These notes are intended to give a summary of relevant concepts from the lectures which are helpful to complete the tutorial sheet. It is not intended to cover the lectures thoroughly. Learning this content is not a replacement for working through the lecture material and the tutorial sheet.

I–map — The set of independencies that a graph G asserts is denoted $\mathcal{I}(G)$. G is said to be an independency map (I–map) for a set of independencies $\mathcal{U}$ if,

$$\mathcal{I}(G) \subseteq \mathcal{U} \tag{1}$$

A complete graph is an I–map since it makes no assertions, this means that an I–map is not necessarily useful.

While the set of "target" independencies $\mathcal{U}$ can be specified in any way, they are often the independencies that a certain distribution p satisfies. This set of independencies is denoted by $\mathcal{I}(p)$.

Minimal I–map — A "sparsified" I-map: A graph G such that if any edge is removed, $\mathcal{I}(G) \not\subseteq \mathcal{U}$.

P–map — G is said to be a perfect map (P–map) for a set of independencies $\mathcal{U}$ if $\mathcal{I}(G) = \mathcal{U}$

Constructing minimal I–maps

Undirected graphs — $\forall x_i \in N$ connect x_i to all variables in $\text{MB}(x_i)$.

Directed graphs — Assume an ordering $\mathbf{x} = (x_1, \ldots, x_d)$, then $\forall x_i \in \mathbf{x}$ set pa_i to π_i, where π_i is a minimal subset of the pre_i such that

$$x_i \perp\!\!\!\perp \{\text{pre}_i \setminus \pi_i\} \mid \pi_i \tag{2}$$

I–equivalence

Undirected graphs — The minimal I-map is unique.

Directed graphs — $\mathcal{I}(G_1)$ and $\mathcal{I}(G_2)$ are I–equivalent *iff* they have the same skeleton and set of immoralities.

- Skeleton – G without arrow heads, i.e. connections irrespective of direction.
- Immoralities – The set of collider nodes without covering edge ("married parents")

Converting I–maps

Directed → undirected graphs — Using the factorisation $p(x_1, \ldots, x_d) = \prod_{i=1}^{d} p(x_i \mid \text{pa}_i)$, form cliques (x_i, pa_i) for all nodes x_i.

Undirected → directed graphs — Read required independencies from the undirected graph (using the local Markov property), build the directed graph using some topological ordering.