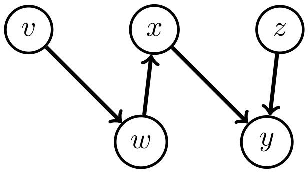
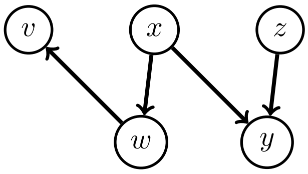
The purpose of this tutorial sheet is to help you better understand the lecture material. Start early and do as many as you have time for. Even if you are unable to make much progress, you should still attend your tutorial.

Exercise 1. ***I-maps***

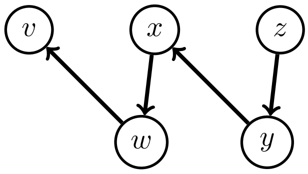
(a) Which of three graphs represent the same set of independencies? Explain.



Graph 1



Graph 2



Graph 3

(b) Assume that the graph G in Figure 1 is a perfect I-map for $p(a, z, q, e, h)$. Determine the minimal directed I-map using the ordering (e, h, q, z, a). Is the obtained graph I-equivalent to G?

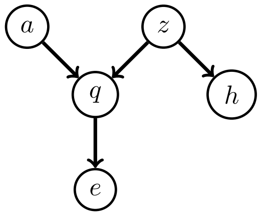


Figure 1: Perfect I-map G for Exercise 1, question (b).

(c) For the collection of random variables (a, z, h, q, e) you are given the following Markov blankets for each variable:

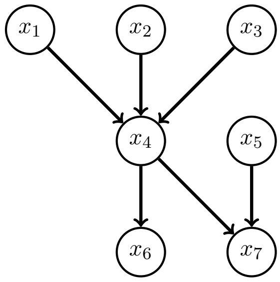
- MB(a) = {q,z}
- MB(z) = {a,q,h}
- MB(h) = {z}
- MB(q) = {a,z,e}
- MB(e) = {q}

(i) Draw the undirected minimal I-map.

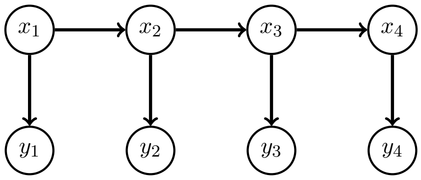
(ii) Indicate a Gibbs distribution that satisfies the independence relations specified by the Markov blankets.

Exercise 2. ***Conversion between graphs***

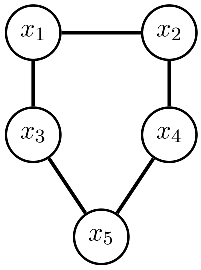
(a) For the DAG G below find the minimal undirected I-map for $\mathcal{I}(G)$.



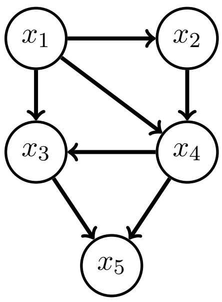
(b) The DAG below is used to define the hidden Markov model. Determine the undirected minimal I-map for the independencies represented by the DAG.



(c) Let $\mathcal{U}$ be the independencies that hold for all distributions that factorise over the graph below. Determine the directed minimal I-map for $\mathcal{U}$ with the variable ordering x_1, x_2, x_3, x_4, x_5.



(d) For the undirected graph from question (c) above, which variable ordering yields the directed minimal I-map below?



Exercise 3. ***Computer exercise***

Distributed separately. Please check the course homepage.