

These notes are intended to give a summary of relevant concepts from the lectures which are helpful to complete the tutorial sheet. It is not intended to cover the lectures thoroughly. Learning this content is not a replacement for working through the lecture material and the tutorial sheet.

Factorisation and independence — For two non-negative functions ϕ_A and ϕ_B :

$$x \perp\!\!\!\perp y \mid z \Leftrightarrow p(x, y, z) = \frac{1}{Z} \phi_A(x, z) \phi_B(y, z) \quad Z = \int_{x, y, z} \phi_A(x, z) \phi_B(y, z) dx dy dz \quad (1)$$

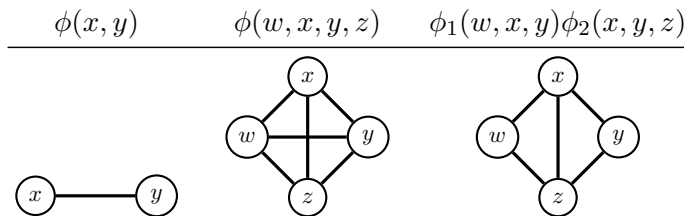
Gibbs distribution — A class of pdfs/pmfs that factorise into factors of sets of variables.

$$p(x_1, \dots, x_d) = \frac{1}{Z} \prod_c \phi_c(\mathcal{X}_c) \quad \mathcal{X}_c \subseteq \{x_1, \dots, x_d\} \quad (2)$$

Energy based model — A model where energy functions are used in place of factors, this is useful as we can work with sums of energies which are in log-space.

$$p(x_1, \dots, x_d) = \frac{1}{Z} \exp \left[- \sum_c E_c(\mathcal{X}_c) \right] \quad E_c(\mathcal{X}_c) = - \log(\phi_c(\mathcal{X}_c)) \quad (3)$$

Undirected graphical model — All variables x_i are associated with one node, each set of variables \mathcal{X}_c for a factor ϕ_c are maximally connected with edges.



Independence and separation in undirected graphical models — Conditioning on a set of variables Z removes the nodes corresponding to those variables from the graph. Two sets of variables X and Y are independent given Z if, after removing the Z -nodes, there is no path between any variable $x \in X$ and $y \in Y$.

Local Markov property — A distribution $p(\mathbf{x})$ satisfies the local Markov property relative to an undirected graph if

$$x \perp\!\!\!\perp X \setminus (x \cup \text{ne}(x)) \mid \text{ne}(x) \quad \forall x \in X \quad (4)$$

holds for p .

Pairwise Markov property — A distribution $p(\mathbf{x})$ satisfies the pairwise Markov property if

$$x_i \perp\!\!\!\perp x_j \mid X \setminus (x_i, x_j) \quad \forall x_i, x_j \in X \quad \text{s.t. } x_i \notin \text{ne}(x_j) \quad (5)$$

holds for p .