

Scheduling

- **Planning with Actions that Require Resources**

Literature

- Malik Ghallab, Dana Nau, and Paolo Traverso. *Automated Planning – Theory and Practice*, chapter 15. Elsevier/Morgan Kaufmann, 2004.
- Michael Pinedo. *Scheduling: Theory, Algorithms and Systems*, Prentice Hall, 2001.
- Peter Brucker. *Scheduling Algorithms*, Springer Verlag, 2004.

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Planning and Scheduling

- solution to planning problem:
 - plan: partially ordered set of actions
 - actions: fully instantiated operators
 - require resources
- resources:
 - can be modelled as parameters of an action
 - problem: planning algorithms tries out all possibilities (inefficient)
 - alternative approach:
 - allow unbound resource variables in plan (planning)
 - find assignment of resources to actions (scheduling)

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Planning and Scheduling

•solution to planning problem:

- plan: partially ordered set of actions**
- actions: fully instantiated operators**
 - require resources**

•resources:

- can be modelled as parameters of an action**
 - problem: planning algorithms tries out all possibilities (inefficient)**
- alternative approach:**
 - allow unbound resource variables in plan (planning)**
 - planning focuses on causal reasoning (what to do)
 - find assignment of resources to actions (scheduling)**
 - scheduling: resource and time allocation (how and when to do it)
 - planning before scheduling (not optimal approach)

Overview

- Scheduling Problems and Schedules
- Searching for Schedules

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Overview

•Scheduling Problems and Schedules

- now: an overview of different types of scheduling problems

➤Searching for Schedules

Actions and Resources

- resources: an entity needed to perform an action
 - state variables: modified by actions in absolute ways
 - example: $\text{move}(r, l, l')$
 - location changes from l to l'
 - resource variables: modified by actions in relative ways
 - example: $\text{move}(r, l, l')$
 - fuel level changes from f to $f-f'$

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Actions and Resources

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Actions with Time Constraints

- Let a be an action in a planning domain:
 - attached time constraints:
 - earliest start time: $s_{min}(a)$ – actual start time: $s(a)$
 - latest end time: $s_{max}(a)$ – actual end time: $e(a)$
 - duration: $d(a)$
 - action types:
 - preemptive actions: cannot be interrupted
 - $d(a) = e(a) - s(a)$
 - non-preemptive actions: can be interrupted
 - resources available to other actions during interruption

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Actions with Time Constraints

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 - earliest start time: $s_{min}(a)$ – actual start time: $s(a)$
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 - action types:
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 - $d(a) = e(a) - s(a)$
 - non-preemptive actions: can be interrupted
 - resources available to other actions during interruption
 - cost: interruption usually has cost associated
 - further constraints: examples:
 - action must be performed at night
 - interruptions must be at least 30 minutes long

Actions with Resource Constraints

- Let a be an action in a planning domain:
 - attached resource constraints:
 - required resource: r
 - quantity of resource required: q
 - reusable: resource will be available to other actions after this action is completed
 - consumable: resource will be consumed when action is complete

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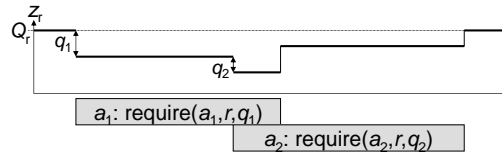
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Actions with Resource Constraints

- Let a be an action in a planning domain:
 - attached resource constraints:
 - required resource: r
 - quantity of resource required: q
 - reusable: resource will be available to other actions after this action is completed
 - tools, machines, HD space, helicopters, docks
 - consumable: resource will be consumed when action is complete
 - petrol, electricity, CPU time, credit
 - time is usually treated differently, as a special case

Reusable Resources

- resource availability:
 - total capacity: Q_r
 - current level at time t : $z_r(t)$
- resource requirements:
 - $\text{require}(a,r,q)$: action a requires q units of resource r while it is active
- resource profile:



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Reusable Resources

•resource availability:

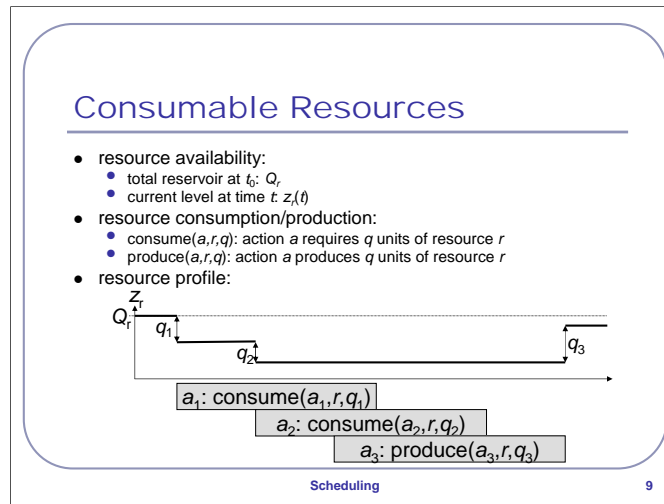
- total capacity: Q_r
- current level at time t : $z_r(t)$

•resource requirements:

- $\text{require}(a,r,q)$: action a requires q units of resource r while it is active

•resource profile:

- [figure]
- actions are overlapping (temporally)
- profile shows availability of resource to other actions
 - returns to full capacity when all actions are completed



Consumable Resources

•resource availability:

•total reservoir at t_0 : Q_r

•current level at time t : $z_r(t)$

•resource consumption/production:

• $\text{consume}(a,r,q)$: action a requires q units of resource r

• $\text{produce}(a,r,q)$: action a produces q units of resource r

•resource profile:

•[figure]

•actions are overlapping (temporally)

•profile shows availability of resource to other actions

•availability at end usually different from beginning

•resource profile as step function: usually not accurate

Other Resource Features

- discrete vs. continuous
 - countable number of units: cranes, bolts
 - real-valued amount: bandwidth, electricity
- unary
 - $Q_r=1$; exactly one resource of this type available
- sharable
 - can be used by several actions at the same time
- resources with states
 - actions may require resources in specific state

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Other Resource Features

•discrete vs. continuous

- countable number of units: cranes, bolts

- real-valued amount: bandwidth, electricity

•unary

- $Q_r=1$; exactly one resource of this type available

•sharable

- can be used by several actions at the same time

•resources with states

- actions may require resources in specific state

- example: freezer with temperature setting

Combining Resource Constraints

- conjunction:
 - action uses multiple resources while being performed
- disjunction:
 - action requires resources as alternatives
 - cost/time may depend on resource used
- resource types:
 - resource-class(s) = $\{r_1, \dots, r_m\}$: require(a, s, q)
 - equivalent to disjunction over identical resources

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Combining Resource Constraints

•conjunction:

- action uses multiple resources while being performed

•disjunction:

- action requires resources as alternatives
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•resource types:

- resource-class(s) = $\{r_1, \dots, r_m\}$: require(a, s, q)
- equivalent to disjunction over identical resources

Cost Functions and Optimization Criteria

- cost function parameters
 - quantity of resource required
 - duration of requirement
- optimization criteria:
 - total schedule cost
 - makespan (end time of last action)
 - weighted completion time
 - (weighted) number of late actions
 - (weighted) maximum tardiness
 - resource usage

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Cost Functions and Optimization Criteria

•cost function parameters

- quantity of resource required
- duration of requirement

•optimization criteria:

- total schedule cost
- makespan (end time of last action)
- weighted completion time
- (weighted) number of late actions
- (weighted) maximum tardiness
- resource usage

Machine Scheduling

- machine: resource of unit capacity
 - either available or not available at time t
 - cannot process two actions at the same time
- job j : partially ordered set of actions a_{j1}, \dots, a_{jk}
 - action a_{ji} requires
 - one resource type
 - for a number of time units
 - actions in same job must be processed sequentially
 - actions in different jobs are independent (not ordered)
- machine scheduling problem:
 - given: n jobs and m machines
 - schedule: mapping from actions to machines + start times

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Machine Scheduling

- class of problems
- machine: resource of unit capacity**
 - either available or not available at time t**
 - cannot process two actions at the same time**
- job j : partially ordered set of actions a_{j1}, \dots, a_{jk}**
 - in general, jobs can have different numbers of activities
 - action a_{ji} requires**
 - one resource type**
 - for a number of time units**
 - actions in same job must be processed sequentially**
 - even if they are only partially ordered: object that is being worked on
 - actions in different jobs are independent (not ordered)**
- machine scheduling problem:**
 - given: n jobs and m machines**
 - schedule: mapping from actions to machines + start times**

Example: Scheduling Problem

- machines:
 - m_1 of resource type r_1
 - m_2, m_3 of resource type r_2
- jobs:
 - $j_1: \langle r_1(3), r_2(3), r_1(3) \rangle$
 - three actions, totally ordered
 - a_{11} requires 3 units of resource type 1, etc.
 - $j_2: \langle r_2(3), r_1(5) \rangle$
 - $j_3: \langle r_1(3), r_1(2), r_2(3), r_1(5) \rangle$

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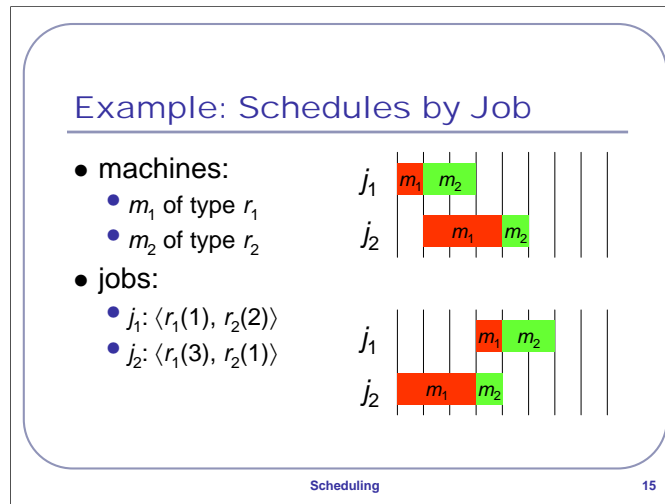
Example: Scheduling Problem

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- $j_3: \langle r_1(3), r_1(2), r_2(3), r_1(5) \rangle$



Example: Schedules by Job

- machines:

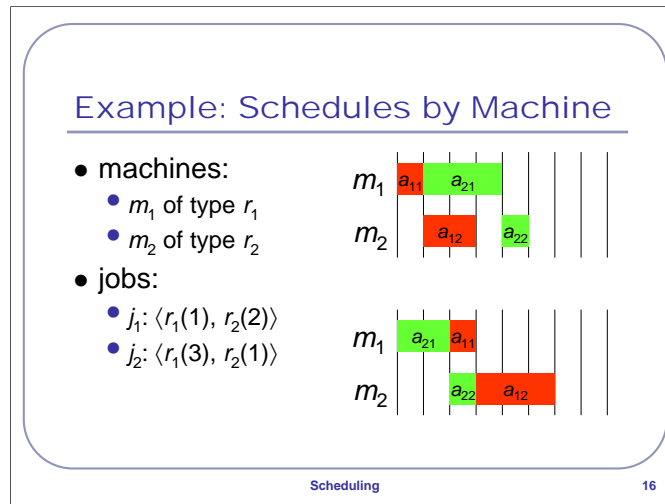
- m_1 of type r_1
- m_2 of type r_2

- jobs:

- $j_1: \langle r_1(1), r_2(2) \rangle$
- $j_2: \langle r_1(3), r_2(1) \rangle$

- [figures]

- schedules showing machines assigned to actions in jobs



Example: Schedules by Machine

- machines:

- m_1 of type r_1
- m_2 of type r_2

- jobs:

- $j_1: \langle r_1(1), r_2(2) \rangle$
- $j_2: \langle r_1(3), r_2(1) \rangle$

- [figures]

- schedules showing actions assigned to machines

Overview

- ◆ Scheduling Problems and Schedules
- Searching for Schedules

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Overview

•Scheduling Problems and Schedules

- just done: an overview of different types of scheduling problems

◆Searching for Schedules

- ◆now: search algorithms that generate schedules

Assignable Actions

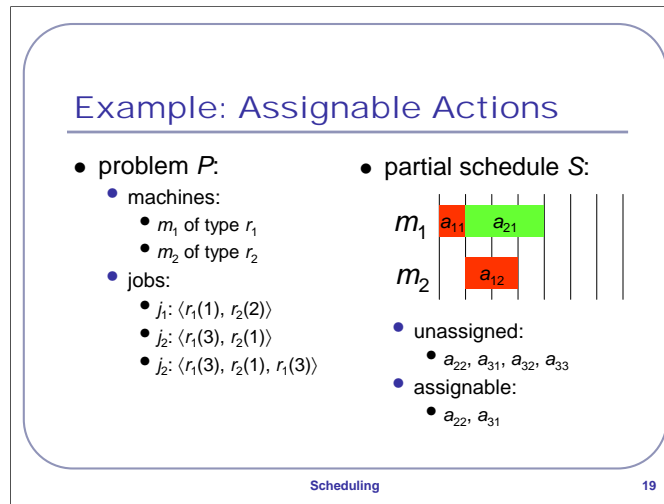
- Let P be a machine scheduling problem. Let S be a partially defined schedule.
- An action a_{j_i} of some job j_i in P is unassigned if it does not appear in S .
- An action a_{j_i} of some job j_i in P is assignable if it has no unassigned predecessors in S .

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Assignable Actions

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- An action a_{j_i} of some job j_i in P is unassigned if it does not appear in S .
- An action a_{j_i} of some job j_i in P is assignable if it has no unassigned predecessors in S .
 - all predecessors in schedule; action is ready to be executed



Example: Assignable Actions

•problem P :

•machines:

• m_1 of type r_1

• m_2 of type r_2

•jobs:

• $j_1: \langle r_1(1), r_2(2) \rangle$

• $j_2: \langle r_1(3), r_2(1) \rangle$

• $j_2: \langle r_1(3), r_2(1), r_1(3) \rangle$

•[figure]

•unassigned:

• $a_{22}, a_{31}, a_{32}, a_{33}$

•assignable:

• a_{22}, a_{31}

Earliest Assignable Time

- Let a_{ji} be an assignable action in S . The earliest assignable time for a_{ji} on machine m in S is:
 - the end of the last action currently scheduled on m in S , or
 - the end of the last predecessor ($a_{j_0} \dots a_{j_{i-1}}$) in S ,whichever comes later.

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Earliest Assignable Time

• Let a_{ji} be an assignable action in S . The earliest assignable time for a_{ji} on machine m in S is:

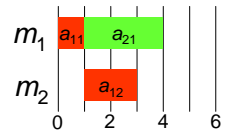
- the end of the last action currently scheduled on m in S ,
or
- the end of the last predecessor ($a_{j_0} \dots a_{j_{i-1}}$) in S ,
- whichever comes later.
- note: assignment not necessarily optimal!

Example: Earliest Assignable Time

- problem P :

- machines:
 - m_1 of type r_1
 - m_2 of type r_2
- jobs:
 - $j_1: \langle r_1(1), r_2(2) \rangle$
 - $j_2: \langle r_1(3), r_2(1) \rangle$
 - $j_3: \langle r_1(3), r_2(1), r_1(3) \rangle$

- partial schedule S :



- earliest assignable time for a_{22} on m_2 : 4
- earliest assignable time for a_{31} on m_1 : 4

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Example: Earliest Assignable Time

- problem P :

- machines:

- m_1 of type r_1
- m_2 of type r_2

- jobs:

- $j_1: \langle r_1(1), r_2(2) \rangle$
- $j_2: \langle r_1(3), r_2(1) \rangle$
- $j_3: \langle r_1(3), r_2(1), r_1(3) \rangle$

- [figure]

- earliest assignable time for a_{22} on m_2 : 4
- earliest assignable time for a_{31} on m_1 : 4

Heuristic Search

```
heuristicScheduler(P,S)
  assignables ← P.getAssignables(S)
  if assignables.isEmpty() then return S
  action ← assignables.selectOne()
  machines ← P.getMachines(action)
  machine ← machines.selectOne()
  time ← S.getEarliestAssignableTime(action, machine)
  S ← S + assign(action, machine, time)
  return heuristicScheduler(P,S)
```

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- **Heuristic Search**
- **heuristicScheduler(*P*,*S*)**
- ***assignables* ← *P*.getAssignables(*S*)**
- **if *assignables*.isEmpty() then return *S***
- ***action* ← *assignables*.selectOne()**
- ***machines* ← *P*.getMachines(*action*)**
- ***machine* ← *machines*.selectOne()**
- ***time* ← *S*.getEarliestAssignableTime(*action*, *machine*)**
- ***S* ← *S* + assign(*action*, *machine*, *time*)**
- **return heuristicScheduler(*P*,*S*)**

Using Local Search

- issues:
 - representing schedules
 - generating a random initial schedule
 - generating neighbours
 - evaluating neighbours (schedules)

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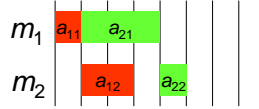
Using Local Search

•issues:

- representing schedules
- generating a random initial schedule
- generating neighbours
- evaluating neighbours (schedules)

Schedule Representation

- representation:
 - totally ordered list of all actions with assigned machines
 - example: $\langle (a_{11}, m_1), (a_{21}, m_1), (a_{12}, m_2), (a_{22}, m_2) \rangle$
- schedule:
 - assign actions in sequence to given machines at earliest assignable times
 - example:



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Schedule Representation

•representation:

- totally ordered list of all actions with assigned machines

- example: $\langle (a_{11}, m_1), (a_{21}, m_1), (a_{12}, m_2), (a_{22}, m_2) \rangle$

•schedule:

- assign actions in sequence to given machines at earliest assignable times

- example:

- [figure]

Initial Schedule and Evaluation

- generating random schedules:
 - randomly choose an assignable action
 - randomly choose a machine of the right resource type for that action
 - append the action-machine pair to the list of assignments
 - do this until all actions are assigned
- evaluating schedules:
 - generate schedule from list
 - apply optimization criterion

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Initial Schedule and Evaluation

•generating random schedules:

- randomly choose an assignable action

- randomly choose a machine of the right resource type for that action

- append the action-machine pair to the list of assignments

- do this until all actions are assigned

•evaluating schedules:

- generate schedule from list

- apply optimization criterion

Generating Neighbours

- machine neighbours:
 - change the machine assigned to an action to any other machine
- position neighbours:
 - change the position of an action a in the list:
 - a_{min} : the latest predecessor of a in the current list
 - a_{max} : the earliest successor of a in the current list
 - move a anywhere between a_{min} and a_{max}

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Generating Neighbours

• machine neighbours:

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• position neighbours:

- change the position of an action a in the list:

- a_{min} : the latest predecessor of a in the current list
- a_{max} : the earliest successor of a in the current list
- move a anywhere between a_{min} and a_{max}

LocalSearchScheduler: Pseudo Code

```
function LocalSearchScheduler(P)
  best ← randomSchedule(P)
  loop MAXLOOP times
    S ← randomSchedule(P)
    do
      succs ← S.getBestNeighbours(P)
      next ← succs.selectOne()
      if S.evaluate() < next.evaluate() then
        S ← next
    while S = next
    if S.evaluate() > best.evaluate() then
      best ← S
  return best
```

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- LocalSearchScheduler: Pseudo Code
- **function LocalSearchScheduler(*P*)**
- ***best* ← randomSchedule(*P*)**
 - will contain best schedule found
- **loop MAXLOOP times**
- ***S* ← randomSchedule(*P*)**
 - best schedule for local search
- **do**
- ***succs* ← *S*.getBestNeighbours(*P*)**
 - returns set of neighbours with highest value for evaluation function
- ***next* ← *succs*.selectOne()**
 - randomly select a (best) neighbour
- **if *S*.evaluate() < *next*.evaluate() then**
- ***S* ← *next***
 - remember best local neighbour
- **while *S* = *next***
 - stop local search when no uphill move possible
- **if *S*.evaluate() > *best*.evaluate() then**
- ***best* ← *S***
 - remember best overall
- **return *best***

Overview

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- Searching for Schedules

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Overview

•Scheduling Problems and Schedules

➡Searching for Schedules

➡just done: search algorithms that generate schedules