These notes are intended to give a summary of relevant concepts from the lectures which are helpful to complete the tutorial sheet. It is not intended to cover the lectures thoroughly. Learning this content is not a replacement for working through the lecture material and the tutorial sheet.

Inverse transform sampling — Given we have a cdf $F_x(\alpha)$ which is invertible, we can generate samples $x^{(i)}$ from our distribution $p_x(x)$ using uniform samples $y^{(i)} \sim \mathcal{U}(0, 1)$,

$$F_x(\alpha) = \mathbb{P}(x \leq \alpha) = \int_{-\infty}^{\alpha} p_x(y) \mathrm{d}y \tag{1}$$

Using the inverse cdf $F_x^{-1}(y)$, a sample $x^{(i)} \sim p_x(x)$ can be generated using

$$x^{(i)} = F_x^{-1}(y^{(i)}) \qquad\qquad y^{(i)} \sim \mathcal{U}(0, 1) \tag{2}$$

Gibbs sampling — Given a multivariate pdf $p(\mathbf{x})$ and an initial state $\mathbf{x}^{(1)} = (x_1^{(1)}, \ldots, x_d^{(1)})$, we obtain multivariate samples $\mathbf{x}^{(k)}$ by sampling from a univariate distribution $p(x_i \mid \mathbf{x}_{\setminus i})$, and updating individual variables many times.

$$\mathbf{x}^{(2)} = (x_1^{(1)}, \ldots, x_{i-1}^{(1)}, x_i^{(2)}, x_{i+1}^{(1)}, \ldots, x_d^{(1)}) \qquad\qquad i \sim \{0, \ldots, d\} \tag{3}$$

$$\vdots$$

$$\mathbf{x}^{(n)} = (x_1^{(n-1)}, \ldots, x_{j-1}^{(n-1)}, x_j^{(n)}, x_{j+1}^{(n-1)}, \ldots, x_d^{(n-1)}) \qquad\qquad j \sim \{0, \ldots, d\} \tag{4}$$

In the multidimensional space of $\mathbf{x}$, the iterative Gibbs sampling process will appear as a path in orthogonal axes. Like other MCMC methods, Gibbs sampling typically exhibits a warm-up period, where the samples are not representative of the distribution $p(\mathbf{x})$ and the samples are not independent from each other. For multi-modal distributions Gibbs sampling may fail to sample from one or more modes, especially if the modes do not overlap when projected onto any of axes.