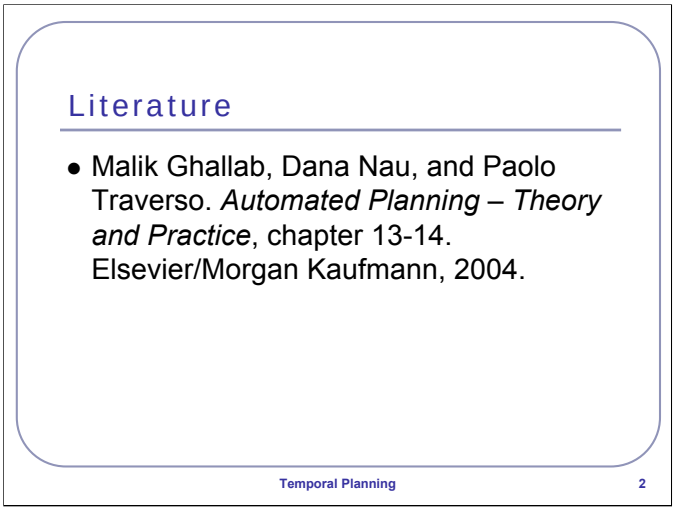
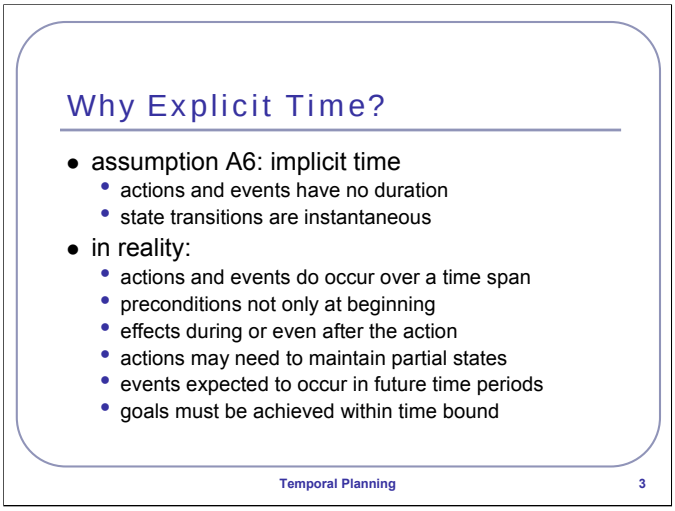
Temporal Planning

- **Planning with Temporal and Concurrent Actions**



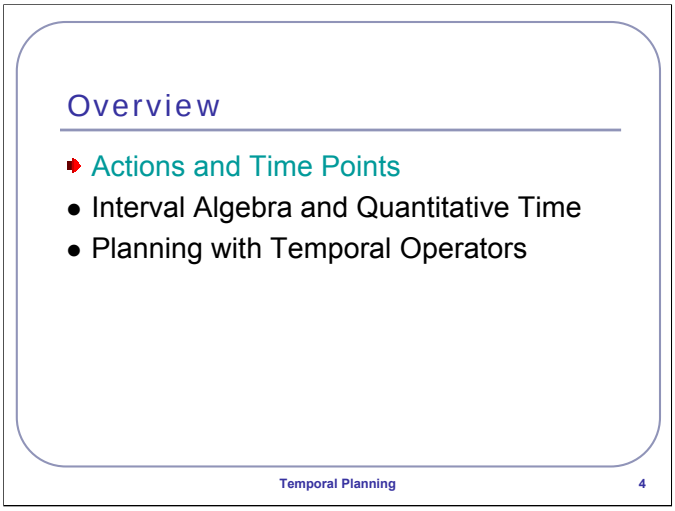
Literature

•Malik Ghallab, Dana Nau, and Paolo Traverso. *Automated Planning – Theory and Practice*, chapter 13-14. Elsevier/Morgan Kaufmann, 2004.



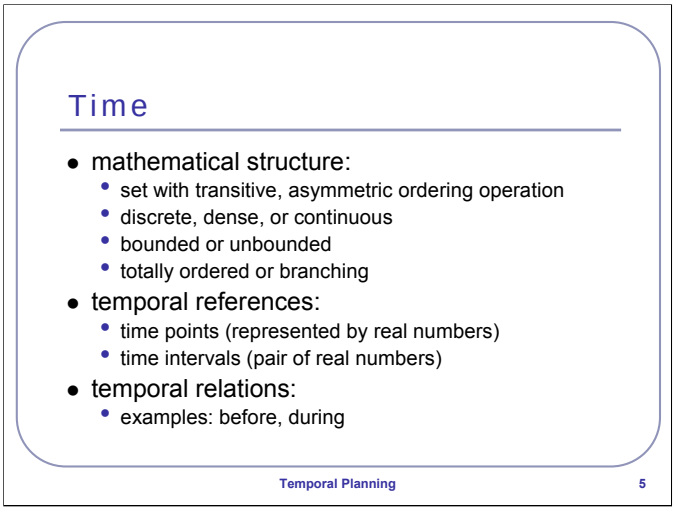
Why Explicit Time?

- **assumption A6: implicit time**
  - **actions and events have no duration**
  - **state transitions are instantaneous**
- **in reality:**
  - **actions and events do occur over a time span**
  - **preconditions not only at beginning**
    - move action: destination must be unoccupied only when robot arrives
  - **effects during or even after the action**
    - move action: origin is no longer occupied after robot has left
    - paint action: painted effect long after action is completed
  - **actions may need to maintain partial states**
  - **events expected to occur in future time periods**
  - **goals must be achieved within time bound**



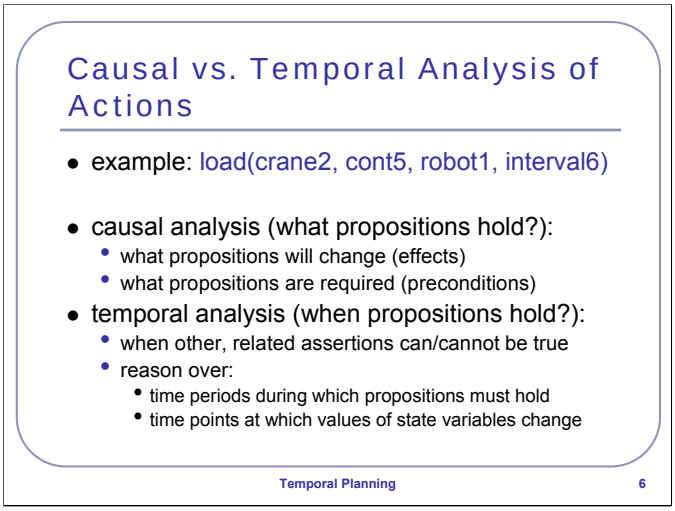
Overview

- **Actions and Time Points**
  - now: maintaining consistency in a network of related time points
- **Interval Algebra and Quantitative Time**
- **Planning with Temporal Operators**



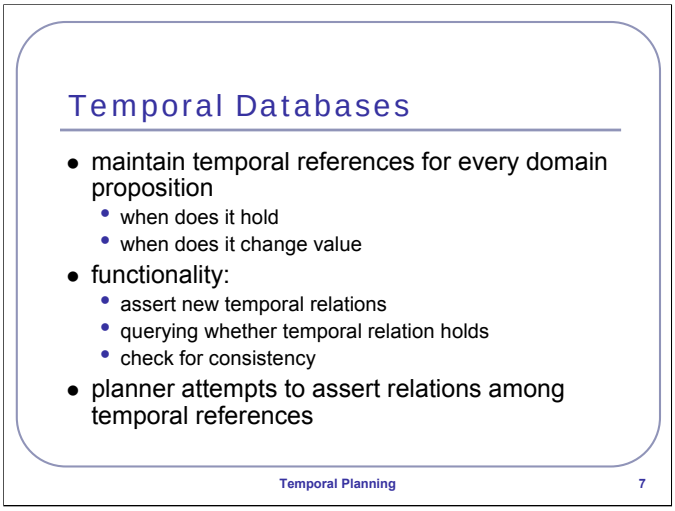
Time

- **mathematical structure:**
  - **set with transitive, asymmetric ordering operation**
  - **discrete, dense, or continuous**
  - **bounded or unbounded**
  - **totally ordered or branching**
- **temporal references:**
  - **time points (represented by real numbers)**
  - **time intervals (pair of real numbers)**
- **temporal relations:**
  - **examples: before, during**



Causal vs. Temporal Analysis of Actions

- **example: load(crane2, cont5, robot1, interval6)**
- **causal analysis (what propositions hold?):**
  - **what propositions will change (effects)**
  - **what propositions are required (preconditions)**
- **temporal analysis (when propositions hold?):**
  - **when other, related assertions can/cannot be true**
  - **reason over:**
    - **time periods during which propositions must hold**
    - **time points at which values of state variables change**



Temporal Databases

•maintain temporal references for every domain proposition

•when does it hold

•when does it change value

•functionality:

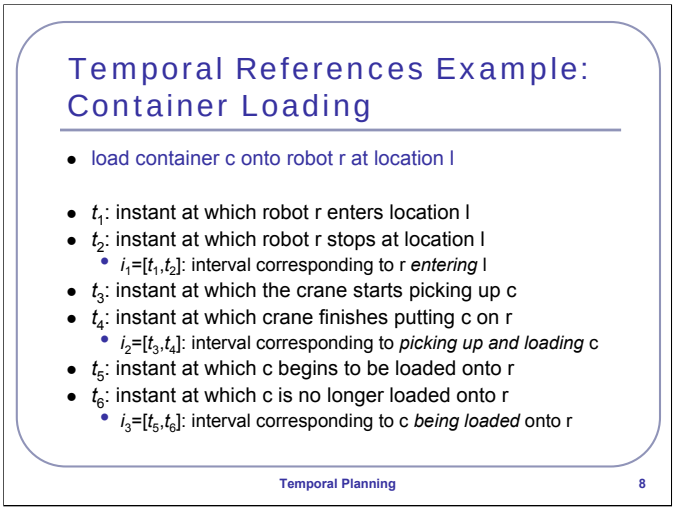
•assert new temporal relations

•querying whether temporal relation holds

•check for consistency

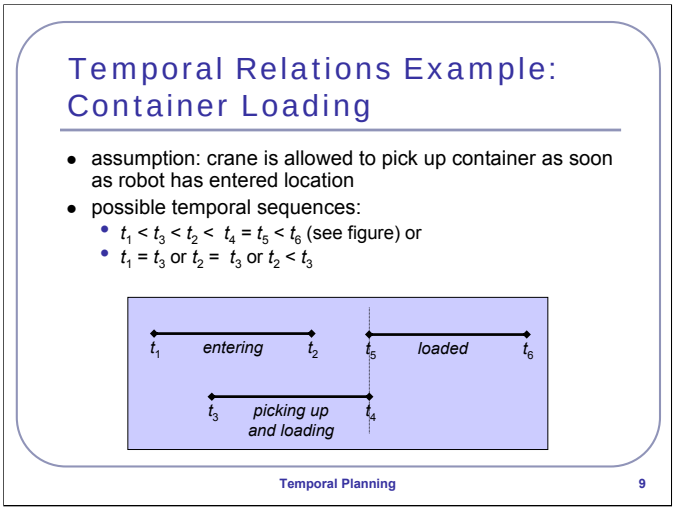
•planner attempts to assert relations among temporal references

•planner uses temporal database like ordering constraints



Temporal References Example: Container Loading

- **load container c onto robot r at location l**
- **t_1: instant at which robot r enters location l**
- **t_2: instant at which robot r stops at location l**
  - **$i_1=[t_1,t_2]$: interval corresponding to r *entering* l**
- **t_3: instant at which the crane starts picking up c**
- **t_4: instant at which crane finishes putting c on r**
  - **$i_2=[t_3,t_4]$: interval corresponding to *picking up and loading* c**
- **t_5: instant at which c begins to be loaded onto r**
- **t_6: instant at which c is no longer loaded onto r**
  - **$i_3=[t_5,t_6]$: interval corresponding to c *being loaded* onto r**
- instants $t_1 \ldots t_6$ and intervals $i_1 \ldots i_3$ are temporal references that specify when domain propositions are true
  - may use just the intervals in this example
- intervals i_1 and i_2 refer to activities taking place, i_3 refers to a proposition holding



Temporal Relations Example: Container Loading

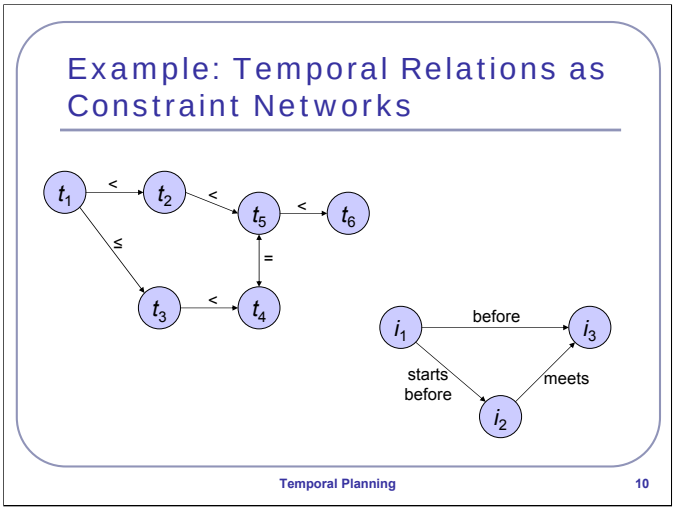
•assumption: crane is allowed to pick up container as soon as robot has entered location

•possible temporal sequences:

- **$t_1 < t_3 < t_2 < t_4 = t_5 < t_6$ (see figure) or**
- **$t_1 = t_3$ or $t_2 = t_3$ or $t_2 < t_3$**

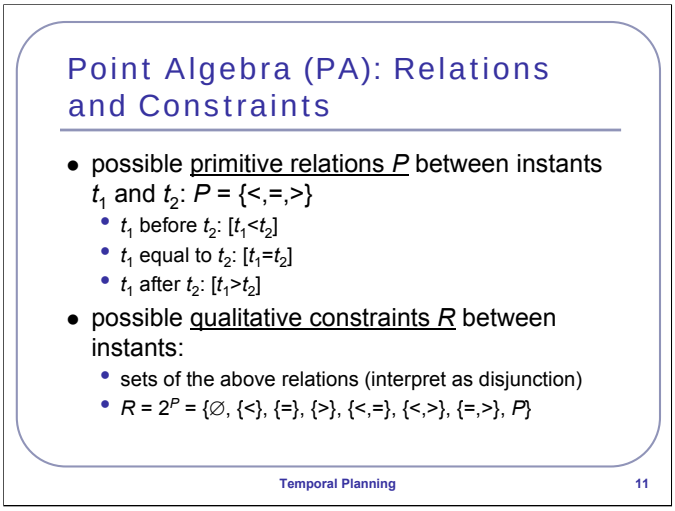
•[figure]

•no absolute information about durations or time positions, only binary constraints between instants or intervals



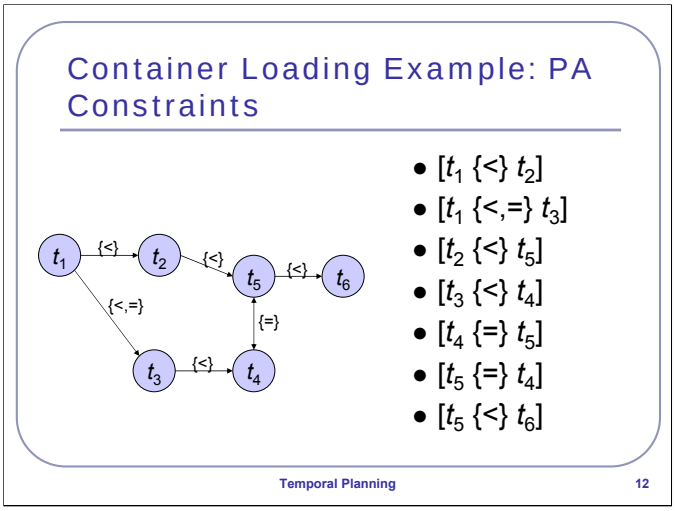
Example: Temporal Relations as Constraint Networks

- top left: instant constraint network
  - variables: instants / domains: real numbers
  - relations: <,=, … (relative positions of time points)
- bottom right: interval constraint network
  - variables: intervals / domains: RxR
  - relations: before, starts before, … (relative positions of intervals)



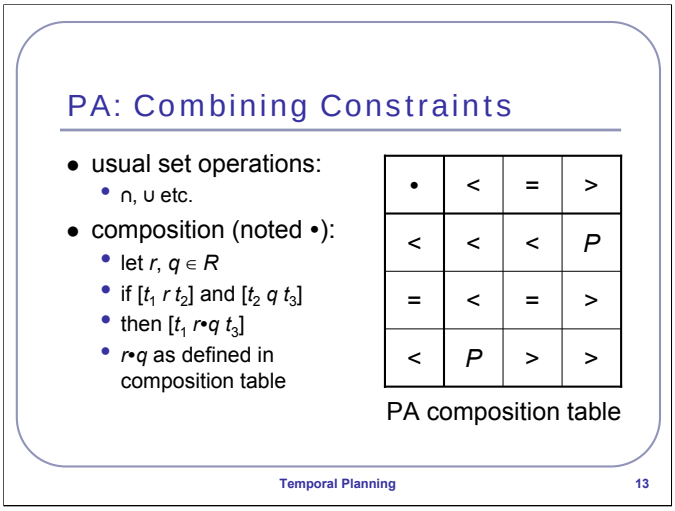
Point Algebra (PA): Relations and Constraints

- possible primitive relations *P* between instants t_1 and t_2: $P = \{<,=,>\}$
  - t_1 before t_2: $[t_1<t_2]$
  - t_1 equal to t_2: $[t_1=t_2]$
  - t_1 after t_2: $[t_1>t_2]$
- possible qualitative constraints *R* between instants:
  - sets of the above relations (interpret as disjunction)
  - $R = 2^P = \{\emptyset, \{<\}, \{=\}, \{>\}, \{<,=\}, \{<,>\}, \{=,>\}, P\}$
    - empty set: no alternatives cannot be satisfied
    - *P*: all possible relations; always satisfied



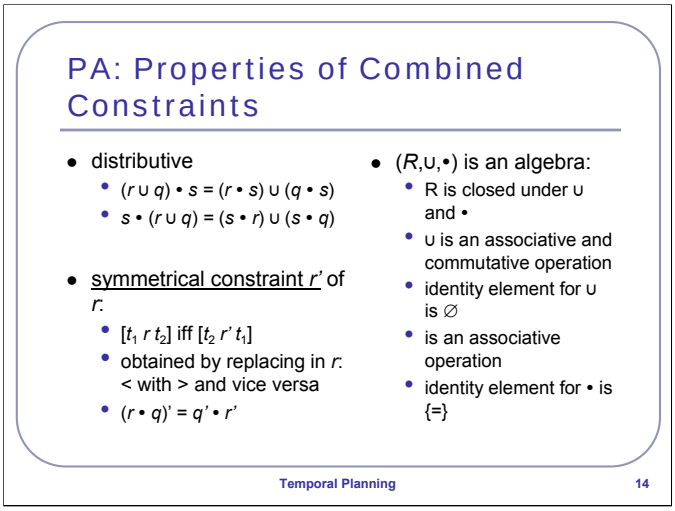
Container Loading Example: PA Constraints

- [figure]
- $[t_1 \{<\} t_2]$
- $[t_1 \{<,=\} t_3]$
- $[t_2 \{<\} t_5]$
- $[t_3 \{<\} t_4]$
- $[t_4 \{=\} t_5]$
- $[t_5 \{=\} t_4]$
- $[t_5 \{<\} t_6]$



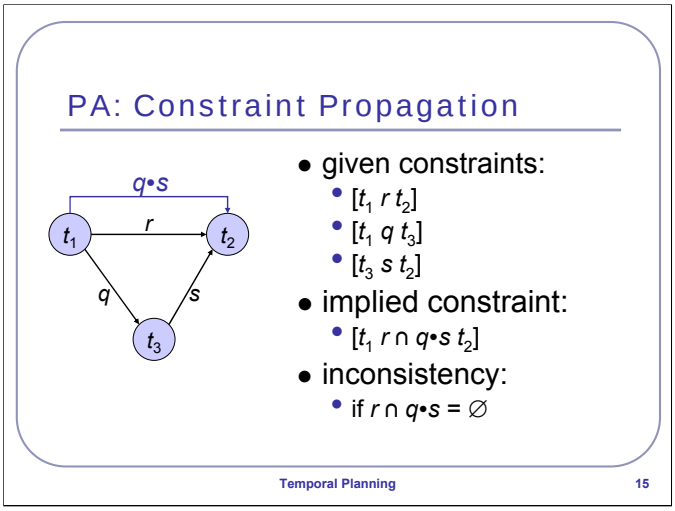
PA: Combining Constraints

- **usual set operations:**
  - **∩, ∪ etc.**
- **composition (noted •):**
  - composition operator handles transitivity
  - **let $r, q \in R$**
  - **if $[t_1\ r\ t_2]$ and $[t_2\ q\ t_3]$**
  - **then $[t_1\ r \bullet q\ t_3]$**
  - **$r \bullet q$ as defined in composition table**
- **[composition table]**



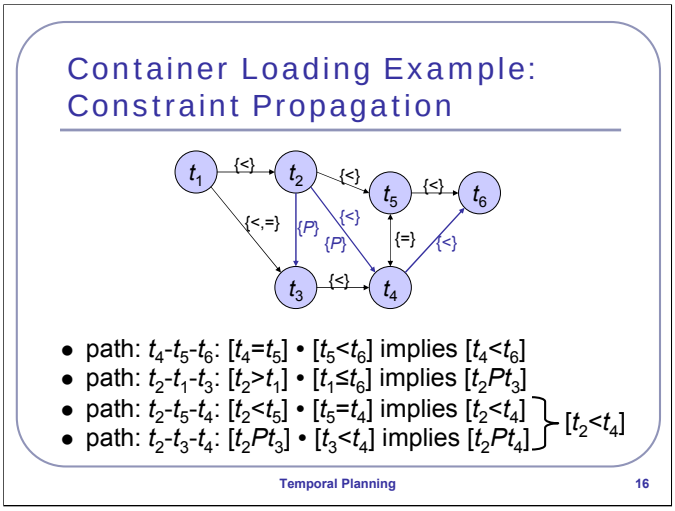
PA: Properties of Combined Constraints

- **distributive**
  - **$(r \cup q) \bullet s = (r \bullet s) \cup (q \bullet s)$**
  - **$s \bullet (r \cup q) = (s \bullet r) \cup (s \bullet q)$**
- **symmetrical constraint *r'* of *r*:**
  - **$[t_1\ r\ t_2]$ iff $[t_2\ r'\ t_1]$**
  - **obtained by replacing in *r*: < with > and vice versa**
  - **$(r \bullet q)' = q' \bullet r'$**
- **$(R,\cup,\bullet)$ is an algebra:**
  - **R is closed under ∪ and •**
  - **∪ is an associative and commutative operation**
  - **identity element for ∪ is ∅**
  - **is an associative operation**
  - **identity element for • is {=}**



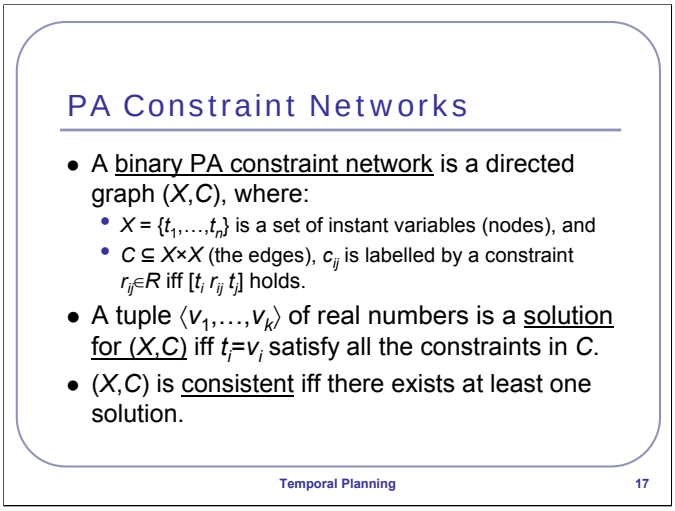
PA: Constraint Propagation

- **given constraints:**
  - **$[t_1\ r\ t_2]$**
  - **$[t_1\ q\ t_3]$**
  - **$[t_3\ s\ t_2]$**
- **implied constraint:**
  - **$[t_1\ r \cap q \bullet s\ t_2]$**
  - important operation: intersection, not union (algebra for union)
- **inconsistency:**
  - **if $r \cap q \bullet s = \varnothing$**
  - necessary condition!



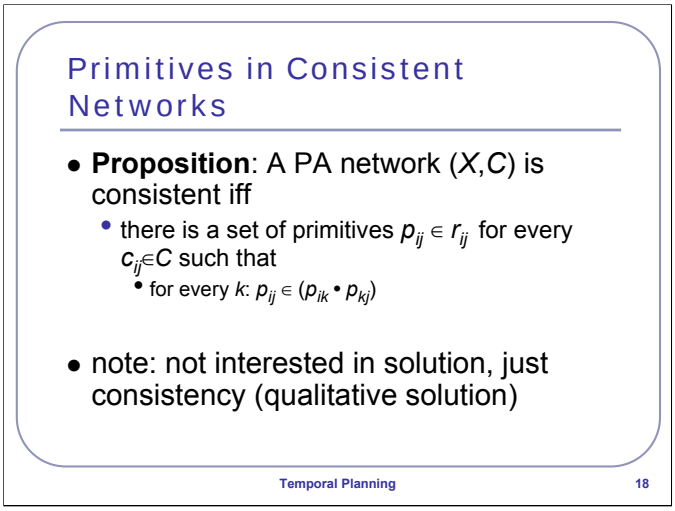
Container Loading Example: Constraint Propagation

- **path: t_4-t_5-t_6: [t_4=t_5] • [t_5<t_6] implies [t_4<t_6]**
- **path: t_2-t_1-t_3: [t_2>t_1] • [t_1≤t_6] implies [$t_2 P t_3$]**
- **path: t_2-t_5-t_4: [t_2<t_5] • [t_5=t_4] implies [t_2<t_4]**
- **path: t_2-t_3-t_4: [$t_2 P t_3$] • [t_3<t_4] implies [$t_2 P t_4$]**
- last two give: **[$t2$<$t4$]**



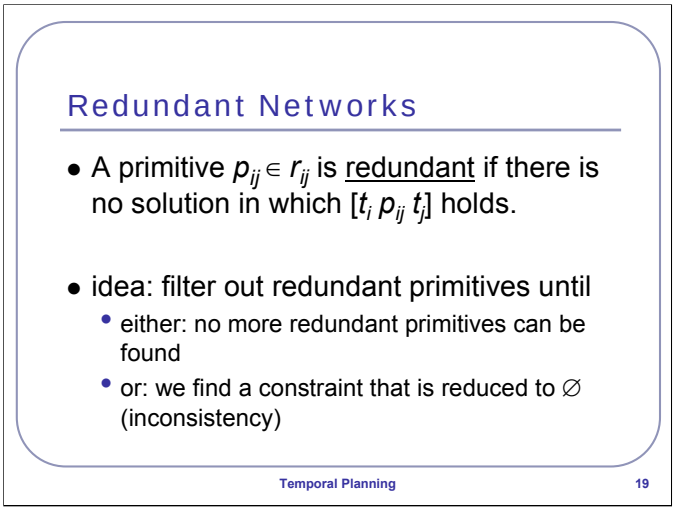
PA Constraint Networks

- **A binary PA constraint network is a directed graph (X,C), where:**
  - **$X = \{t_1,\ldots,t_n\}$ is a set of instant variables (nodes), and**
  - **$C \subseteq X \times X$ (the edges), c_{ij} is labelled by a constraint $r_{ij} \in R$ iff $[t_i\ r_{ij}\ t_j]$ holds.**
    - if $c_{ij} \in C$ labelled with $r_{ij} \in R$ then c_{ji} must be labelled with the symmetric constraint r_{ij}'
    - if there is no $c_{ij} \in C$ then we can add c_{ij} labelled with $r_{ij} = P$
- **A tuple $\langle v_1,\ldots,v_k\rangle$ of real numbers is a solution for (X,C) iff $t_i = v_i$ satisfy all the constraints in C.**
- **(X,C) is consistent iff there exists at least one solution.**



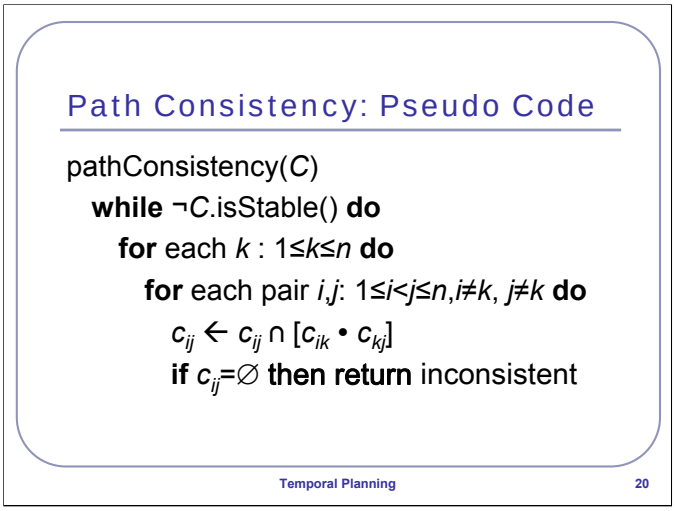
Primitives in Consistent Networks

- **Proposition: A PA network (X,C) is consistent iff**
  - **there is a set of primitives $p_{ij} \in r_{ij}$ for every $c_{ij} \in C$ such that**
    - **for every k: $p_{ij} \in (p_{ik} \bullet p_{kj})$**
  - note: in a solution every pair t_i, t_j will be related by a single primitive relation $p_{ij} \in r_{ij}$
- **note: not interested in solution, just consistency (qualitative solution)**



Redundant Networks

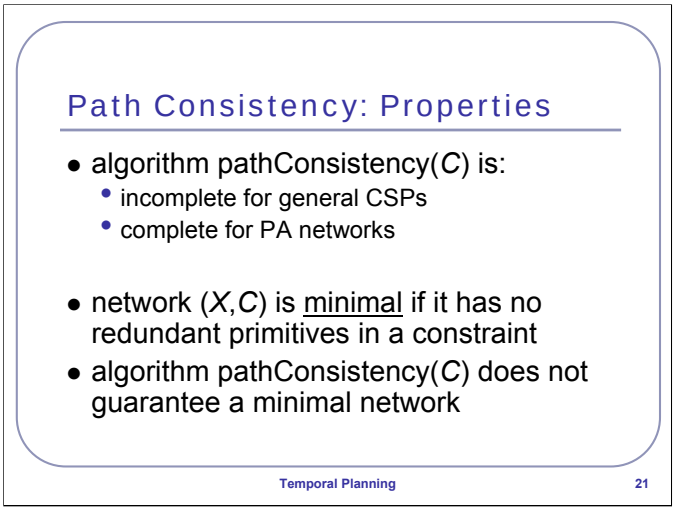
- **A primitive $p_{ij} \in r_{ij}$ is redundant if there is no solution in which $[t_i\ p_{ij}\ t_j]$ holds.**
- **idea: filter out redundant primitives until**
  - **either: no more redundant primitives can be found**
  - **or: we find a constraint that is reduced to ∅ (inconsistency)**
  - use path consistency algorithm for CSP problems



Path Consistency: Pseudo Code

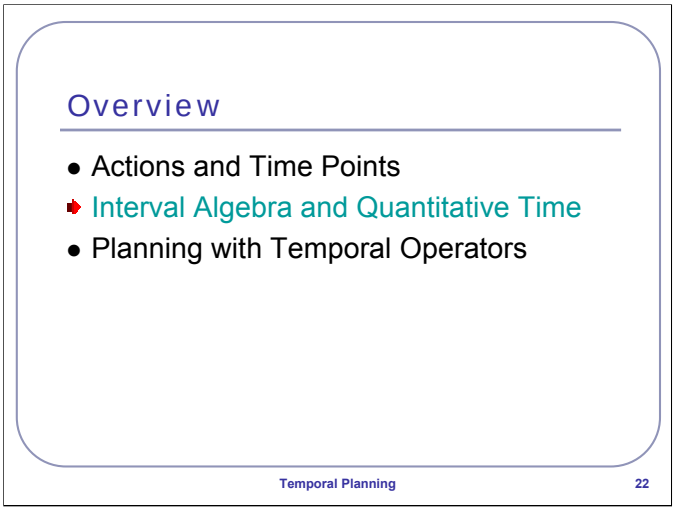
- **pathConsistency(*C*)**
  - given time point network
- **while ¬*C*.isStable() do**
- **for each *k* : 1≤*k*≤*n* do**
  - "middle node"
- **for each pair *i*,*j*: 1≤*i*<*j*≤*n*,*i*≠*k*, *j*≠*k* do**
  - surrounding nodes
- **$c_{ij} \leftarrow c_{ij} \cap [c_{ik} \bullet c_{kj}]$**
  - update direct link using transitivity property
- **if $c_{ij}=\varnothing$ then return inconsistent**

- use as constraint manager during planning: incremental version of the algorithm



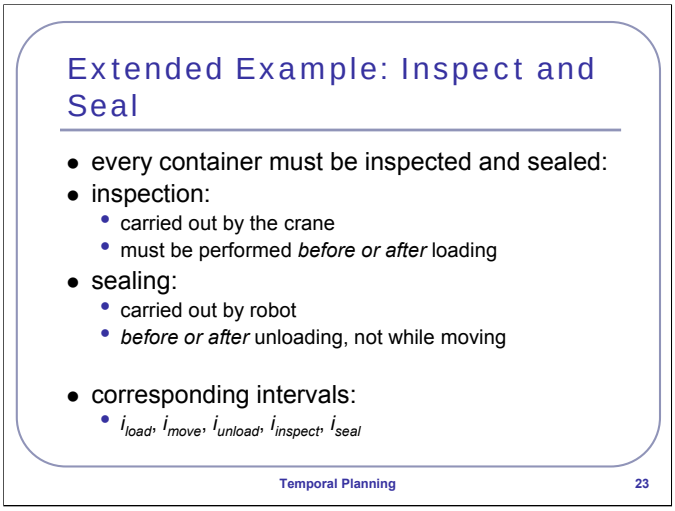
Path Consistency: Properties

- algorithm pathConsistency(*C*) is:
  - incomplete for general CSPs
  - complete for PA networks
- network (*X*,*C*) is minimal if it has no redundant primitives in a constraint
- algorithm pathConsistency(*C*) does not guarantee a minimal network



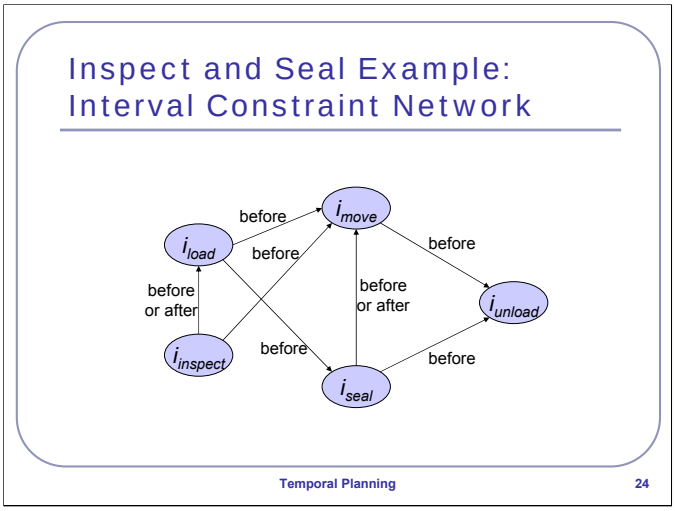
Overview

- **Actions and Time Points**
  - just done: maintaining consistency in a network of related time points
- **Interval Algebra and Quantitative Time**
  - now: reasoning about more complex structures
- **Planning with Temporal Operators**



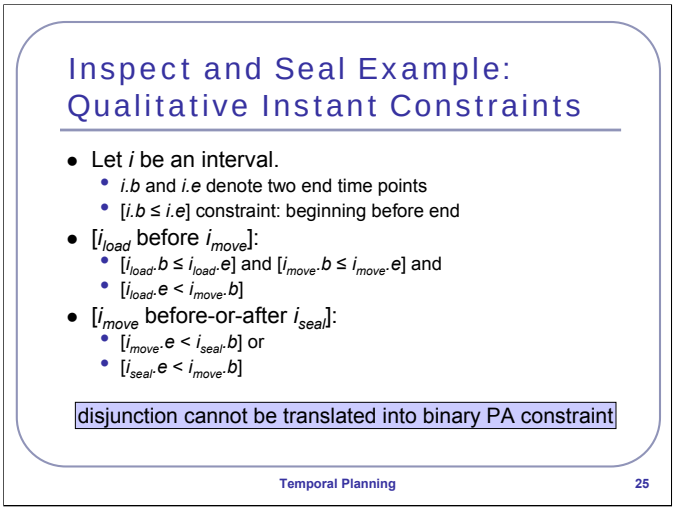
Extended Example: Inspect and Seal

- every container must be inspected and sealed:
- inspection:
  - carried out by the crane
  - must be performed *before or after* loading
- sealing:
  - carried out by robot
  - *before or after* unloading, not while moving
- corresponding intervals:
  - i_{load}, i_{move}, i_{unload}, $i_{inspect}$, i_{seal}



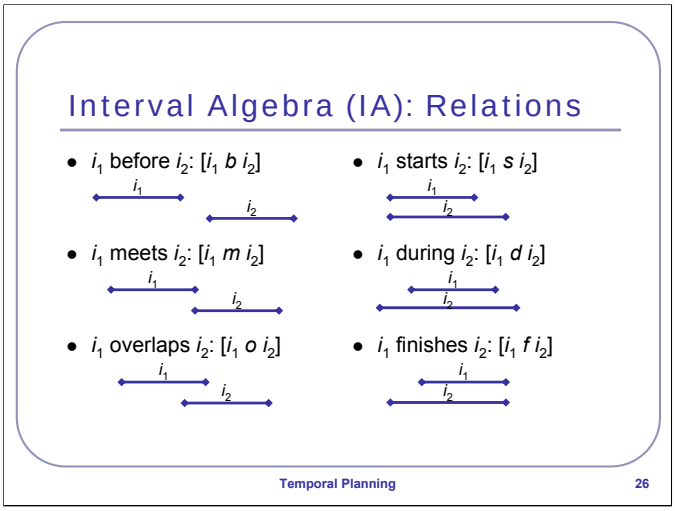
Inspect and Seal Example: Interval Constraint Network

- "before or after" = must not overlap



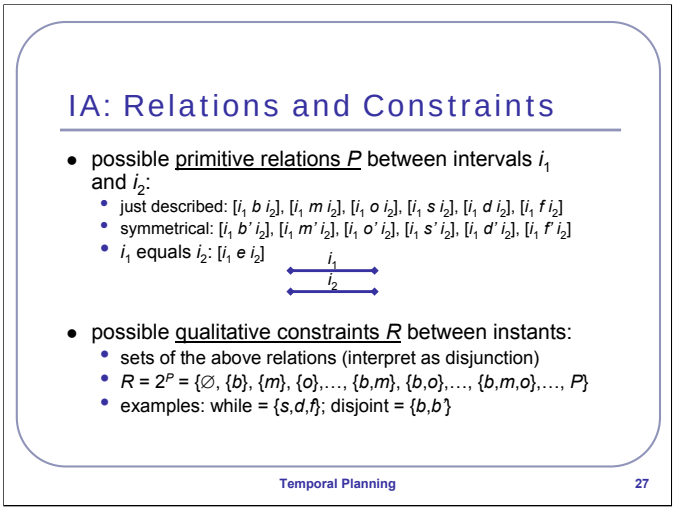
Inspect and Seal Example: Qualitative Instant Constraints

- **Let *i* be an interval.**
  - ***i.b* and *i.e* denote two end time points**
  - **[*i.b* ≤ *i.e*] constraint: beginning before end**
- **[i_{load} before i_{move}]:**
  - **[$i_{load}.b \leq i_{load}.e$] and [$i_{move}.b \leq i_{move}.e$] and**
  - **[$i_{load}.e < i_{move}.b$]**
- **[i_{move} before-or-after i_{seal}]:**
  - **[$i_{move}.e < i_{seal}.b$] or**
  - **[$i_{seal}.e < i_{move}.b$]**
- **disjunction cannot be translated into binary PA constraint**
- translation of interval constraint network into (binary) instant constraint network not possible in general



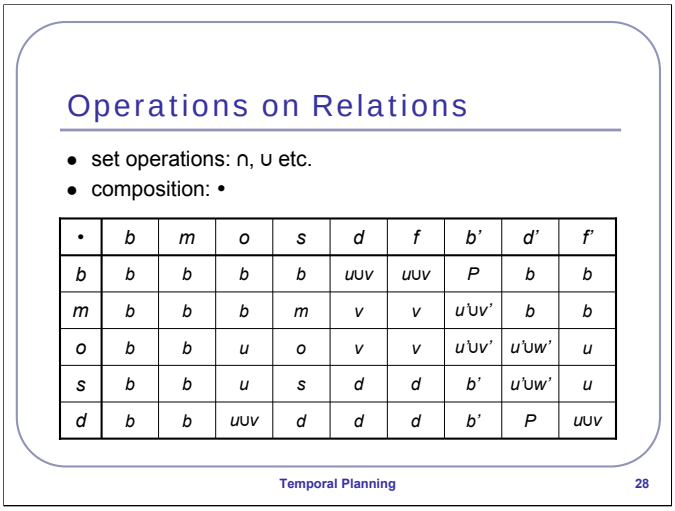
Interval Algebra (IA): Relations

•interval algebra: similar to point algebra, but related objects are intervals



IA: Relations and Constraints

- **possible primitive relations *P* between intervals i_1 and i_2:**
  - **just described: [i_1 *b* i_2], [i_1 *m* i_2], [i_1 *o* i_2], [i_1 *s* i_2], [i_1 *d* i_2], [i_1 *f* i_2]**
  - **symmetrical: [i_1 *b'* i_2], [i_1 *m'* i_2], [i_1 *o'* i_2], [i_1 *s'* i_2], [i_1 *d'* i_2], [i_1 *f'* i_2]**
  - **i_1 equals i_2: [i_1 *e* i_2]**
  - 13 possible ways of relating the two end points
- **possible qualitative constraints *R* between instants:**
  - **sets of the above relations (interpret as disjunction)**
  - **$R = 2^P$ = {∅, {*b*}, {*m*}, {*o*},…, {*b*,*m*}, {*b*,*o*},…, {*b*,*m*,*o*},…, *P*}**
  - constraints: 2^{13} possible constraints
    - empty set: no alternatives cannot be satisfied
    - *P*: all possible relations; always satisfied
  - **examples: while = {*s*,*d*,*f*}; disjoint = {*b*,*b'*}**



•	*b*	*m*	*o*	*s*	*d*	*f*	*b'*	*d'*	*f'*
b	*b*	*b*	*b*	*b*	*u*∪*v*	*u*∪*v*	*P*	*b*	*b*
m	*b*	*b*	*b*	*m*	*v*	*v*	*u*∪*v'*	*b*	*b*
o	*b*	*b*	*u*	*o*	*v*	*v*	*u*∪*v'*	*u*∪*w'*	*u*
s	*b*	*b*	*u*	*s*	*d*	*d*	*b'*	*u*∪*w'*	*u*
d	*b*	*b*	*u*∪*v*	*d*	*d*	*d*	*b'*	*P*	*u*∪*v*

Operations on Relations

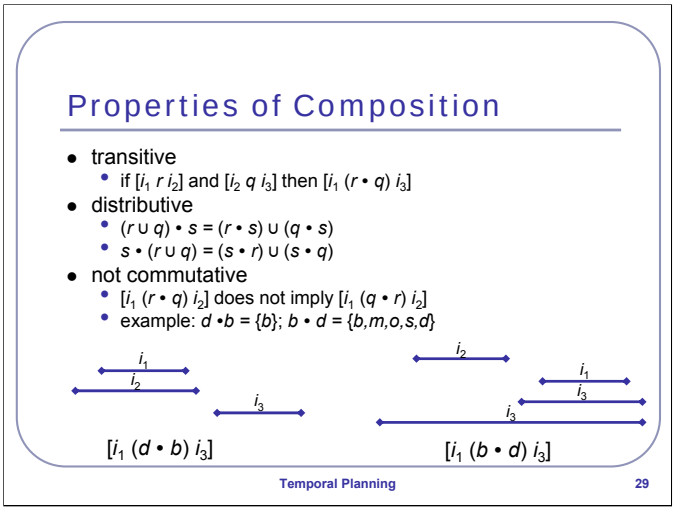
•set operations: ∩, ∪ etc.

•composition: •

•[table]

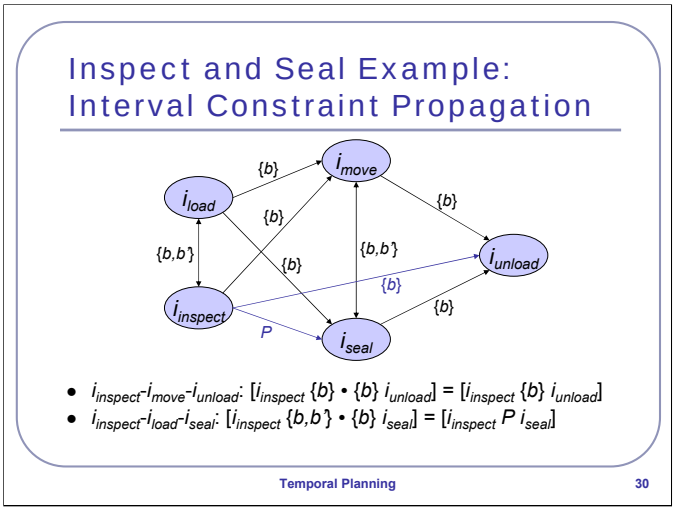
•*u*={*b,m,o*}; *v*={*o,s,d*}; *w*={*d,f*}

•composition is transitive and distributive



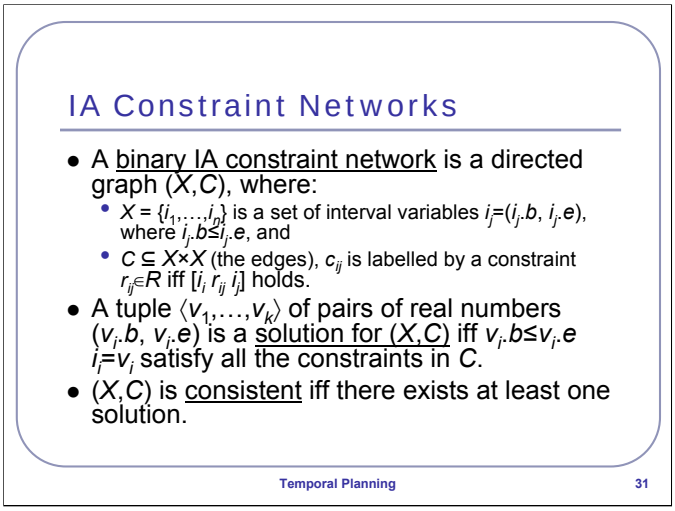
Properties of Composition

- **transitive**
  - **if [i_1 r i_2] and [i_2 q i_3] then [i_1 (r • q) i_3]**
- **distributive**
  - **(r ∪ q) • s = (r • s) ∪ (q • s)**
  - **s • (r ∪ q) = (s • r) ∪ (s • q)**
- **not commutative**
  - **[i_1 (r • q) i_2] does not imply [i_1 (q • r) i_2]**
  - **example: d •b = {b}; b • d = {b,m,o,s,d}**
- **[figure]**
- for [i_1 (b · d) i_3]: i_3 could start before, at the same time as, during, at the end of, or after i_2



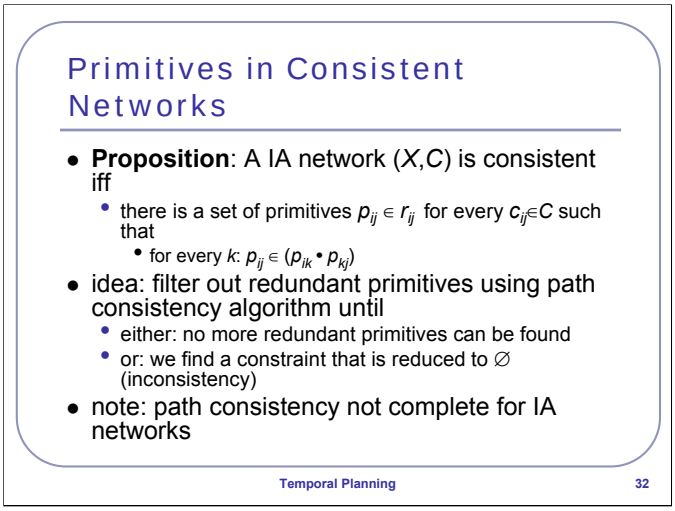
Inspect and Seal Example: Interval Constraint Propagation

- [figure]
- $i_{inspect}$-i_{move}-i_{unload}: [$i_{inspect}$ {b} • {b} i_{unload}] = [$i_{inspect}$ {b} i_{unload}]
- $i_{inspect}$-i_{load}-i_{seal}: [$i_{inspect}$ {b,b'} • {b} i_{seal}] = [$i_{inspect}$ P i_{seal}]



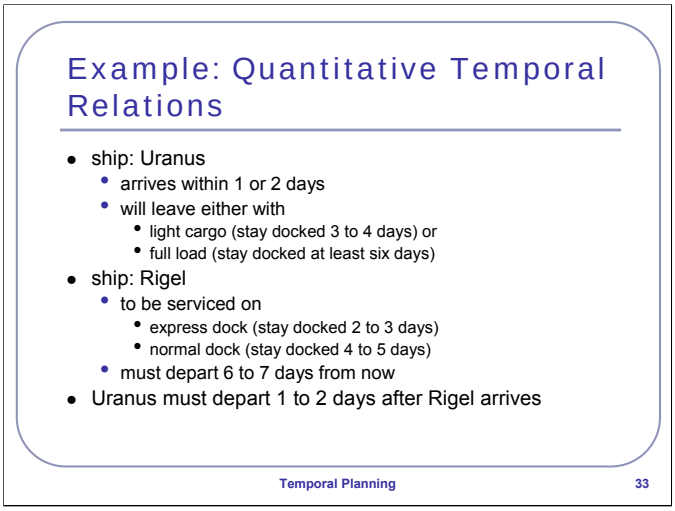
IA Constraint Networks

- **A binary IA constraint network is a directed graph (X,C), where:**
  - **$X = \{i_1,\ldots,i_n\}$ is a set of interval variables $i_j=(i_j.b, i_j.e)$, where $i_j.b \leq i_j.e$, and**
    - domain of each interval variable is a half plane due to $i_j.b \leq i_j.e$
  - **$C \subseteq X \times X$ (the edges), c_{ij} is labelled by a constraint $r_{ij} \in R$ iff $[i_i\ r_{ij}\ i_j]$ holds.**
- **A tuple $\langle v_1,\ldots,v_k\rangle$ of pairs of real numbers $(v_i.b, v_i.e)$ is a solution for (X,C) iff $v_i.b \leq v_i.e$ $i_i=v_i$ satisfy all the constraints in C.**
- **(X,C) is consistent iff there exists at least one solution.**



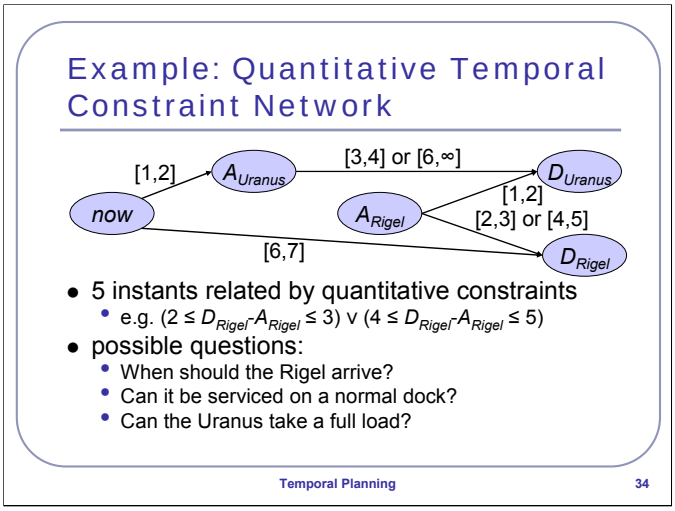
Primitives in Consistent Networks

- **Proposition: A IA network (*X*,*C*) is consistent iff**
  - **there is a set of primitives $p_{ij} \in r_{ij}$ for every $c_{ij} \in C$ such that**
    - **for every *k*: $p_{ij} \in (p_{ik} \bullet p_{kj})$**
- **idea: filter out redundant primitives using path consistency algorithm until**
  - **either: no more redundant primitives can be found**
  - **or: we find a constraint that is reduced to ∅ (inconsistency)**
- **note: path consistency not complete for IA networks**
  - consistency checking for IP networks is an NP-complete problem



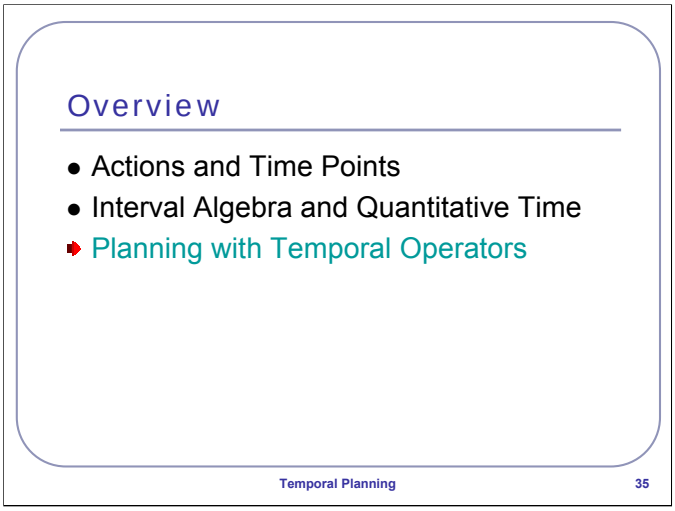
Example: Quantitative Temporal Relations

- ship: Uranus
  - arrives within 1 or 2 days
  - will leave either with
    - light cargo (stay docked 3 to 4 days) or
    - full load (stay docked at least six days)
- ship: Rigel
  - to be serviced on
    - express dock (stay docked 2 to 3 days)
    - normal dock (stay docked 4 to 5 days)
  - must depart 6 to 7 days from now
- Uranus must depart 1 to 2 days after Rigel arrives



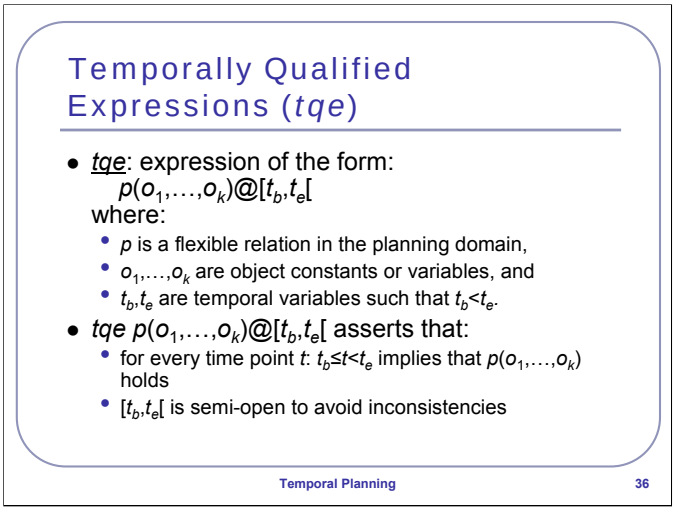
Example: Quantitative Temporal Constraint Network

- **5 instants related by quantitative constraints**
  - **e.g. (2 ≤ D_{Rigel}-A_{Rigel} ≤ 3) ∨ (4 ≤ D_{Rigel}-A_{Rigel} ≤ 5)**
- **possible questions:**
  - **When should the Rigel arrive?**
  - **Can it be serviced on a normal dock?**
  - **Can the Uranus take a full load?**



Overview

- **Actions and Time Points**
- **Interval Algebra and Quantitative Time**
  - just done: reasoning about more complex structures
- **Planning with Temporal Operators**
  - now: integrating reasoning about time into a planning algorithm



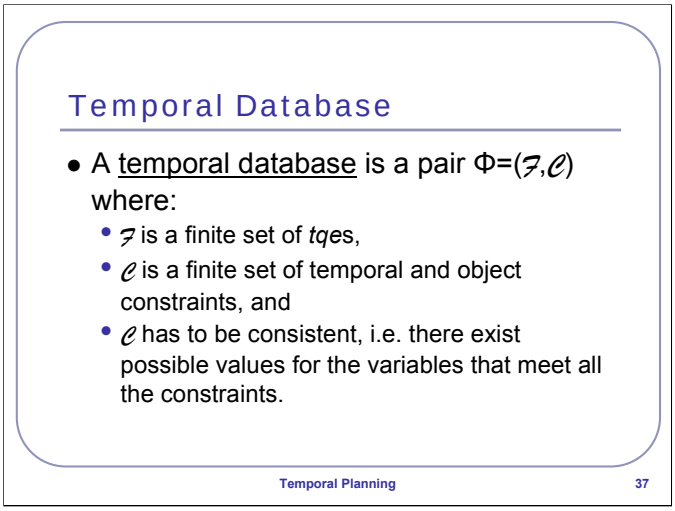
Temporally Qualified Expressions (*tqe*)

- ***tqe*: expression of the form:**
  $p(o_1,\ldots,o_k)@[t_b,t_e[$

where:

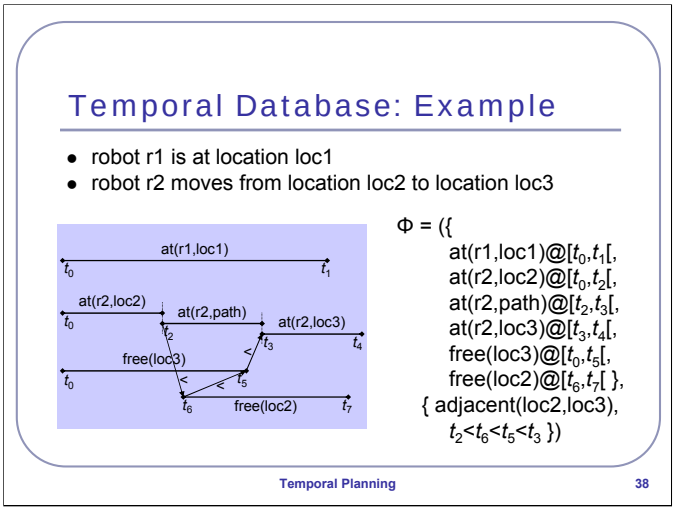
- **p is a flexible relation in the planning domain,**
- **$o_1,\ldots,o_k$ are object constants or variables, and**
  - object constants: objects in the planning domain: robots, containers, etc.
  - object variables: (possibly typed) variables with object constants as possible values
- **t_b,t_e are temporal variables such that $t_b<t_e$.**

- ***tqe* $p(o_1,\ldots,o_k)@[t_b,t_e[$ asserts that:**
  - **for every time point t: $t_b\leq t<t_e$ implies that $p(o_1,\ldots,o_k)$ holds**
  - **$[t_b,t_e[$ is semi-open to avoid inconsistencies**
  - semi-open interval:
    - asserted relation holds at t_b, but not at t_e
    - suppose two *tqe*s with inconsistent relations meet at time point t; inconsistency at t!



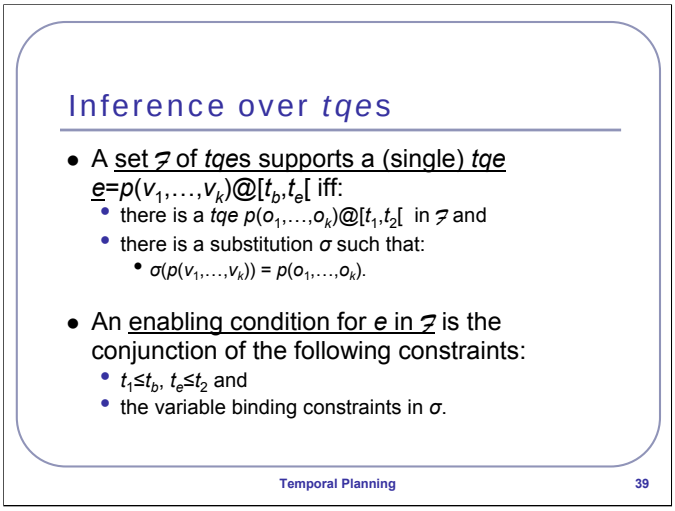
Temporal Database

- **A temporal database is a pair Φ=(ℱ,𝒞) where:**
  - Φ - phi
  - **ℱ is a finite set of *tq*es,**
  - **𝒞 is a finite set of temporal and object constraints, and**
    - temporal constraints: see time point algebra
    - object constraints: rigid relations
  - **𝒞 has to be consistent, i.e. there exist possible values for the variables that meet all the constraints.**



Temporal Database: Example

- robot r1 is at location loc1
- robot r2 moves from location loc2 to location loc3
- [figure]
- Φ = (
- {at(r1,loc1)@[t_0,t_1[, at(r2,loc2)@[t_0,t_2[, at(r2,path)@[t_2,t_3[, at(r2,loc3)@[t_3,t_4[, free(loc3)@[t_0,t_5[, free(loc2)@[t_6,t_7[},
- {adjacent(loc2,loc3), $t_2<t_6<t_5<t_3$ })



Inference over *tqe*s

•A set ℱ of *tqe*s supports a (single) *tqe e*=$p(v_1,\ldots,v_k)@[t_b,t_e[$ iff:

•there is a *tqe* $p(o_1,\ldots,o_k)@[t_1,t_2[$ in ℱ and

•there is a substitution σ such that:

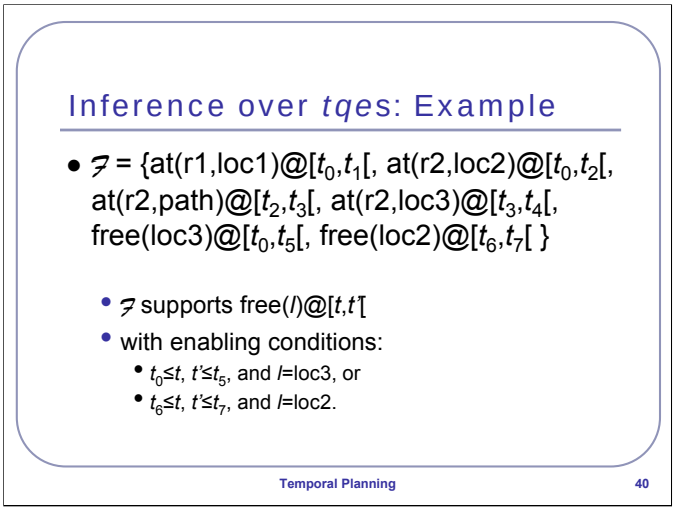
•$\sigma(p(v_1,\ldots,v_k)) = p(o_1,\ldots,o_k)$.

•An enabling condition for *e* in ℱ is the conjunction of the following constraints:

•$t_1 \leq t_b$, $t_e \leq t_2$ and

•the variable binding constraints in σ.

•note: if $p(o_1,\ldots,o_k)$ holds during $[t_1,t_2[$ it must also hold over any sub-interval



Inference over *tqe*s: Example

•$\mathcal{F}$ = {at(r1,loc1)@[t_0,t_1[, at(r2,loc2)@[t_0,t_2[, at(r2,path)@[t_2,t_3[, at(r2,loc3)@[t_3,t_4[, free(loc3)@[t_0,t_5[, free(loc2)@[t_6,t_7[}

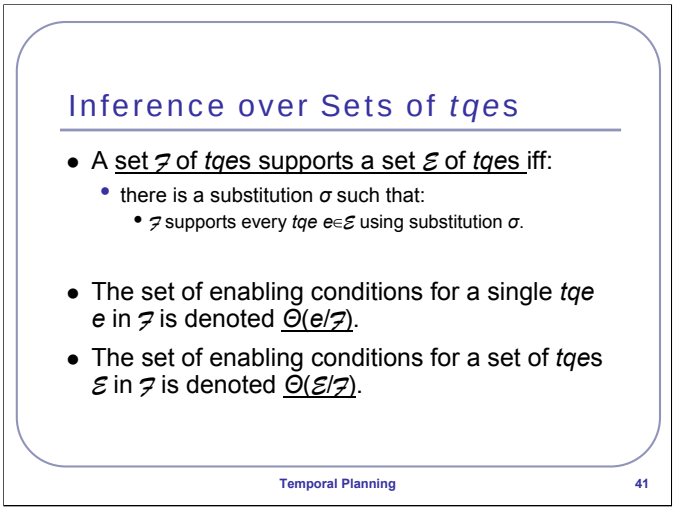
•$\mathcal{F}$ supports free(*l*)@[*t*,*t'*[

•with enabling conditions:

•$t_0 \le t$, $t' \le t_5$, and *l*=loc3, or

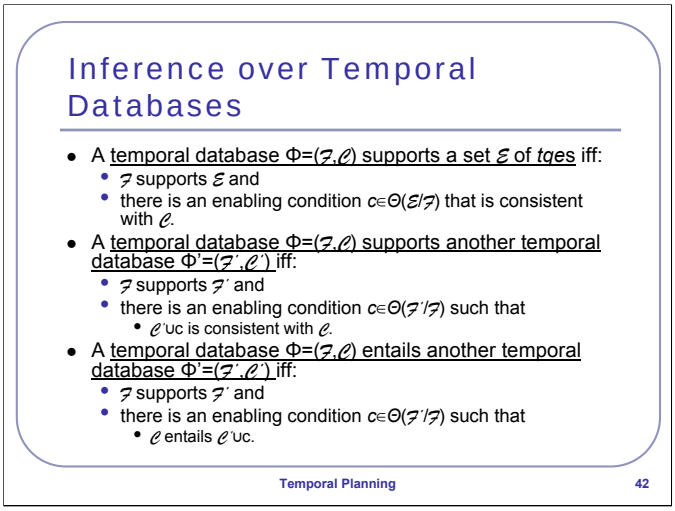
•$t_6 \le t$, $t' \le t_7$, and *l*=loc2.

•note: enabling conditions are not necessarily unique



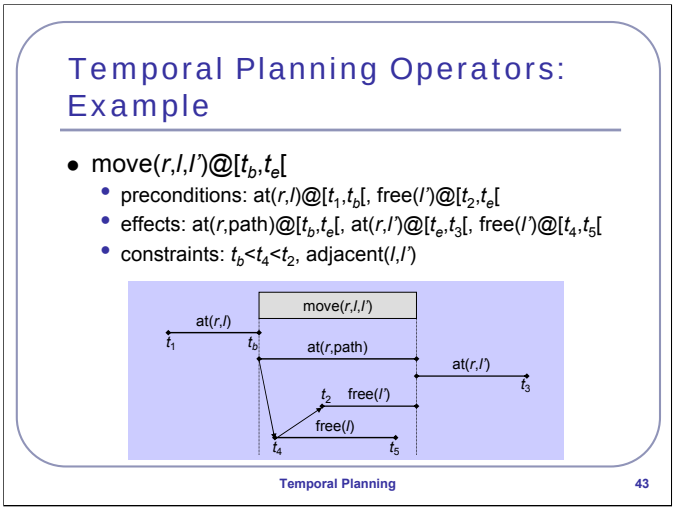
Inference over Sets of *tqe*s

- **A set $\mathcal{F}$ of *tqe*s supports a set $\mathcal{E}$ of *tqe*s iff:**
  - **there is a substitution σ such that:**
    - **$\mathcal{F}$ supports every *tqe* $e \in \mathcal{E}$ using substitution σ.**
- **The set of enabling conditions for a single *tqe* *e* in $\mathcal{F}$ is denoted $\Theta(e/\mathcal{F})$.**
- **The set of enabling conditions for a set of *tqe*s $\mathcal{E}$ in $\mathcal{F}$ is denoted $\Theta(\mathcal{E}/\mathcal{F})$.**
  - Θ = theta



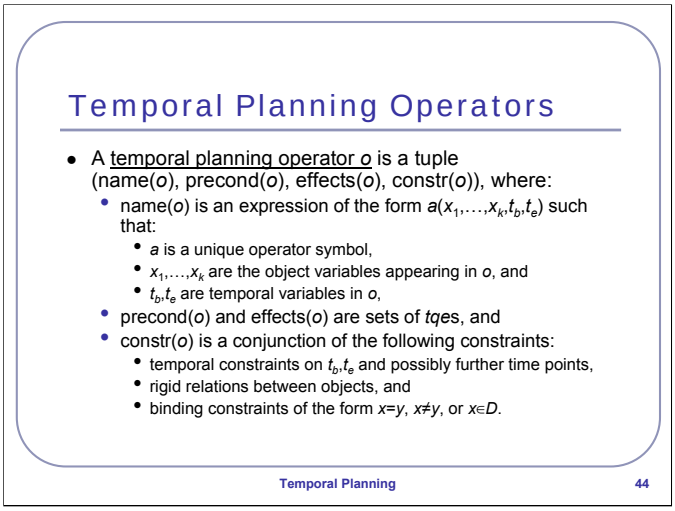
Inference over Temporal Databases

- A temporal database $\Phi=(\mathcal{F},\mathcal{C})$ supports a set $\mathcal{E}$ of *tqes* iff:
  - $\mathcal{F}$ supports $\mathcal{E}$ and
  - there is an enabling condition $c \in \Theta(\mathcal{E}/\mathcal{F})$ that is consistent with $\mathcal{C}$.
- A temporal database $\Phi=(\mathcal{F},\mathcal{C})$ supports another temporal database $\Phi'=(\mathcal{F}',\mathcal{C}')$ iff:
  - $\mathcal{F}$ supports $\mathcal{F}'$ and
  - there is an enabling condition $c \in \Theta(\mathcal{F}'/\mathcal{F})$ such that
    - $\mathcal{C}' \cup c$ is consistent with $\mathcal{C}$.
- A temporal database $\Phi=(\mathcal{F},\mathcal{C})$ entails another temporal database $\Phi'=(\mathcal{F}',\mathcal{C}')$ iff:
  - $\mathcal{F}$ supports $\mathcal{F}'$ and
  - there is an enabling condition $c \in \Theta(\mathcal{F}'/\mathcal{F})$ such that
    - $\mathcal{C}$ entails $\mathcal{C}' \cup c$.



Temporal Planning Operators: Example

- move(r,l,l')@[t_b,t_e[
  - preconditions: at(r,l)@[t_1,t_b[, free(l')@[t_2,t_e[
  - effects: at(r,path)@[t_b,t_e[, at(r,l')@[t_e,t_3[, free(l')@[t_4,t_5[
  - constraints: $t_b<t_4<t_2$, adjacent(l,l')



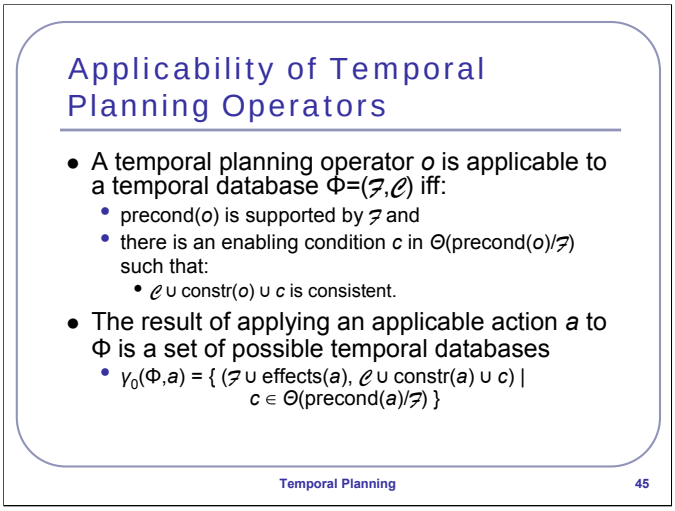
Temporal Planning Operators

•A temporal planning operator *o* is a tuple (name(*o*), precond(*o*), effects(*o*), constr(*o*)), where:

- **•name(*o*) is an expression of the form $a(x_1,\ldots,x_k,t_b,t_e)$ such that:**
  - **•*a* is a unique operator symbol,**
  - **•$x_1,\ldots,x_k$ are the object variables appearing in *o*, and**
  - **•t_b,t_e are temporal variables in *o*,**
- **•precond(*o*) and effects(*o*) are sets of *tqe*s, and**
- **•constr(*o*) is a conjunction of the following constraints:**
  - **•temporal constraints on t_b,t_e and possibly further time points,**
  - **•rigid relations between objects, and**
  - **•binding constraints of the form $x=y$, $x\neq y$, or $x\in D$.**

•note: operators are usually written as in the example in the previous slide

•no negative effects: handled by domain axioms or state-variable representation



Applicability of Temporal Planning Operators

•A temporal planning operator *o* is applicable to a temporal database Φ=(ℱ,𝒞) iff:

•precond(*o*) is supported by ℱ and

•there is an enabling condition *c* in Θ(precond(*o*)/ℱ) such that:

•𝒞 ∪ constr(*o*) ∪ *c* is consistent.

•The result of applying an applicable action *a* to Φ is a set of possible temporal databases

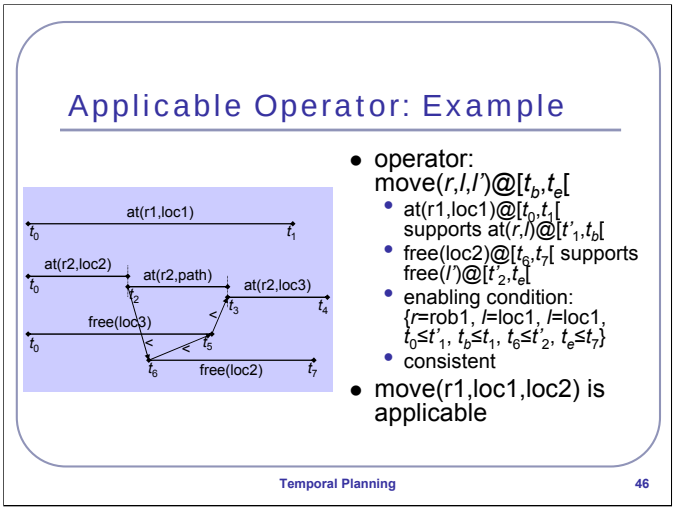
•action: partially instantiated operator

•$\gamma_0(\Phi,a) = \{ (\mathcal{F} \cup \text{effects}(a), \mathcal{C} \cup \text{constr}(a) \cup c) \mid c \in \Theta(\text{precond}(a)/\mathcal{F}) \}$

•provisional definition; see Automated Planning book for complete definition

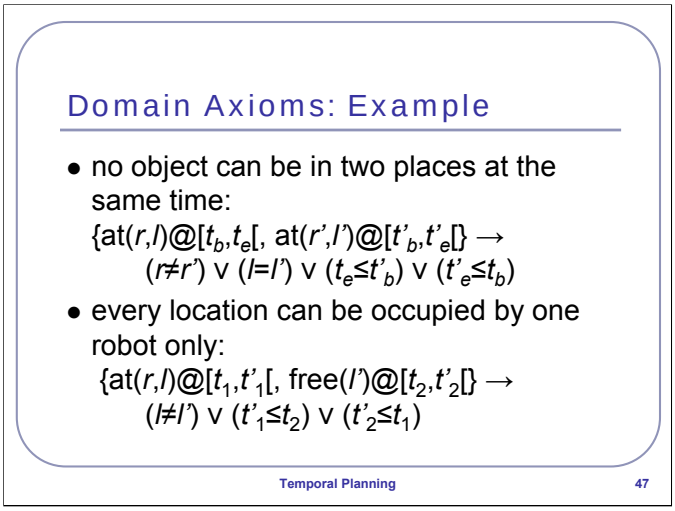
•result is set: not due to non-determinism but different ways in which *a* can be inserted

•note: applying an action *a* does not remove anything from the temporal database



Applicable Operator: Example

- **[figure]**
- **operator: move(*r*,*l*,*l'*)@[t_b,t_e[**
  - time points with prime are from operator definition
  - **at(r1,loc1)@[t_0,t_1[supports at(*r*,*l*)@[t'_1,t_b[**
  - **free(loc2)@[t_6,t_7[supports free(*l'*)@[t'_2,t_e[**
  - **enabling condition: {*r*=rob1, *l*=loc1, *l*=loc1, $t_0 \le t'_1$, $t_b \le t_1$, $t_6 \le t'_2$, $t_e \le t_7$}**
  - **consistent**
- **move(r1,loc1,loc2) is applicable**
- other possibility: move(r2,loc3,loc2)



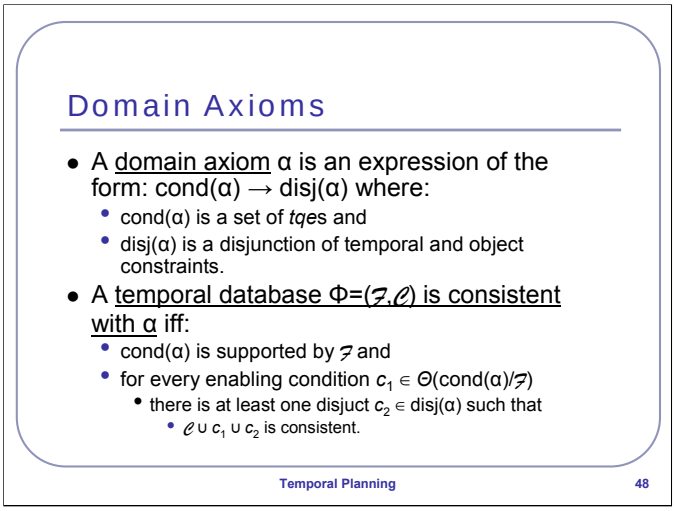
Domain Axioms: Example

- **no object can be in two places at the same time:**

$$\{at(r,l)@[t_b,t_e[, at(r',l')@[t'_b,t'_e[\} \rightarrow (r \neq r') \vee (l = l') \vee (t_e \leq t'_b) \vee (t'_e \leq t_b)$$

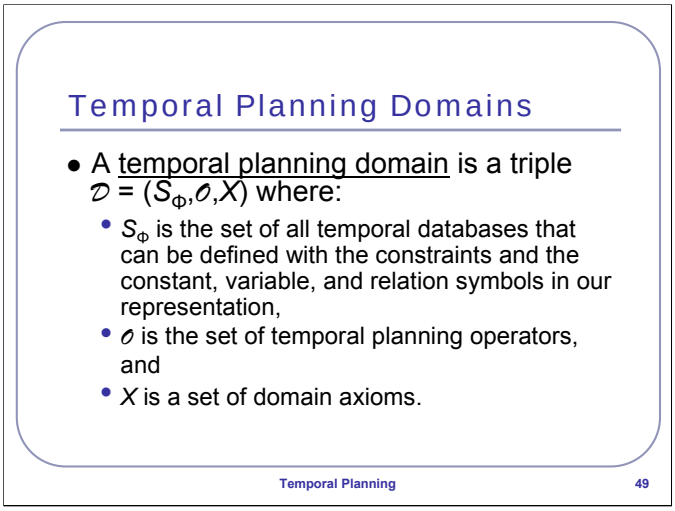
- **every location can be occupied by one robot only:**

$$\{at(r,l)@[t_1,t'_1[, free(l')@[t_2,t'_2[\} \rightarrow (l \neq l') \vee (t'_1 \leq t_2) \vee (t'_2 \leq t_1)$$



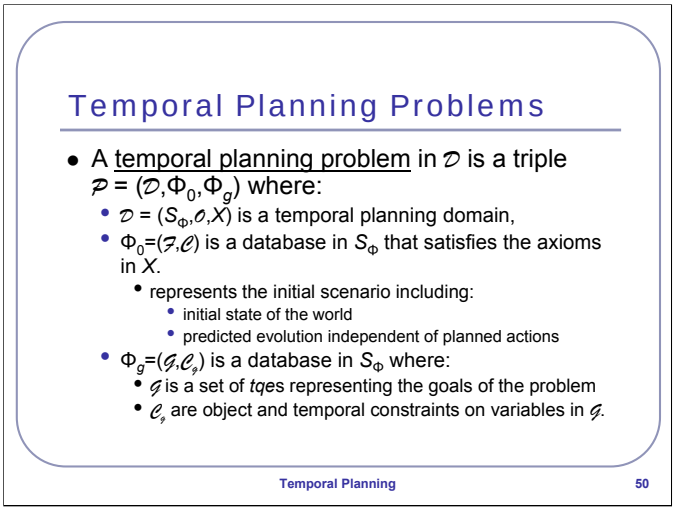
Domain Axioms

- A domain axiom α is an expression of the form: cond(α) → disj(α) where:
  - cond(α) is a set of *tqe*s and
  - disj(α) is a disjunction of temporal and object constraints.
- A temporal database $\Phi=(\mathcal{F},\mathcal{C})$ is consistent with α iff:
  - cond(α) is supported by $\mathcal{F}$ and
  - for every enabling condition $c_1 \in \Theta(\text{cond}(\alpha)/\mathcal{F})$
    - there is at least one disjuct $c_2 \in$ disj(α) such that
      - $\mathcal{C} \cup c_1 \cup c_2$ is consistent.



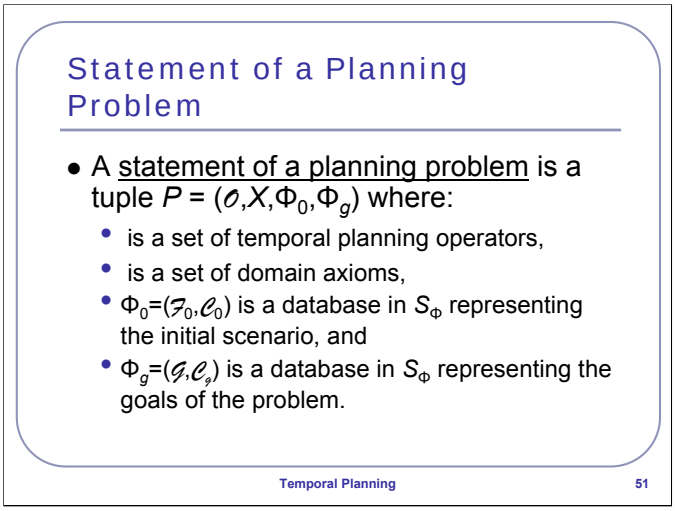
Temporal Planning Domains

- **A temporal planning domain is a triple $\mathcal{D} = (S_\Phi, \mathcal{O}, X)$ where:**
  - **S_Φ is the set of all temporal databases that can be defined with the constraints and the constant, variable, and relation symbols in our representation,**
  - **$\mathcal{O}$ is the set of temporal planning operators, and**
  - **X is a set of domain axioms.**



Temporal Planning Problems

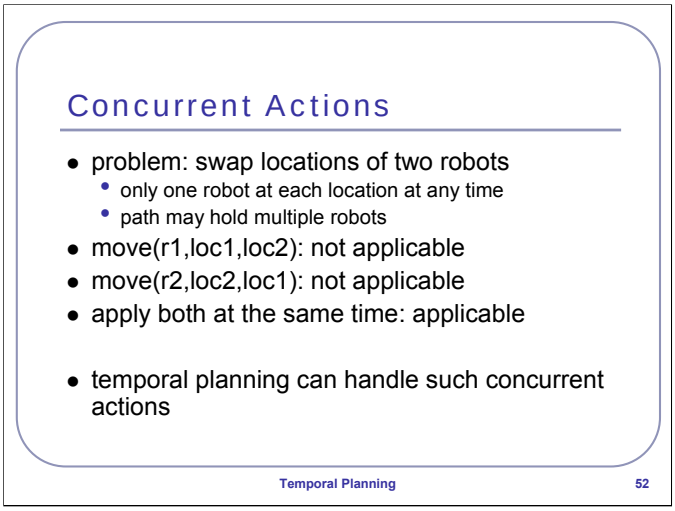
- A temporal planning problem in $\mathcal{D}$ is a triple $\mathcal{P} = (\mathcal{D},\Phi_0,\Phi_g)$ where:
  - $\mathcal{D} = (S_\Phi,\mathcal{O},X)$ is a temporal planning domain,
  - $\Phi_0=(\mathcal{F},\mathcal{C})$ is a database in S_Φ that satisfies the axioms in X.
    - represents the initial scenario including:
      - initial state of the world
      - predicted evolution independent of planned actions
  - $\Phi_g=(\mathcal{G},\mathcal{C}_g)$ is a database in S_Φ where:
    - $\mathcal{G}$ is a set of *tq*es representing the goals of the problem
    - $\mathcal{C}_g$ are object and temporal constraints on variables in $\mathcal{G}$.



Statement of a Planning Problem

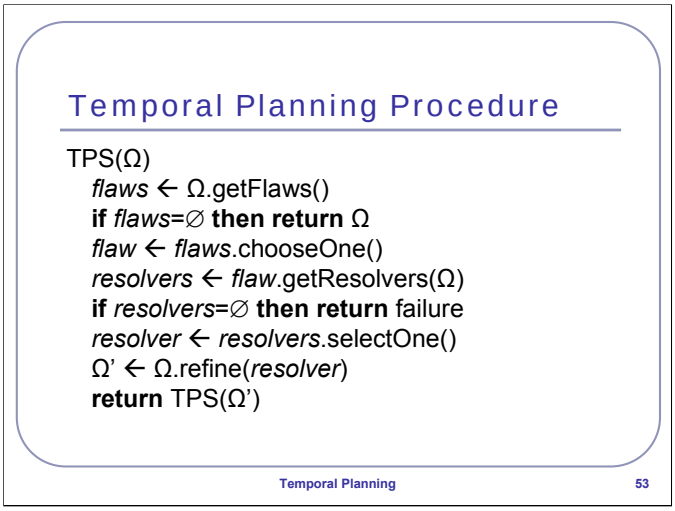
•A statement of a planning problem is a tuple $P = (\mathcal{O},X,\Phi_0,\Phi_g)$ where:

- **is a set of temporal planning operators,**
- **is a set of domain axioms,**
- **$\Phi_0=(\mathcal{F}_0,\mathcal{C}_0)$ is a database in S_Φ representing the initial scenario, and**
- **$\Phi_g=(\mathcal{G},\mathcal{C}_g)$ is a database in S_Φ representing the goals of the problem.**



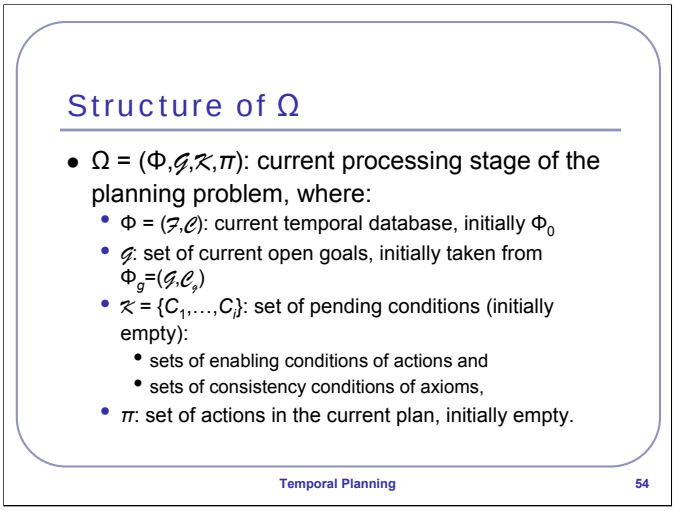
Concurrent Actions

- **problem: swap locations of two robots**
  - **only one robot at each location at any time**
  - **path may hold multiple robots**
- **move(r1,loc1,loc2): not applicable**
- **move(r2,loc2,loc1): not applicable**
- **apply both at the same time: applicable**
- **temporal planning can handle such concurrent actions**
  - higher expressiveness!



Temporal Planning Procedure

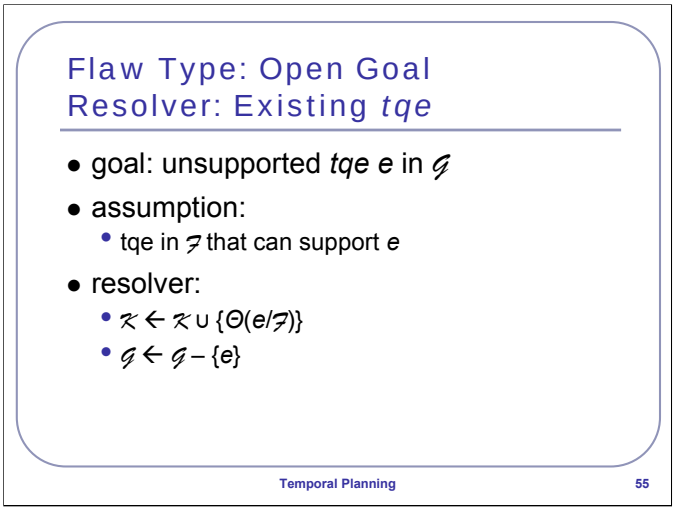
- **TPS(Ω)**
- ***flaws* ← Ω.getFlaws()**
- **if *flaws*=∅ then return Ω**
- ***flaw* ← *flaws*.chooseOne()**
- ***resolvers* ← *flaw*.getResolvers(Ω)**
- **if *resolvers*=∅ then return failure**
- ***resolver* ← *resolvers*.selectOne()**
- **Ω' ← Ω.refine(*resolver*)**
- **return TPS(Ω')**



Structure of Ω

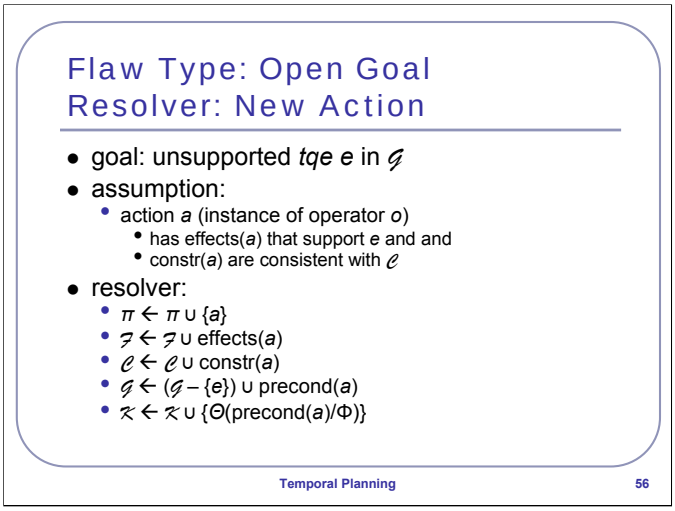
•Ω = (Φ,𝒢,𝒦,π): current processing stage of the planning problem, where:

- **•Φ = (ℱ,𝒞): current temporal database, initially Φ_0**
- **•𝒢: set of current open goals, initially taken from $\Phi_g=(\mathcal{G},\mathcal{C}_g)$**
- **•𝒦 = {C_1,…,C_i}: set of pending conditions (initially empty):**
  - **•sets of enabling conditions of actions and**
  - **•sets of consistency conditions of axioms,**
- **•π: set of actions in the current plan, initially empty.**



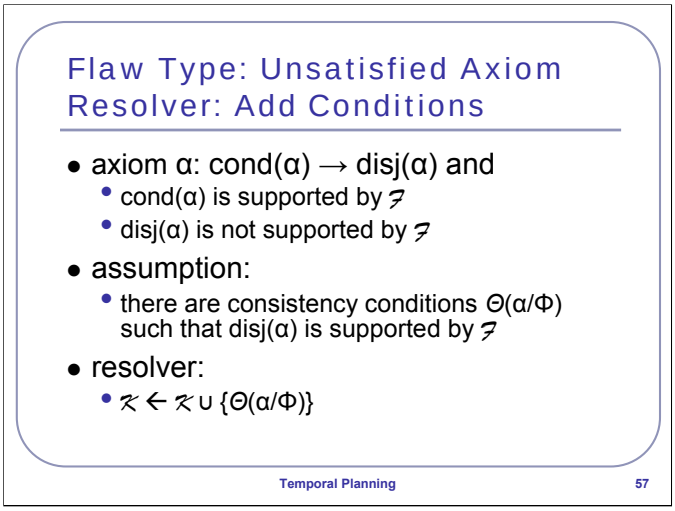
Flaw Type: Open Goal
Resolver: Existing *tqe*

- **goal: unsupported *tqe* e in $\mathcal{G}$**
- **assumption:**
  - **tqe in $\mathcal{F}$ that can support e**
- **resolver:**
  - **$\mathcal{K} \leftarrow \mathcal{K} \cup \{\Theta(e/\mathcal{F})\}$**
  - **$\mathcal{G} \leftarrow \mathcal{G} - \{e\}$**



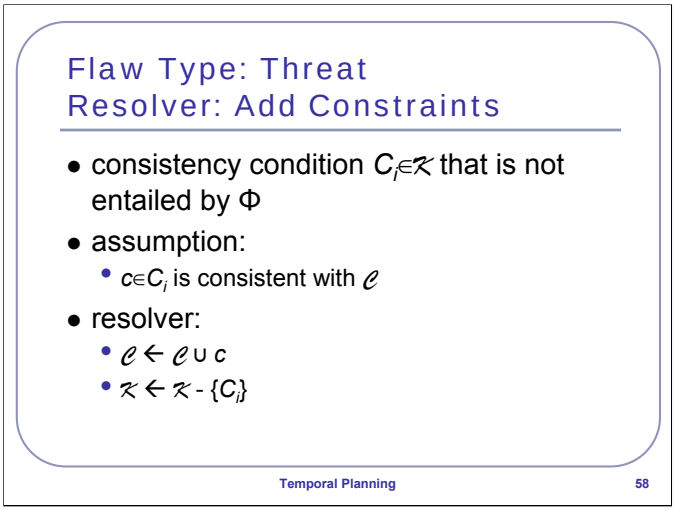
Flaw Type: Open Goal
Resolver: New Action

- **goal: unsupported *tqe* *e* in $\mathcal{G}$**
- **assumption:**
  - **action *a* (instance of operator *o*)**
    - **has effects(*a*) that support *e* and and**
    - **constr(*a*) are consistent with $\mathcal{C}$**
- **resolver:**
  - **$\pi \leftarrow \pi \cup \{a\}$**
  - **$\mathcal{F} \leftarrow \mathcal{F} \cup$ effects(*a*)**
  - **$\mathcal{C} \leftarrow \mathcal{C} \cup$ constr(*a*)**
  - **$\mathcal{G} \leftarrow (\mathcal{G} - \{e\}) \cup$ precond(*a*)**
  - **$\mathcal{K} \leftarrow \mathcal{K} \cup \{\Theta(\text{precond}(a)/\Phi)\}$**



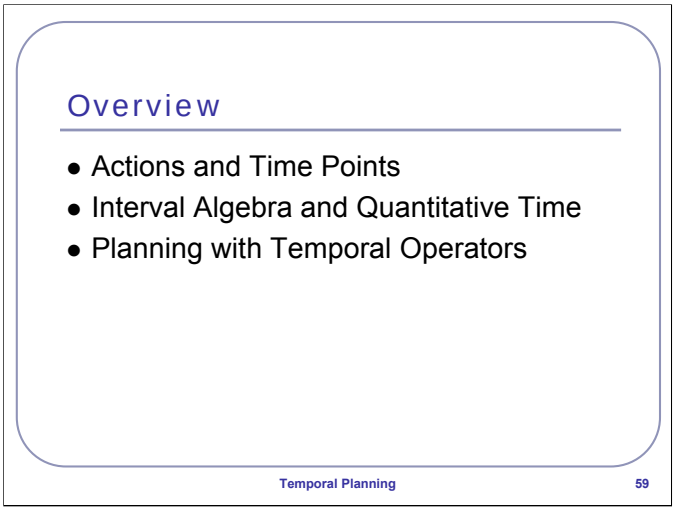
Flaw Type: Unsatisfied Axiom
Resolver: Add Conditions

- **axiom α: cond(α) → disj(α) and**
  - **cond(α) is supported by $\mathcal{F}$**
  - **disj(α) is not supported by $\mathcal{F}$**
- **assumption:**
  - **there are consistency conditions $\Theta(\alpha/\Phi)$ such that disj(α) is supported by $\mathcal{F}$**
- **resolver:**
  - **$\mathcal{K} \leftarrow \mathcal{K} \cup \{\Theta(\alpha/\Phi)\}$**



Flaw Type: Threat
Resolver: Add Constraints

- **consistency condition $C_i \in \mathcal{K}$ that is not entailed by Φ**
- **assumption:**
  - **$c \in C_i$ is consistent with $\mathcal{C}$**
- **resolver:**
  - **$\mathcal{C} \leftarrow \mathcal{C} \cup c$**
  - **$\mathcal{K} \leftarrow \mathcal{K} - \{C_i\}$**



Overview

Actions and Time Points

•Interval Algebra and Quantitative Time

•Planning with Temporal Operators

- •just now: integrating reasoning about time into a planning algorithm