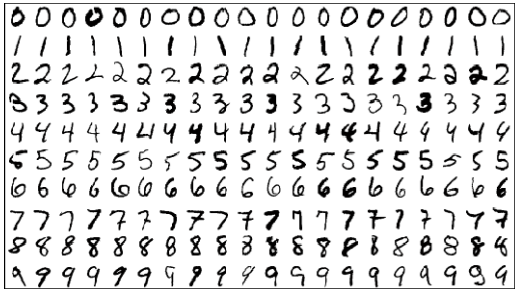
- We started the course with the basic observation that variability is part of nature.
- Variability leads to uncertainty when analysing or drawing conclusions from data.
- This motivates taking a probabilistic approach to modelling and reasoning.







Course recap

- Probabilistic modelling:
  - Identify the quantities that relate to the aspects of reality that you wish to capture with your model.
  - Consider them to be random variables, e.g. $\mathbf{x}, \mathbf{y}, \mathbf{z}$, with a joint pdf (pmf) $p(\mathbf{x}, \mathbf{y}, \mathbf{z})$.
- Probabilistic reasoning:
  - Assume you know that $\mathbf{y} \in \mathcal{E}$ (measurement, evidence)
  - Probabilistic reasoning about $\mathbf{x}$ then consists in computing

    $$p(\mathbf{x}|\mathbf{y} \in \mathcal{E})$$

    or related quantities like its maximiser or posterior expectations.

Course recap

- Principled framework but naive implementation quickly runs into computational issues.
- For example,

$$p(\mathbf{x}|\mathbf{y}_o) = \frac{\sum_{\mathbf{z}} p(\mathbf{x}, \mathbf{y}_o, \mathbf{z})}{\sum_{\mathbf{x},\mathbf{z}} p(\mathbf{x}, \mathbf{y}_o, \mathbf{z})}$$

  cannot be computed if $\mathbf{x}, \mathbf{y}, \mathbf{z}$ each are $d = 500$ dimensional, and if each element of the vectors can take $K = 10$ values.
- The course had four main topics.

  Topic 1: Representation We discussed reasonably weak assumptions to efficiently represent $p(\mathbf{x}, \mathbf{y}, \mathbf{z})$.

  - Two classes of assumptions: independence and parametric assumptions.
  - Directed and undirected graphical models
  - Independencies encoded by the graphs
  - Expressive power of the graphical models
  - Factor graphs

Course recap

Topic 2: Exact inference We have seen that the independence assumptions allow us, under certain conditions, to efficiently compute the posterior probability or derived quantities.

- Variable elimination for general factor graphs
- Inference when the model can be represented as a factor tree (message passing algorithms)
- Application to Hidden Markov models

Topic 3: Learning We discussed methods to learn probabilistic models from data by introducing parameters and learning them from data.

- Learning by Bayesian inference
- Learning by parameter estimation
- Likelihood function
- Factor analysis and independent component analysis

Course recap

Topic 4: Approximate inference and learning We discussed that intractable integrals may hinder inference and likelihood-based learning.

- Intractable integrals may be due to unobserved variables or intractable partition functions.
- Changing perspective sometimes allows us to deal with intractability: Score matching as alternative criteria for learning when the partition function is intractable.
- Monte Carlo integration and sampling
- Variational approaches to learning and inference
- Learning of deep latent variable models and variational autoencoders
- EM algorithm and its application to hidden Markov models