

*The purpose of this additional sheet is to provide more practice and exam preparation material. N.B. The tutors are not required to work through this material in the tutorial.*

**Exercise 1. Maximum likelihood estimation for a Gaussian**

The Gaussian pdf parametrised by mean  $\mu$  and standard deviation  $\sigma$  is given by

$$p(x; \boldsymbol{\theta}) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp \left[ -\frac{(x - \mu)^2}{2\sigma^2} \right], \quad \boldsymbol{\theta} = (\mu, \sigma).$$

- (a) Given iid data  $\mathcal{D} = \{x_1, \dots, x_n\}$ , what is the likelihood function  $L(\boldsymbol{\theta})$  for the Gaussian model?
- (b) What is the log-likelihood function  $\ell(\boldsymbol{\theta})$ ?
- (c) Show that the maximum likelihood estimates for the mean  $\mu$  and standard deviation  $\sigma$  are the sample mean

$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i \tag{1}$$

and the square root of the sample variance

$$S^2 = \frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2. \tag{2}$$

**Exercise 2. Posterior of the mean of a Gaussian with known variance**

Given iid data  $\mathcal{D} = \{x_1, \dots, x_n\}$ , compute  $p(\mu | \mathcal{D}, \sigma^2)$  for the Bayesian model

$$p(x | \mu) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp \left[ -\frac{(x - \mu)^2}{2\sigma^2} \right] \quad p(\mu; \mu_0, \sigma_0^2) = \frac{1}{\sqrt{2\pi\sigma_0^2}} \exp \left[ -\frac{(\mu - \mu_0)^2}{2\sigma_0^2} \right] \tag{3}$$

where  $\sigma^2$  is a fixed known quantity.

*Hint: You will need the result from Tutorial 5 for taking the product of Gaussians.*