These notes are intended to give a summary of relevant concepts from the lectures which are helpful to complete the tutorial sheet. It is not intended to cover the lectures thoroughly. Learning this content is not a replacement for working through the lecture material and the tutorial sheet.

Inverse transform sampling — Given we have a cdf $F_x(\alpha)$ which is invertible, we can generate samples x^i from our distribution $p_x(x)$ using uniform samples $y^i \sim \mathcal{U}(0,1)$,

$$F_x(\alpha) = \mathbb{P}(x \leq \alpha) = \int_{-\infty}^{\alpha} p_x(y)\mathrm{d}y \tag{1}$$

Using the inverse cdf $F_x^{-1}(y)$, a sample $x^i \sim p_x(x)$ can be generated using

$$x^i = F_x^{-1}(y^i) \qquad\qquad y^i \sim \mathcal{U}(0,1) \tag{2}$$

Gibbs sampling — Given a multivariate pdf $p(\mathbf{x})$ and an initial state $\mathbf{x}^1 = (x_1^1, \ldots, x_d^1)$, we obtain multivariate samples $\mathbf{x}^k$ by sampling from a univariate distribution $p(x_i \mid \mathbf{x}_{\backslash i})$, and updating individual variables many times.

$$\mathbf{x}^2 = (x_1^1, \ldots, x_{i-1}^1, x_i^2, x_{i+1}^1, \ldots, x_d^1) \qquad\qquad i \sim \{0, \ldots, d\} \tag{3}$$

$$\vdots$$

$$\mathbf{x}^n = (x_1^{n-1}, \ldots, x_{j-1}^{n-1}, x_j^n, x_{j+1}^{n-1}, \ldots, x_d^{n-1}) \qquad\qquad j \sim \{0, \ldots, d\} \tag{4}$$

In the multidimensional space of $\mathbf{x}$, the iterative Gibbs sampling process will appear as a path in orthogonal axes. Like other MCMC methods, Gibbs sampling typically exhibits a warm-up period, where the samples are not representative of the distribution $p(\mathbf{x})$. For multi-modal distributions Gibbs sampling may fail to sample from one or more modes, especially if the modes do not overlap when projected onto any of axes.