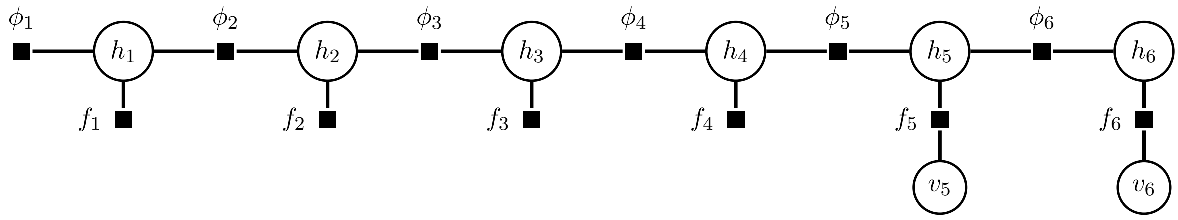
The purpose of this additional sheet is to provide more practice and exam preparation material. N.B. The tutors are not required to work through this material in the tutorial.

Exercise 1. ***Hidden Markov model – beta-recursion***

We consider the following factor graph from the lecture on hidden Markov models.



The factor graph corresponds to the conditional pmf

$$p(h_1, \ldots, h_6, v_5, v_6 \mid v_{1:4})$$

and the factors are defined as

$$f_t(h_t) = p(v_t|h_t) \ \ (t \leq 4) \qquad f_t(v_t, h_t) = p(v_t|h_t) \ \ (t > 4) \tag{1}$$

$$\phi_1(h_1) = p(h_1) \qquad \phi_t(h_t, h_{t-1}) = p(h_t|h_{t-1}) \ \ (t > 1) \tag{2}$$

We define $\beta(h_s) = \mu_{\phi_{s+1} \to h_s}(h_s)$, which is the message from a factor node "back" to a variable node.

(a) Show that $\beta(h_4) = \mu_{\phi_5 \to h_4}(h_4) = 1$.

(b) Use sum-product message passing to show that the beta-recursion holds

$$\beta(h_4) = 1 \tag{3}$$

$$\beta(h_s) = \sum_{h_{s+1}} p(h_{s+1}|h_s) p(v_{s+1}|h_{s+1}) \beta(h_{s+1}) \quad (s < 4) \tag{4}$$