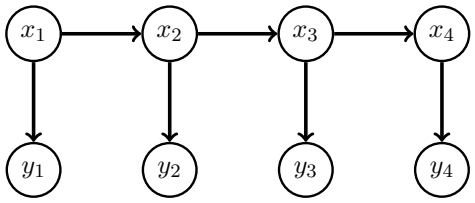
The purpose of this tutorial sheet is to help you better understand the lecture material. Start early and do as many as you have time for. Even if you are unable to make much progress, you should still attend your tutorial.

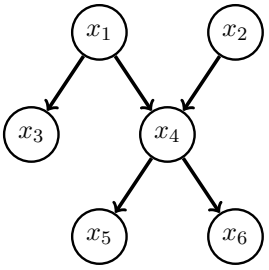
Exercise 1. ***Conversion to factor graphs***

(a) Draw an undirected graph and an undirected factor graph for $p(x_1, x_2, x_3) = p(x_1)p(x_2)p(x_3|x_1, x_2)$

(b) Draw an undirected factor graph for the directed graphical model defined by the graph below.

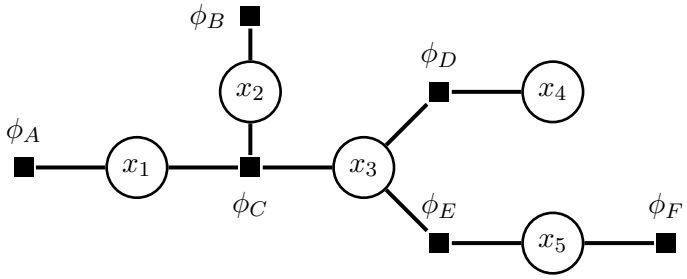


(c) Draw the moralised graph and an undirected factor graph for directed graphical models defined by the graph below (this kind of graph is called a polytree: there are no loops but a node may have more than one parent).



Exercise 2. ***Sum-product message passing***

We here re-consider the factor tree from the lecture on exact inference.



Let all variables be binary, $x_i \in \{0, 1\}$, and the factors be defined as follows:

x_1	ϕ_A
0	2
1	4

x_2	ϕ_B
0	4
1	4

x_1	x_2	x_3	ϕ_C
0	0	0	4
1	0	0	2
0	1	0	2
1	1	0	6
0	0	1	2
1	0	1	6
0	1	1	6
1	1	1	4

x_3	x_4	ϕ_D
0	0	8
1	0	2
0	1	2
1	1	6

x_3	x_5	ϕ_E
0	0	3
1	0	6
0	1	6
1	1	3

x_5	ϕ_F
0	1
1	8

(a) Mark the graph with arrows indicating all messages that need to be computed for the computation of $p(x_1)$.

(b) Compute the messages that you have identified.

Assuming that the computation of the messages is scheduled according to a common clock, group the messages together so that all messages in the same group can be computed in parallel during a clock cycle.

(c) What is $p(x_1 = 1)$?

(d) Draw the factor graph corresponding to $p(x_1, x_3, x_4, x_5|x_2 = 1)$ and provide the numerical values for all factors.

(e) Compute $p(x_1 = 1|x_2 = 1)$, re-using messages that you have already computed for the evaluation of $p(x_1 = 1)$.