Performance Modelling — Lecture 7
Stochastic Petri Nets

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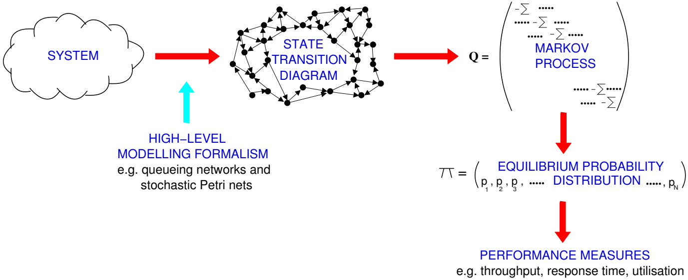
Motivation



As discussed in a previous lecture, high level modelling formalisms are used to make the job of constructing the state transition diagram and/or infinitesimal generator matrix easier. The second such language we consider is Stochastic Petri Nets.

Petri Nets

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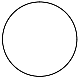
Petri nets were introduced in the 1960s for modelling a variety of concurrent systems, but their use for performance modelling originates from 1980s.

Basic Definitions

The primitives of the notation are the following:



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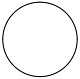

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TRANSITIONS

Transitions are used to describe events that occur in the system; these will usually result in a modification to the system state.

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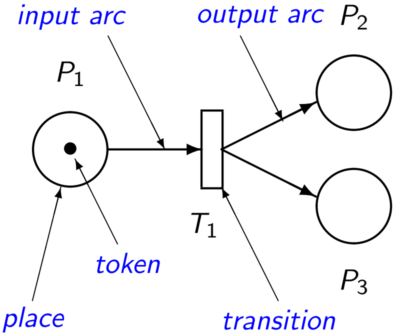
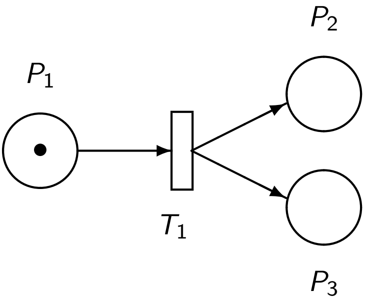
ARCS

Arcs specify the relationships between local states or conditions (places) and events (transitions). An arc from a place to a transition is termed an input arc. This indicates the local state in which the event can occur. An arc to a place from a transition is termed an output arc.

Anatomy of a Petri net

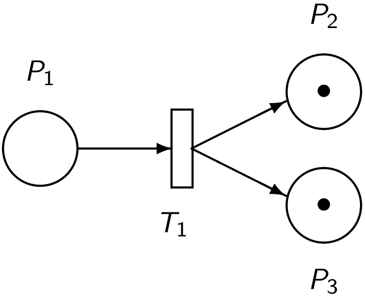


Example firing



When a transition fires tokens from input places are absorbed and tokens are created on each of the output places.

Example firing



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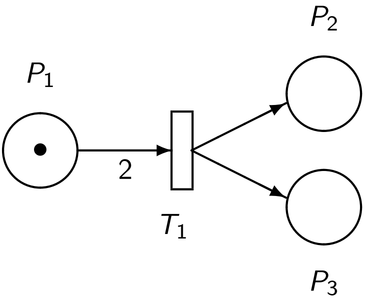
This is instantaneous.

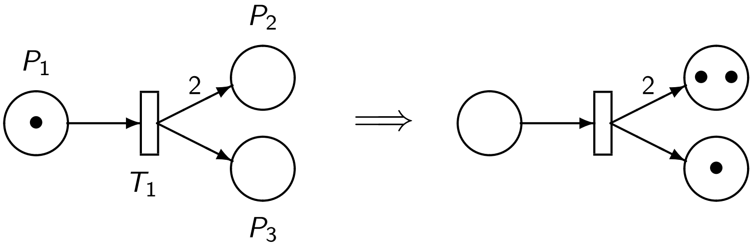
Arcs with multiplicities



If a multiplicity is associated with an arc, the firing rule is adjusted to reflect the multiplicity by altering the number of tokens required or produced.

For example, the net above cannot fire.

Arcs with multiplicities



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Petri net marking

The state of the Petri net system at any time, is characterised by the distribution of tokens over the places, generally termed a marking: $m : P \longrightarrow \mathbb{N}_0$, where $m(p) = n$ means that there are n tokens on place p.

The firing rule more formally

A transition t is said to be enabled in a marking m, written as $m[t\rangle$, if all the pre-places of t (those connected by an input arc) have a marking that is greater than or equal to the multiplicity of that input arc.

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When t in m fires, a new marking m' is reach, written as $m[t\rangle m'$.

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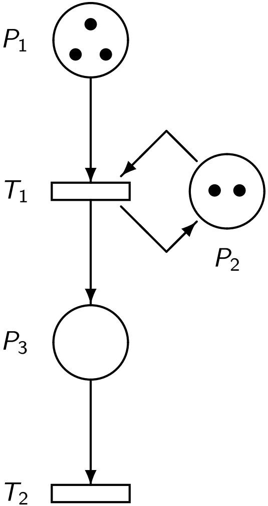
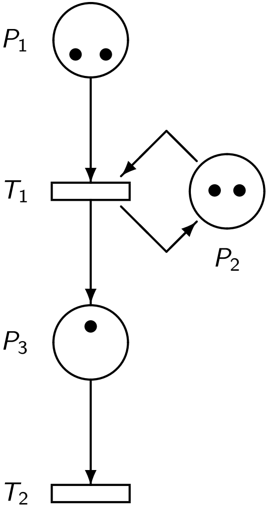
If, when playing the token game, as well as all the states we come across, we record the transitions between those states, we obtain the reachability graph.

Example: reachability graph



$(3, 2, 0)$

Example: reachability graph



$$(3,2,0) \xrightarrow{T_1} (2,2,1)$$

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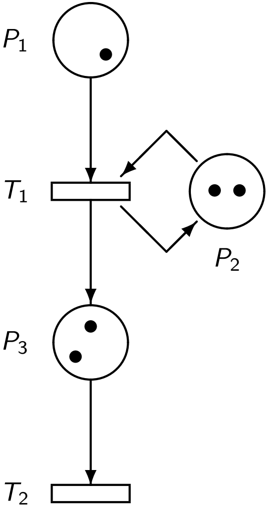

$$(3,2,0) \xrightarrow{T_1} (2,2,1) \xrightarrow{T_1} (1,2,2)$$

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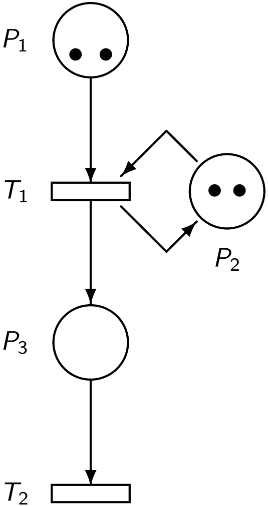

$$
\begin{array}{ccc}
(3,2,0) & & \\
\downarrow T_1 & & \\
(2,2,1) & \xrightarrow{T_2} & \color{red}{(2,2,0)} \\
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(1,2,2) & &
\end{array}
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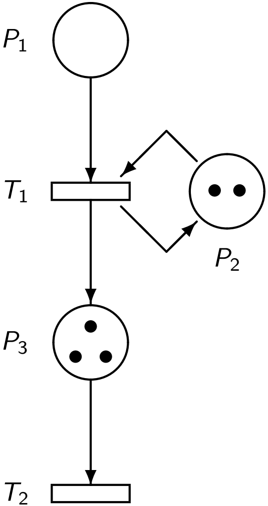

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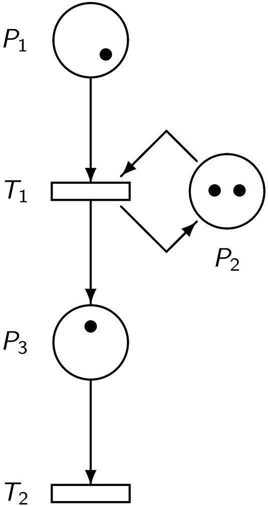

$$\begin{tikzcd}
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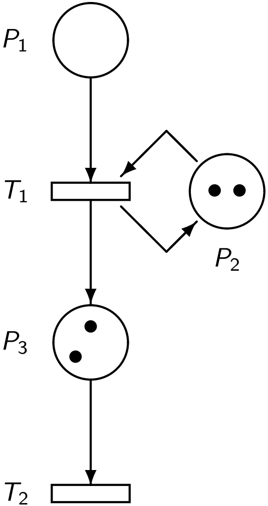

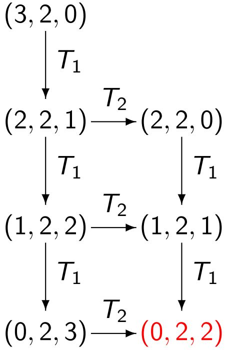

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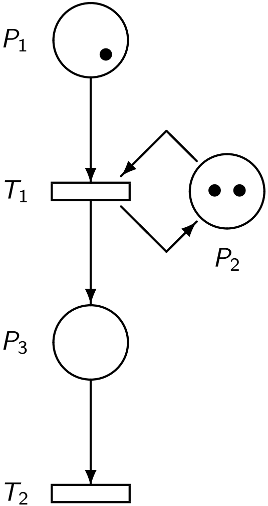

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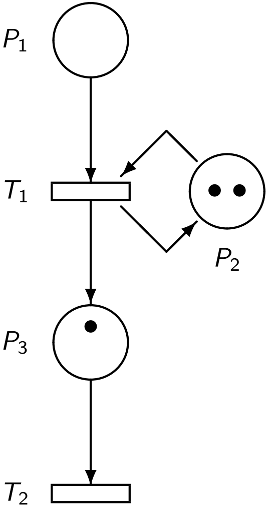

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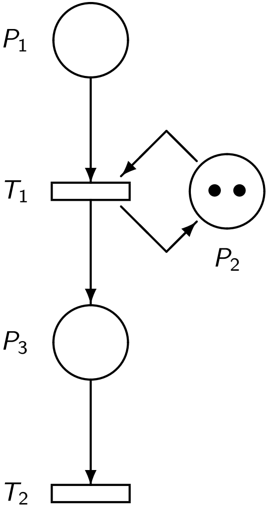

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It is straightforward to show that the reachability graph of a SPN forms the state transition diagram of an underlying Markov process.

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We consider again the simple multi-processor system, initially just with one processor.

We make a slight modification to the system: we now explicitly represent the processor requesting and gaining access to the common memory.

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A processor executes locally for some time (mean duration $1/\lambda$), and then requests access to common memory (gaining access has mean duration $1/r$). Once it has gained access, the duration of common memory access is assumed to be $1/\mu$ on average.

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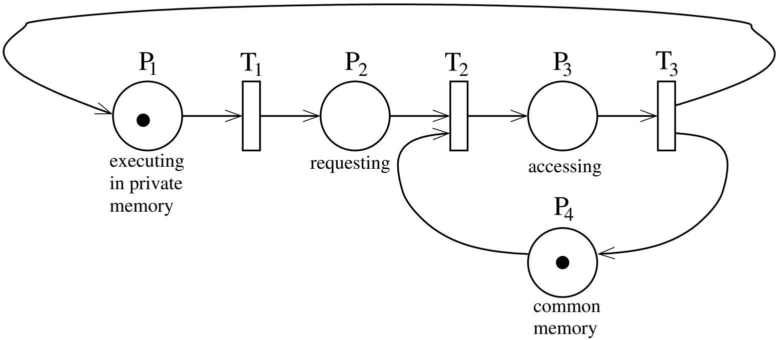

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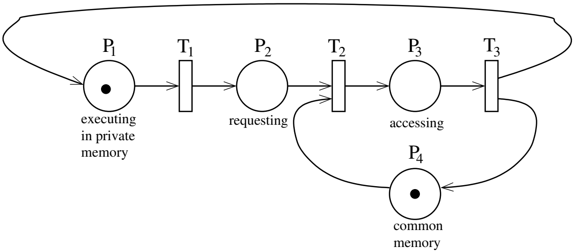

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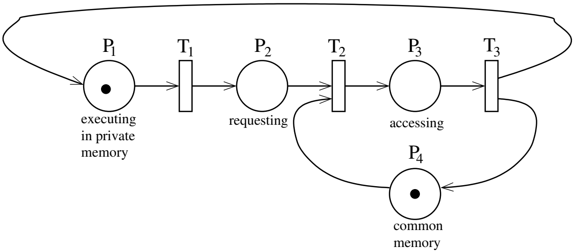

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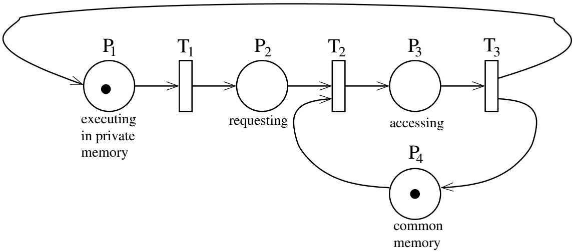

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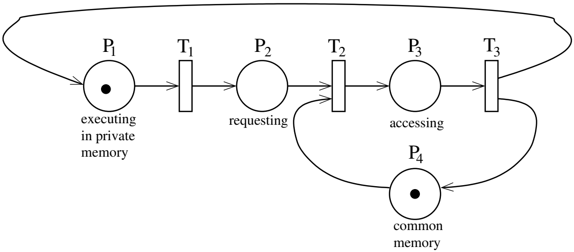

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- P_3 represents the state when the process is using the common memory;
- P_4 represents the common memory when it is not in use;

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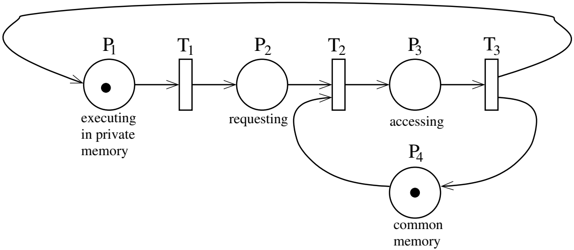

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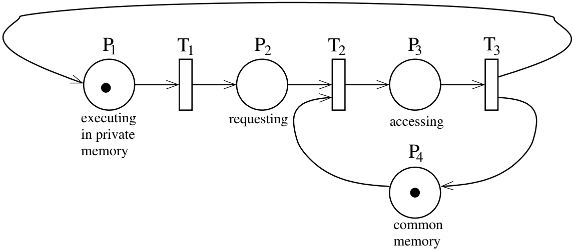

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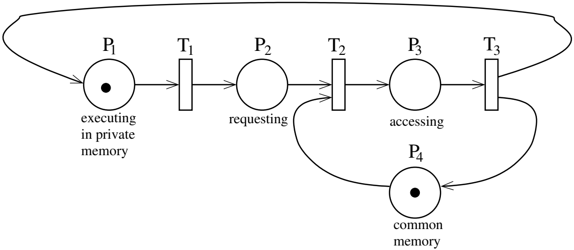

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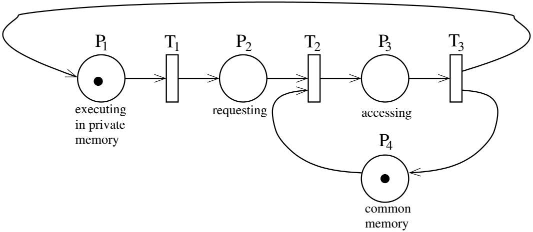

The reachability set of this model is
$\{(1,0,0,1),(0,1,0,1),(0,0,1,0)\}$,

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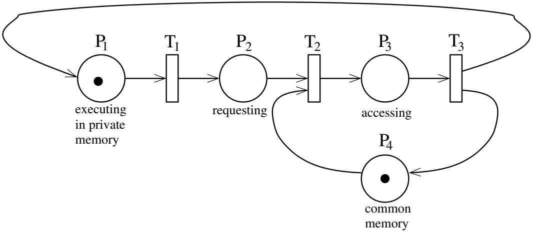

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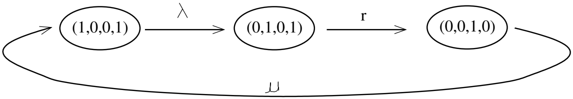
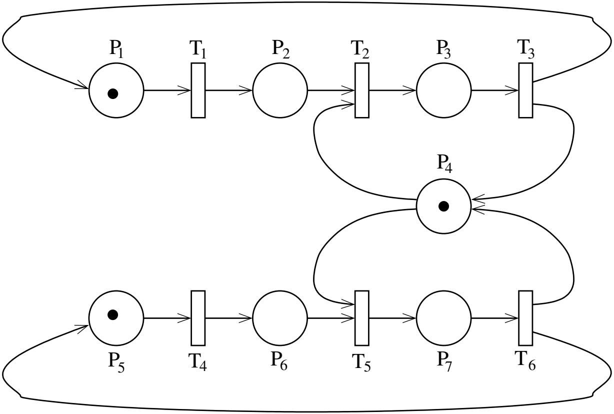


The two processor system



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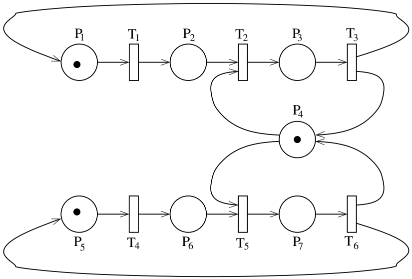

Now we see why the action of gaining access needs to be explicitly represented in the SPN model. Without it we could not enforce the necessary mutual exclusion between the processors.

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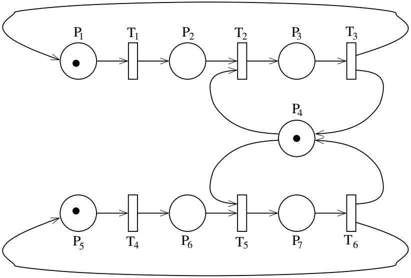

The reachability set is now

$$\left\{ \begin{array}{ccc} (1,0,0,1,1,0,0), & (1,0,0,1,0,1,0), & (1,0,0,0,0,0,1), \\ (0,1,0,1,1,0,0), & (0,1,0,1,0,1,0), & (0,1,0,0,0,0,1), \\ & (0,0,1,0,1,0,0), & (0,0,1,0,0,1,0) \end{array} \right\}$$

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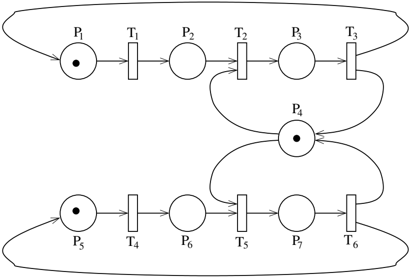

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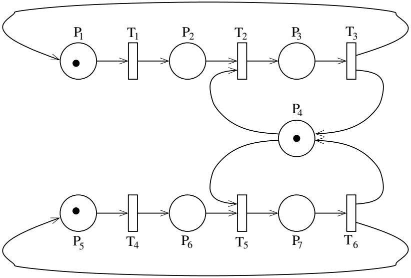
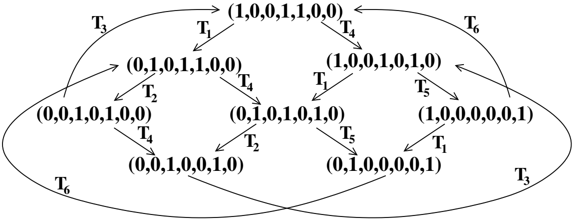
Note, we have eight distinct states rather than the five states we had in the Markov process when we modelled the system directly.

The two processor system





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One of the reasons for this is that actions which would not be explicitly represented if we were working directly at the Markov process level have to be represented by transitions.

This has the effect of increasing the state space since we now have to consider the state of waiting for these actions to occur, whereas we would prefer to abstract away from these actions.

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Two new primitives are added to the notation, all others remaining the same and keeping the same interpretation. These new primitives are immediate transitions and inhibitor arcs.

Immediate Transitions

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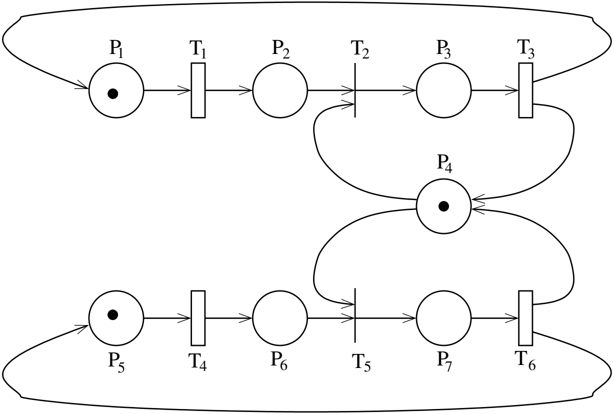
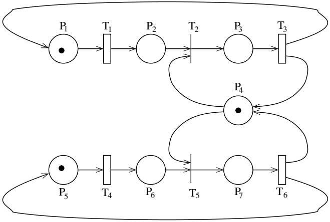
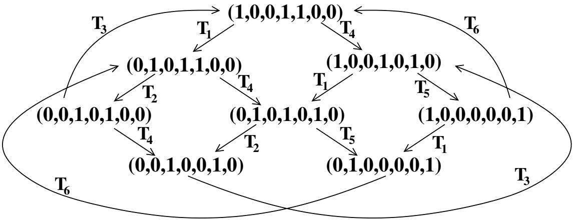
Logical actions arise when the system makes a choice between two or more alternatives. In an SPN these actions must be represented as actions which take time. However it is more natural to think of them occurring instantaneously.

Multi-processor example again

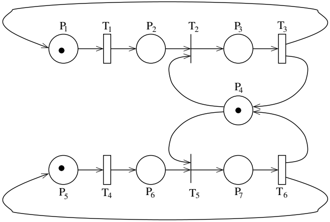


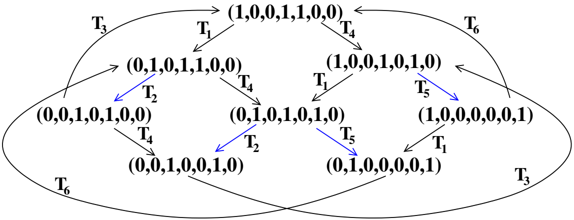
Multi-processor example again



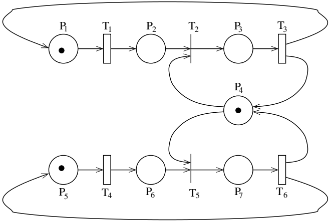


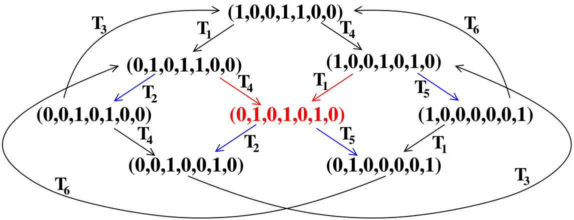
Multi-processor example again





Multi-processor example again





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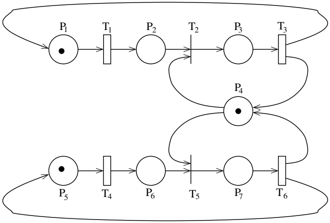

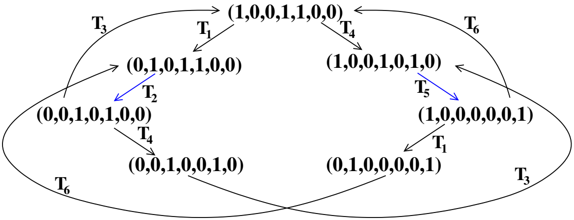

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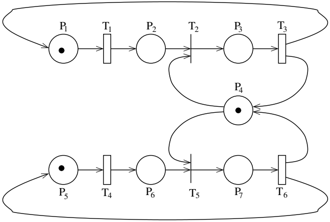

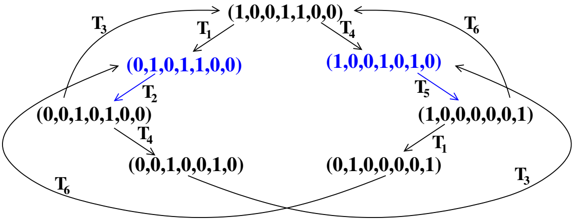

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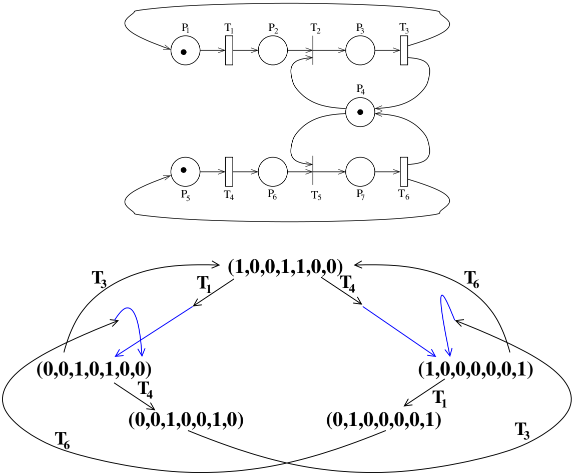

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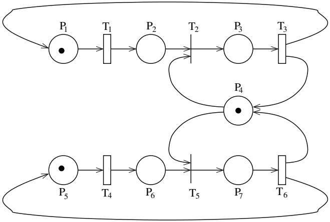

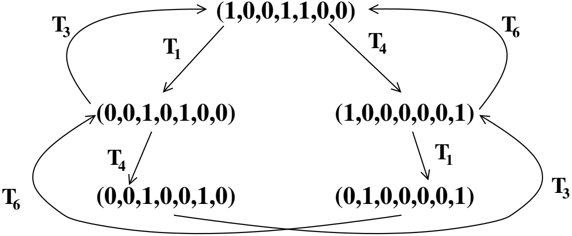

Inhibitor Arcs

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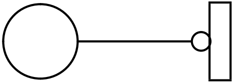

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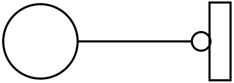

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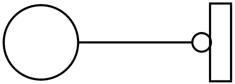

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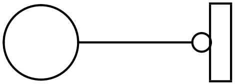

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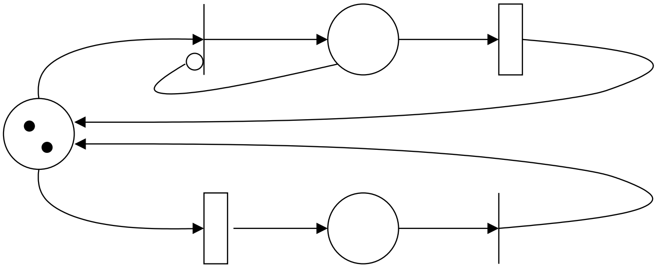
The inhibitor arcs impose additional constraints to the usual firing rule.

Example with inhibitor arc



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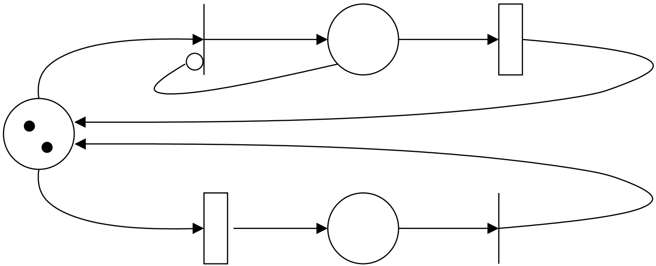

Exercise: Construct the reachability graph for this GSPN with and without the inhibitor arc.