These notes are intended to give a summary of relevant concepts from the lectures which are helpful to complete the tutorial sheet. It is not intended to cover the lectures thoroughly. Learning this content is not a replacement for working through the lecture material and the tutorial sheet.

Note the difference between the notations $p(\mathbf{x};\boldsymbol{\theta})$ and $p(\mathbf{x} \mid \boldsymbol{\theta})$. The former is a pdf/pmf of a random variable $\mathbf{x}$ that is parametrised by a vector of numbers (parameters) $\boldsymbol{\theta}$. The latter is a *conditional* pdf/pmf of a random variable $\mathbf{x}$ given information of another *random variable* $\boldsymbol{\theta}$.

Likelihood $L(\boldsymbol{\theta})$ — The chance that the model generates data like the observed one when using parameter configuration $\boldsymbol{\theta}$. For *iid* data $\mathcal{D} = \{\mathbf{x}_1, \ldots, \mathbf{x}_n\}$, the likelihood of the parameters $\boldsymbol{\theta}$ is

$$L(\boldsymbol{\theta}) = p(\mathcal{D};\boldsymbol{\theta}) = \prod_{i=1}^{n} p(\mathbf{x}_i;\boldsymbol{\theta}) \tag{1}$$

Prior $p(\boldsymbol{\theta})$ — Beliefs about the plausibility of parameter values before we see any data.

Posterior $p(\boldsymbol{\theta} \mid \mathcal{D})$ — Beliefs about the parameters after having seen the data. This is proportional to the likelihood function $L(\boldsymbol{\theta})$ weighted by our prior beliefs about the parameters $p(\boldsymbol{\theta})$

$$p(\boldsymbol{\theta} \mid \mathcal{D}) \propto L(\boldsymbol{\theta}) p(\boldsymbol{\theta}) \tag{2}$$

Parametric statistical model — A set of pdfs/pmfs indexed by parameters $\boldsymbol{\theta}$,

$$\{p(\mathbf{x};\boldsymbol{\theta})\}_{\boldsymbol{\theta}} \tag{3}$$

- **Parameter estimation** Using $\mathcal{D}$ to pick the "best" parameter value $\hat{\boldsymbol{\theta}}$ among the possible $\boldsymbol{\theta}$ – i.e. pick the "best" pdf/pmf $p(\mathbf{x};\hat{\boldsymbol{\theta}})$ from the set of pdfs/pmfs $\{p(\mathbf{x};\boldsymbol{\theta})\}_{\boldsymbol{\theta}}$,

Bayesian model — Considers $p(\mathbf{x};\boldsymbol{\theta})$ to be conditional $p(\mathbf{x} \mid \boldsymbol{\theta})$. Models the probability of the parameters $\boldsymbol{\theta}$, as well as the random variable $\mathbf{x}$

$$p(\mathbf{x}, \boldsymbol{\theta}) = p(\mathbf{x} \mid \boldsymbol{\theta}) p(\boldsymbol{\theta}) \tag{4}$$

- **Bayesian inference** Determine the plausibility of all possible $\boldsymbol{\theta}$ in light of the observed data – i.e. compute the posterior $p(\boldsymbol{\theta} \mid \mathcal{D})$.

Maximum likelihood — The parameters $\hat{\boldsymbol{\theta}}$ that give the largest likelihood (or log-likelihood)

$$\hat{\boldsymbol{\theta}} = \underset{\boldsymbol{\theta}}{\operatorname{argmax}}\, \ell(\boldsymbol{\theta}) = \underset{\boldsymbol{\theta}}{\operatorname{argmax}}\, L(\boldsymbol{\theta}) \tag{5}$$

Sometimes this can be computed directly (as in the tutorials). However, numerical methods are often needed for this optimisation problem, which leads to local optima.