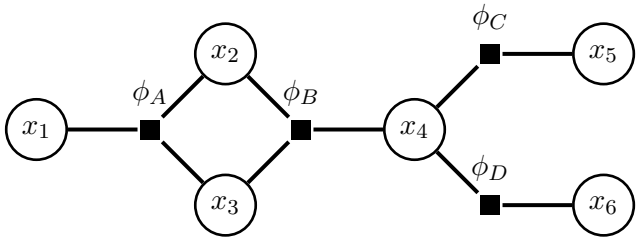
The purpose of this additional sheet is to provide more practice and exam preparation material. N.B. The tutors are not required to work through this material in the tutorial.

Exercise 1. ***Choice of elimination order in factor graphs***

Consider the following factor graph, which contains a loop:



Let all variables be binary, $x_i \in \{0, 1\}$, and the factors be defined as follows:

x_1	x_2	x_3	ϕ_A
0	0	0	4
1	0	0	2
0	1	0	2
1	1	0	6
0	0	1	2
1	0	1	6
0	1	1	6
1	1	1	4

x_2	x_3	x_4	ϕ_B
0	0	0	2
1	0	0	2
0	1	0	4
1	1	0	2
0	0	1	6
1	0	1	8
0	1	1	4
1	1	1	2

x_4	x_5	ϕ_C
0	0	8
1	0	2
0	1	2
1	1	6

x_4	x_6	ϕ_D
0	0	3
1	0	6
0	1	6
1	1	3

(a) Draw the factor graph corresponding to $p(x_2, x_3, x_4, x_5 \mid x_1 = 0, x_6 = 1)$ and give the tables defining the new factors $\phi_A^{x_1=0}(x_2, x_3)$ and $\phi_D^{x_6=1}(x_4)$ that you obtain.

(b) Find $p(x_2 \mid x_1 = 0, x_6 = 1)$ using the elimination ordering (x_4, x_5, x_3):

(i) Draw the graph for $p(x_2, x_3, x_5 \mid x_1 = 0, x_6 = 1)$ by marginalising x_4
Compute the table for the new factor $\tilde{\phi}_4(x_2, x_3, x_5)$

(ii) Draw the graph for $p(x_2, x_3 \mid x_1 = 0, x_6 = 1)$ by marginalising x_5
Compute the table for the new factor $\tilde{\phi}_{45}(x_2, x_3)$

(iii) Draw the graph for $p(x_2 \mid x_1 = 0, x_6 = 1)$ by marginalising x_3
Compute the table for the new factor $\tilde{\phi}_{453}(x_2)$

(c) Note that the previous variable ordering involved computing a new factor $\tilde{\phi}_4$ that depends on three variables x_2, x_3, and x_5, this involved computing 2^3 numbers (i.e. the rows in the table for $\tilde{\phi}_4$). Instead, now find $p(x_2 \mid x_1 = 0, x_6 = 1)$ using the elimination ordering (x_5, x_4, x_3),

(i) Draw the graph for $p(x_2, x_3, x_4, \mid x_1 = 0, x_6 = 1)$ by marginalising x_5
Compute the table for the new factor $\tilde{\phi}_5(x_4)$

(ii) Draw the graph for $p(x_2, x_3 \mid x_1 = 0, x_6 = 1)$ by marginalising x_4
Compute the table for the new factor $\tilde{\phi}_{54}(x_2, x_3)$

(iii) Draw the graph for $p(x_2 \mid x_1 = 0, x_6 = 1)$ by marginalising x_3
Compute the table for the new factor $\tilde{\phi}_{543}(x_2)$