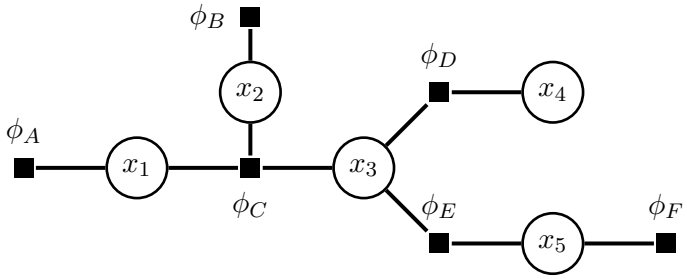
The purpose of the tutorials is twofold: First, they help you better understand the lecture material. Secondly, they provide exam preparation material. You are not expected to complete all questions before the tutorial sessions. Start early and do as many as you have time for.

Exercise 1. ***Sum-product message passing***

We here re-consider the factor tree from the lecture on exact inference.



Let all variables be binary, $x_i \in \{0, 1\}$, and the factors be defined as follows:

x_1	ϕ_A
0	2
1	4

x_2	ϕ_B
0	4
1	4

x_1	x_2	x_3	ϕ_C
0	0	0	4
1	0	0	2
0	1	0	2
1	1	0	6
0	0	1	2
1	0	1	6
0	1	1	6
1	1	1	4

x_3	x_4	ϕ_D
0	0	8
1	0	2
0	1	2
1	1	6

x_3	x_5	ϕ_E
0	0	3
1	0	6
0	1	6
1	1	3

x_5	ϕ_F
0	1
1	8

(a) Mark the graph with arrows indicating all messages that need to be computed for the computation of $p(x_1)$.

(b) Compute the messages that you have identified.

Assuming that the computation of the messages is scheduled according to a common clock, group the messages together so that all messages in the same group can be computed in parallel during a clock cycle.

(c) What is $p(x_1 = 1)$?

(d) Draw the factor graph corresponding to $p(x_1, x_3, x_4, x_5|x_2 = 1)$ and provide the numerical values for all factors.

(e) Compute $p(x_1 = 1|x_2 = 1)$, re-using messages that you have already computed for the evaluation of $p(x_1 = 1)$.