The Graphplan Planner

Searching the Planning Graph

Literature

- Malik Ghallab, Dana Nau, and Paolo Traverso. *Automated Planning – Theory and Practice*, chapter 6. Elsevier/Morgan Kaufmann, 2004.

Neoclassical Planning

- concerned with restricted state-transition systems
- representation is usually restricted to propositional STRIPS
- neoclassical vs. classical planning
  - classical planning: search space consists of nodes containing partial plans
  - neoclassical planning: nodes can be seen as sets of partial plans
- resulted in significant speed-up and revival of planning research

Overview

- The Propositional Representation
- The Planning-Graph Structure
- The Graphplan Algorithm

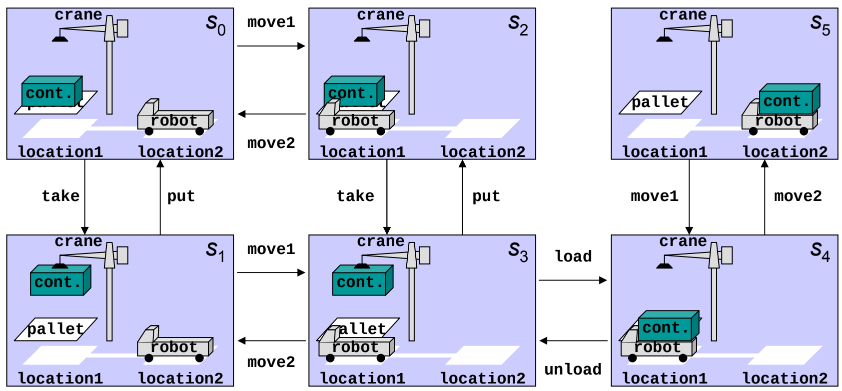
Classical Representations

- propositional representation
  - world state is set of propositions
  - action consists of precondition propositions, propositions to be added and removed
- STRIPS representation
  - like propositional representation, but first-order literals instead of propositions
- state-variable representation
  - state is tuple of state variables $\{x_1,\ldots,x_n\}$
  - action is partial function over states

Propositional Planning Domains

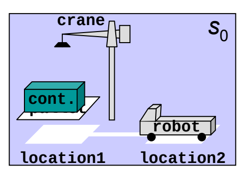
- Let $L=\{p_1,\ldots,p_n\}$ be a finite set of proposition symbols. A propositional planning domain on L is a restricted state-transition system $\Sigma=(S,A,\gamma)$ such that:
  - $S \subseteq 2^L$, i.e. each state s is a subset of L
  - $A \subseteq 2^L \times 2^L \times 2^L$, i.e. each action a is a triple (precond(a), effects$^-$(a), effects$^+$(a)) where effects$^-$(a) and effects$^+$(a) must be disjoint
  - $\gamma: S \times A \rightarrow 2^L$ where
    - $\gamma(s,a)=(s$ - effects$^-(a)) \cup$ effects$^+(a)$ if precond$(a) \subseteq s$
    - $\gamma(s,a)$=undefined otherwise
  - S is closed under γ

DWR Example: State Space



DWR Example: Propositional States

- L={onpallet,onrobot,holding,at1,at2}
- $S=\{s_0,\ldots,s_5\}$
  - s_0={onpallet,at2}
  - s_1={holding,at2}
  - s_2={onpallet,at1}
  - s_3={holding,at1}
  - s_4={onrobot,at1}
  - s_5={onrobot,at2}



DWR Example: Propositional Actions

a	precond(a)	effects$^-$(a)	effects$^+$(a)
take	{onpallet}	{onpallet}	{holding}
put	{holding}	{holding}	{onpallet}
load	{holding,at1}	{holding}	{onrobot}
unload	{onrobot,at1}	{onrobot}	{holding}
move1	{at2}	{at2}	{at1}
move2	{at1}	{at1}	{at2}

DWR Example: Propositional State Transitions

	s_0	s_1	s_2	s_3	s_4	s_5
take	s_1		s_3			
put		s_0		s_2		
load				s_4		
unload					s_3	
move1			s_0	s_1		s_4
move2	s_2	s_3			s_5	

Propositional Planning Problems

- A <u>propositional planning problem</u> is a triple $\mathcal{P}=(\Sigma,s_i,g)$ where:
  - $\Sigma=(S,A,\gamma)$ is a propositional planning domain on $L=\{p_1,\ldots,p_n\}$
  - $s_i \in S$ is the initial state
  - $g \subseteq L$ is a set of <u>goal propositions</u> that define the set of goal states $S_g=\{s \in S \mid g \subseteq s\}$

DWR Example: Propositional Planning Problem

- Σ: propositional planning domain for DWR domain
- s_i: any state
  - example: initial state = $s_0 \in S$
- g: any subset of L
  - example: $g=$\{onrobot,at2\}, i.e. $S_g=\{s_5\}$

Classical Plans

- A plan is any sequence of actions $\pi=\langle a_1,\ldots,a_k\rangle$, where $k\geq 0$.
  - The length of plan π is $|\pi|=k$, the number of actions.
  - If $\pi_1=\langle a_1,\ldots,a_k\rangle$ and $\pi_2=\langle a'_1,\ldots,a'_j\rangle$ are plans, then their concatenation is the plan $\pi_1\bullet\pi_2=\langle a_1,\ldots,a_k,a'_1,\ldots,a'_j\rangle$.
  - The extended state transition function for plans is defined as follows:
    - $\gamma(s,\pi)=s$ if $k=0$ (π is empty)
    - $\gamma(s,\pi)=\gamma(\gamma(s,a_1),\langle a_2,\ldots,a_k\rangle)$ if $k>0$ and a_1 applicable in s
    - $\gamma(s,\pi)=$undefined otherwise

Classical Solutions

- Let $\mathcal{P}=(\Sigma,s_i,g)$ be a propositional planning problem. A plan π is a solution for $\mathcal{P}$ if $g\subseteq\gamma(s_i,\pi)$.
  - A solution π is redundant if there is a proper subsequence of π is also a solution for $\mathcal{P}$.
  - π is minimal if no other solution for $\mathcal{P}$ contains fewer actions than π.

DWR Example: Plans and Solutions

plan π	$\mid\pi\mid$	$\gamma(s_i,\pi)$	sol.	red.	min.
$\langle\rangle$	0	s_0	no	-	-
$\langle$move2,move2$\rangle$	2	undef.	no	-	-
$\langle$take,move1$\rangle$	2	s_3	no	-	-
$\langle$take,move1,put,move2, take,move1,load,move2$\rangle$	8	s_5	yes	yes	no
$\langle$take,move1,load,move2$\rangle$	4	s_5	yes	no	yes
$\langle$move1,take,load,move2$\rangle$	4	s_5	yes	no	yes

Reachable Successor States

- The successor function $\Gamma^m:2^S\rightarrow 2^S$ for a propositional domain $\Sigma=(S,A,\gamma)$ is defined as:
  - $\Gamma(s)=\{\gamma(s,a) \mid a\in A$ and a applicable in $s\}$ for $s\in S$
  - $\Gamma(\{s_1,\ldots,s_n\})= \bigcup_{(k\in[1,n])}\Gamma(s_k)$
  - $\Gamma^0(\{s_1,\ldots,s_n\})= \{s_1,\ldots,s_n\}$
  - $\Gamma^m(\{s_1,\ldots,s_n\})= \Gamma(\Gamma^{m-1}(\{s_1,\ldots,s_n\}))$

  $\Big\}$ $s_1,\ldots,s_n\in S$
- The transitive closure of Γ defines the set of all reachable states:
  - $\Gamma^>(s)= \bigcup_{(k\in[0,\infty])}\Gamma^k(\{s\})$ for $s\in S$

Relevant Actions and Regression Sets

- Let $\mathcal{P}=(\Sigma,s_i,g)$ be a propositional planning problem. An action $a \in A$ is <u>relevant for g</u> if
  - $g \cap \text{effects}^+(a) \neq \{\}$ and
  - $g \cap \text{effects}^-(a) = \{\}$.
- The <u>regression set</u> of g for a relevant action $a \in A$ is:
  - $\gamma^{-1}(g,a)=(g - \text{effects}^+(a)) \cup \text{precond}(a)$
  - note: $\gamma(s,a) \in S_g$ iff $\gamma^{-1}(g,a) \subseteq s$

Regression Function

- The <u>regression function</u> Γ^{-m} for a propositional domain $\Sigma=(S,A,\gamma)$ on L is defined as:
  - $\Gamma^{-1}(g)=\{\gamma^{-1}(g,a) \mid a \in A \text{ is relevant for } g\}$ for $g \in 2^L$
  - $\Gamma^{0}(\{g_1,\ldots,g_n\})= \{g_1,\ldots,g_n\}$
  - $\Gamma^{-1}(\{g_1,\ldots,g_n\})= \cup_{(k \in [1,n])}\Gamma^{-1}(g_k)$
  - $\Gamma^{-m}(\{g_1,\ldots,g_n\})= \Gamma^{-1}(\Gamma^{-(m-1)}(\{g_1,\ldots,g_n\}))$

  $\left.\right\}$ $g_1,\ldots,g_n \in 2^L$
- The transitive closure of Γ^{-1} defines the <u>set of all regression sets</u>:
  - $\Gamma^{<}(g)= \cup_{(k \in [0,\infty])}\Gamma^{-k}(\{g\})$ for $g \in 2^L$

Statement of a Propositional Planning Problem

- A statement of a propositional planning problem is a triple $P=(A,s_i,g)$ where:
  - A is a set of actions in an appropriate propositional planning domain $\Sigma=(S,A,\gamma)$ on L
  - s_i is the initial state in an appropriate propositional planning problem $\mathcal{P}=(\Sigma,s_i,g)$
  - g is a set of goal propositions in the same propositional planning problem $\mathcal{P}$

Example: Ambiguity in Statement of a Planning Problem

- statement: $P=(\{a_1\}, s_i, g)$ where $a_1=(\{p_1\},\{p_1\},\{p_2\})$, $s_i=\{p_1\}$, and $g=\{p_2\}$

- $\mathcal{P}_1=(\Sigma_1,s_i,g)$ where
- $\Sigma_1=($
  - $\{\{p_1\},\{p_2\}\}$,
  - $\{a_1\}$,
  - $\{(\{p_1\},a_1)\rightarrow\{p_2\}\})$ on
- $L_1=\{p_1,p_2\}$

- $\mathcal{P}_2=(\Sigma_2,s_i,g)$ where
- $\Sigma_2=($
  - $\{\{p_1\},\{p_2\},\{p_1,p_3\},\{p_2,p_3\}\}$,
  - $\{a_1\}$,
  - $\{(\{p_1\},a_1)\rightarrow\{p_2\}$, $(\{p_1,p_3\},a_1)\rightarrow\{p_2,p_3\}\})$ on
- $L_2=\{p_1,p_2,p_3\}$

Statement Ambiguity

- **Proposition**: Let $\mathcal{P}_1$ and $\mathcal{P}_2$ be two propositional planning problems that have the same statement. Then both, $\mathcal{P}_1$ and $\mathcal{P}_2$, have
  - the same set of reachable states $\Gamma^{>}(\{s_i\})$ and
  - the same set of solutions.

Properties of the Propositional Representation

- **Expressiveness**: For every propositional planning domain there is a corresponding state-transition system, but what about vice versa?
- **Conciseness**: propositional action representation is concise because it does not mention what does not change
- **Consistency**: not every assignment of truth values to propositions must correspond to a state in the underlying state-transition system

Grounding a STRIPS Planning Problem

- Let $P=(O,s_i,g)$ be the statement of a STRIPS planning problem and C the set of all the constant symbols that are mentioned in s_i. Let ground(O) be the set of all possible instantiations of operators in O with constant symbols from C consistently replacing variables in preconditions and effects.
- Then P'=(ground(O),s_i,g) is a statement of a STRIPS planning problem and P' has the same solutions as P.

Translation: Propositional Representation to Ground STRIPS

- Let $P=(A,s_i,g)$ be a statement of a propositional planning problem. In the actions A:
  - replace every action (precond(a), effects$^-$(a), effects$^+$(a)) with an operator o with
    - some unique name(o),
    - precond(o) = precond(a), and
    - effects(o) = effects$^+$(a) $\cup$ $\{\neg p \mid p \in$ effects$^-$(a)$\}$.

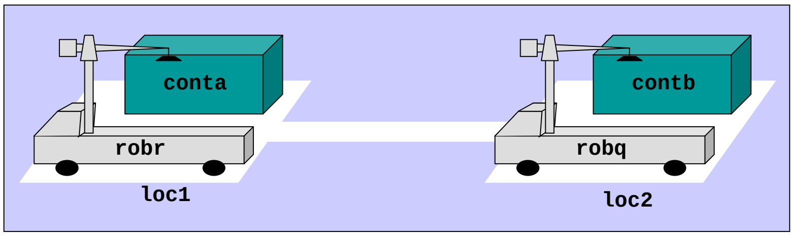
Translation: Ground STRIPS to Propositional Representation

- Let $P=(O,s_i,g)$ be a ground statement of a classical planning problem.
  - In the operators O, in the initial state s_i, and in the goal g replace every atom $P(v_1,\ldots,v_n)$ with a propositional atom $Pv_1,\ldots,v_n$.
  - In every operator o:
    - for all $\neg p$ in precond(o), replace $\neg p$ with p',
    - if p in effects(o), add $\neg p'$ to effects(o),
    - if $\neg p$ in effects(o), add p' to effects(o).
  - In the goal replace $\neg p$ with p'.
  - For every operator o create an action (precond(o), effects$^-$(a), effects$^+$(a)).

Overview

- The Propositional Representation
- ➨ The Planning-Graph Structure
- The Graphplan Algorithm

Example: Simplified DWR Problem



- robots can load and unload autonomously
- locations may contain unlimited number of robots and containers
- problem: swap locations of containers

Simplified DWR Problem: STRIPS Actions

- move(r,l,l')
  - precond: at(r,l), adjacent(l,l')
  - effects: at(r,l'), ¬at(r,l)
- load(c,r,l)
  - precond: at(r,l), in(c,l), unloaded(r)
  - effects: loaded(r,c), ¬in(c,l), ¬unloaded(r)
- unload(c,r,l)
  - precond: at(r,l), loaded(r,c)
  - effects: unloaded(r), in(c,l), ¬loaded(r,c)

Simplified DWR Problem: State Proposition Symbols

- robots:
  - *r1* and *r2*: at(robr,loc1) and at(robr,loc2)
  - *q1* and *q2*: at(robq,loc1) and at(robq,loc2)
  - *ur* and *uq*: unloaded(robr) and unloaded(robq)
- containers:
  - *a1*, *a2*, *ar*, and *aq*: in(conta,loc1), in(conta,loc2), loaded(conta,robr), and loaded(conta,robq)
  - *b1*, *b2*, *br*, and *bq*: in(contb,loc1), in(contb,loc2), loaded(contb,robr), and loaded(contb,robq)
- initial state: {*r1*, *q2*, *a1*, *b2, ur, uq*}

Simplified DWR Problem: Action Symbols

- move actions:
  - Mr12: move(robr,loc1,loc2), Mr21: move(robr,loc2,loc1), Mq12: move(robq,loc1,loc2), Mq21: move(robq,loc2,loc1)
- load actions:
  - Lar1: load(conta,robr,loc1); Lar2, Laq1, Laq2, Lar1, Lbr2, Lbq1, and Lbq2 correspondingly
- unload actions:
  - Uar1: unload(conta,robr,loc1); Uar2, Uaq1, Uaq2, Uar1, Ubr2, Ubq1, and Ubq2 correspondingly

Solution Existence

- **Proposition**: A propositional planning problem $\mathcal{P}=(\Sigma,s_i,g)$ has a solution iff $S_g \cap \Gamma^{>}(\{s_i\}) \neq \{\}$.

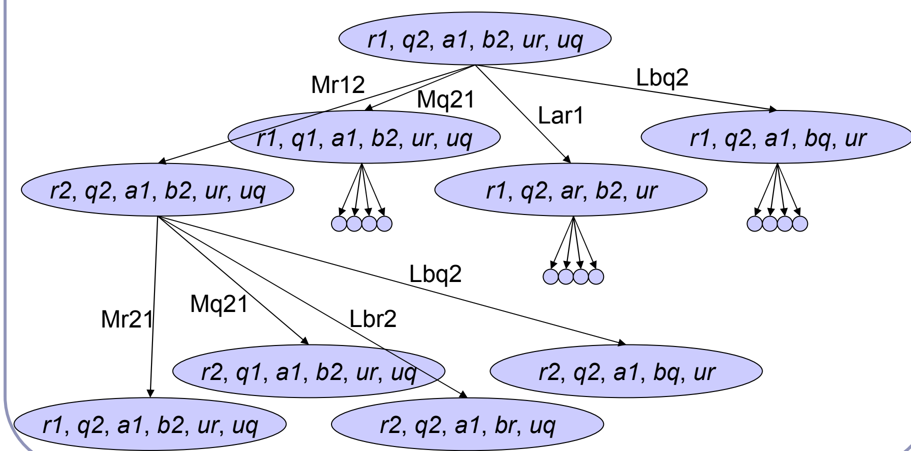
- **Proposition**: A propositional planning problem $\mathcal{P}=(\Sigma,s_i,g)$ has a solution iff $\exists s \in \Gamma^{<}(\{g\}) : s \subseteq s_i$.

Reachability Tree

- tree structure, where:
  - root is initial state s_i
  - children of node s are $\Gamma(\{s\})$
  - arcs are labelled with actions
- all nodes in reachability tree are $\Gamma^{>}(\{s_i\})$
  - all nodes to depth d are $\Gamma^{d}(\{s_i\})$
  - solves problems with up to d actions in solution

- problem: $O(k^d)$ nodes;
  k = applicable actions per state

DWR Example: Reachability Tree



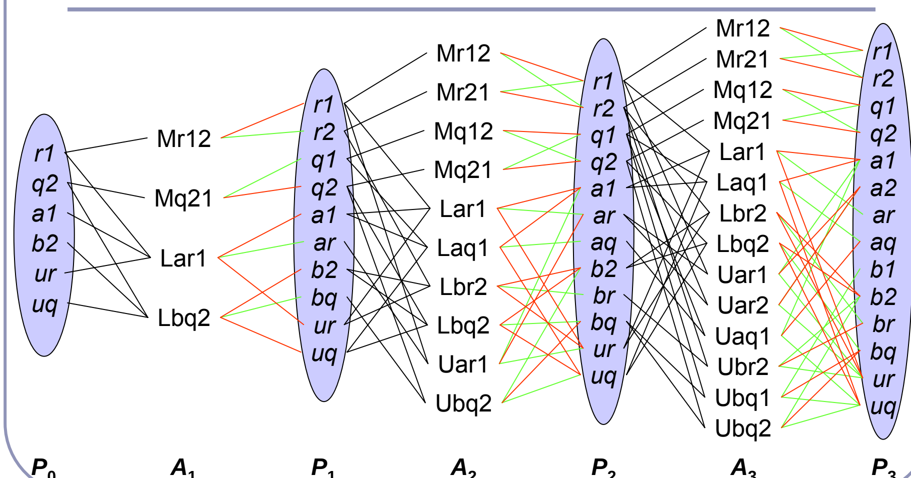
Planning Graph: Nodes

- layered directed graph $G=(N,E)$:
  - $N = P_0 \cup A_1 \cup P_1 \cup A_2 \cup P_2 \cup \ldots$
    - state proposition layers: P_0, P_1, …
    - action layers: A_1, A_2, …
- first proposition layer P_0:
  - propositions in initial state s_i: $P_0=s_i$
- action layer A_j:
  - all actions a where: precond(a)$\subseteq P_{j-1}$
- proposition layer P_j:
  - all propositions p where: $p \in P_{j-1}$ or $\exists a \in A_j$: $p \in$effects$^+$(a)

Planning Graph: Arcs

- from proposition $p \in P_{j-1}$ to action $a \in A_j$:
  - if: $p \in$ precond(a)
- from action $a \in A_j$ to layer $p \in P_j$:
  - positive arc if: $p \in$ effects$^+$(a)
  - negative arc if: $p \in$ effects$^-$(a)
- no arcs between other layers

Planning Graph Example

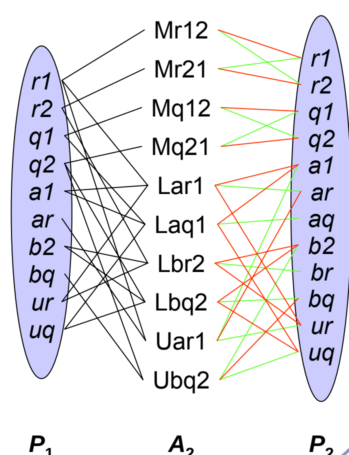


Reachability in the Planning Graph

- reachability analysis:
  - if a goal g is reachable from initial state s_i
  - then there will be a proposition layer P_g in the planning graph such that $g \subseteq P_g$
- necessary condition, but not sufficient
- low complexity:
  - planning graph is of polynomial size and
  - can be computed in polynomial time

Independent Actions: Examples

- Mr12 and Lar1:
  - cannot occur together
  - Mr12 deletes precondition *r1* of Lar1
- Mr12 and Mr21:
  - cannot occur together
  - Mr12 deletes positive effect *r1* of Mr21
- Mr12 and Mq21:
  - may occur in same action layer



Independent Actions

- Two actions a_1 and a_2 are independent iff:
  - $\text{effects}^-(a_1) \cap (\text{precond}(a_2) \cup \text{effects}^+(a_2)) = \{\}$ and
  - $\text{effects}^-(a_2) \cap (\text{precond}(a_1) \cup \text{effects}^+(a_1)) = \{\}$.
- A set of actions π is independent iff every pair of actions $a_1,a_2 \in \pi$ is independent.

Pseudo Code: independent

function independent(a_1,a_2)
  for all $p \in \text{effects}^-(a_1)$
    if $p \in \text{precond}(a_2)$ **or** $p \in \text{effects}^+(a_2)$ **then**
      return false
  for all $p \in \text{effects}^-(a_2)$
    if $p \in \text{precond}(a_1)$ **or** $p \in \text{effects}^+(a_1)$ **then**
      return false
  return true

Applying Independent Actions

- A set π of independent actions is <u>applicable</u> to a state s iff
  $\bigcup_{a \in \pi} \text{precond}(a) \subseteq s$.
- The <u>result</u> of applying the set π in s is defined as:
  $\gamma(s,\pi) = (s - \text{effects}^-(\pi)) \cup \text{effects}^+(\pi)$, where:
  - $\text{precond}(\pi) = \bigcup_{a \in \pi} \text{precond}(a)$,
  - $\text{effects}^+(\pi) = \bigcup_{a \in \pi} \text{effects}^+(a)$, and
  - $\text{effects}^-(\pi) = \bigcup_{a \in \pi} \text{effects}^-(a)$.

Execution Order of Independent Actions

- **Proposition**: If a set π of independent actions is applicable in state s then, for any permutation $\langle a_1,\ldots,a_k \rangle$ of the elements of π:
  - the sequence $\langle a_1,\ldots,a_k \rangle$ is applicable to s, and
  - the state resulting from the application of π to s is the same as from the application of $\langle a_1,\ldots,a_k \rangle$, i.e.:
    $\gamma(s,\pi) = \gamma(s,\langle a_1,\ldots,a_k \rangle)$.

Layered Plans

- Let $P = (A,s_i,g)$ be a statement of a propositional planning problem and $G = (N,E)$, $N = P_0 \cup A_1 \cup P_1 \cup A_2 \cup P_2 \cup \ldots$, the corresponding planning graph.
- A layered plan over G is a sequence of sets of actions: $\prod = \langle \pi_1,\ldots,\pi_k \rangle$ where:
  - $\pi_i \subseteq A_i \subseteq A$,
  - π_i is applicable in state P_{i-1}, and
  - the actions in π_i are independent.

Layered Solution Plan

- A layered plan $\prod = \langle \pi_1,\ldots,\pi_k \rangle$ is a solution to a to a planning problem $P=(A,s_i,g)$ iff:
  - π_1 is applicable in s_i,
  - for $j \in \{2\ldots k\}$, π_j is applicable in state $\gamma(\ldots\gamma(\gamma(s_i,\pi_1), \pi_2), \ldots \pi_{j-1})$, and
  - $g \subseteq \gamma(\ldots\gamma(\gamma(s_i,\pi_1), \pi_2), \ldots, \pi_k)$.

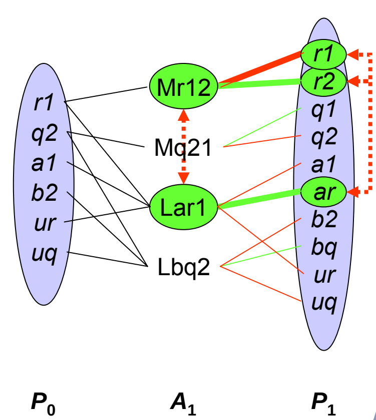
Execution Order in Layered Solution Plans

- **Proposition**: If $\prod = \langle \pi_1,\ldots,\pi_k \rangle$ is a solution to a to a planning problem $P=(A,s_i,g)$, then:
  - a sequence of actions corresponding to any permutation of the elements of π_1,
  - followed by a sequence of actions corresponding to any permutation of the elements of π_2,
  - …
  - followed by a sequence of actions corresponding to any permutation of the elements of π_k

  is a path from s_i to a goal state.

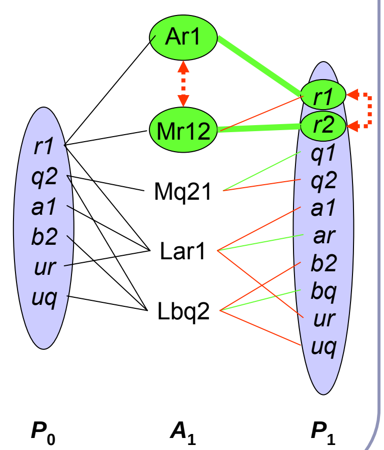
Problem: Dependent Propositions: Example

- *r2* and *ar*:
  - *r2*: positive effect of Mr12
  - *ar*: positive effect of Lar1
  - but: Mr12 and Lar1 not independent
  - hence: *r2* and *ar* incompatible in P_1
- *r1* and *r2*:
  - positive and negative effects of same action: Mr12
  - hence: *r1* and *r2* incompatible in P_1



No-Operation Actions

- No-Op for proposition *p*:
  - name: Ap
  - precondition: *p*
  - effect: p
- *r1* and *r2*:
  - *r1*: positive effect of Ar1
  - *r2*: positive effect of Mr12
  - but: Ar1 and Mr12 not independent
  - hence: *r1* and *r2* incompatible in P_1
- only one incompatibility test



Mutex Propositions

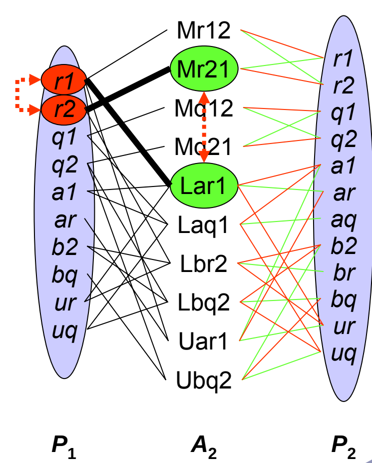
- Two propositions *p* and *q* in proposition layer P_j are <u>mutex</u> (mutually exclusive) if:
  - every action in the preceding action layer A_j that has *p* as a positive effect (incl. no-op actions) is mutex with every action in A_j that has *q* as a positive effect, and
  - there is no single action in A_j that has both, *p* and *q*, as positive effects.
- notation: $\mu P_j = \{ (p,q) \mid p,q \in P_j \text{ are mutex}\}$

Pseudo Code: mutex for Propositions

function mutex(p_1,p_2,μA_j)
  for all $a_1 \in p_1$.producers()
    for all $a_2 \in p_2$.producers()
      if $(a_1,a_2) \notin \mu A_j$ **then**
        return false
  return true

Mutex Actions: Example

- *r1* and *r2* are mutex in P_1
- *r1* is precondition for Lar1 in A_2
- *r2* is precondition for Mr21 in A_2
- hence: Lar1 and Mr21 are mutex in A_2



Mutex Actions

- Two actions a_1 and a_2 in action layer A_j are mutex if:
  - a_1 and a_2 are dependent, or
  - a precondition of a_1 is mutex with a precondition of a_2.
- notation:
  $\mu A_j = \{ (a_1,a_2) \mid a_1,a_2 \in A_j \text{ are mutex}\}$

Pseudo Code: mutex for Actions

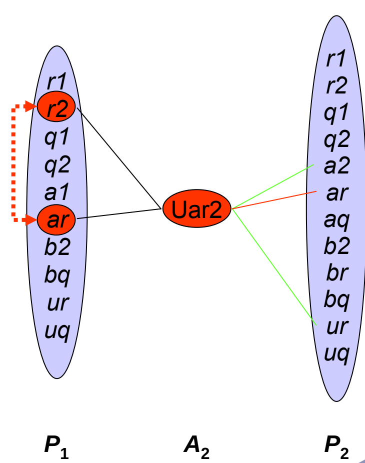
function mutex(a_1,a_2,μP)
  if ¬independent(a_1,a_2) **then**
    return true
  for all $p_1 \in$precond(a_1)
    for all $p_2 \in$precond(a_2)
      if $(p_1,p_2) \in \mu P$ **then return** true
  return false

Decreasing Mutex Relations

- **Proposition**: If $p,q \in P_{j-1}$ and $(p,q) \notin \mu P_{j-1}$ then $(p,q) \notin \mu P_j$.
  - Proof:
    - if $p,q \in P_{j-1}$ then Ap,Aq$\in A_j$
    - if $(p,q) \notin \mu P_{j-1}$ then (Ap,Aq)$\notin \mu A_j$
    - since Ap,Aq$\in A_j$ and (Ap,Aq)$\notin \mu A_j$, $(p,q) \notin \mu P_j$ must hold
- **Proposition**: If $a_1,a_2 \in A_{j-1}$ and $(a_1,a_2) \notin \mu A_{j-1}$ then $(a_1,a_2) \notin \mu A_j$.
  - Proof:
    - if $a_1,a_2 \in A_{j-1}$ and $(a_1,a_2) \notin \mu A_{j-1}$ then
      - a_1 and a_2 are independent and
      - their preconditions in P_{j-1} are not mutex
    - both properties remain true for P_j
    - hence: $a_1,a_2 \in A_j$ and $(a_1,a_2) \notin \mu A_j$

Removing Impossible Actions

- actions with mutex preconditions *p* and *q* are impossible
  - example: preconditions *r2* and *ar* of Uar2 in A_2 are mutex
- can be removed from the graph
  - example: remove Uar2 from A_2



Reachability in Planning Graphs

- **Proposition**: Let $P = (A,s_i,g)$ be a propositional planning problem and $G = (N,E)$, $N = P_0 \cup A_1 \cup P_1 \cup A_2 \cup P_2 \cup \ldots$, the corresponding planning graph. If
  - g is reachable from s_i

  then
  - there is a proposition layer P_g such that
    - $g \subseteq P_g$ and
    - $\neg\exists\ g_1,g_2 \in g$: $(g_1,g_2) \in \mu P_g$.

Overview

- The Propositional Representation
- The Planning-Graph Structure
- The Graphplan Algorithm

The Graphplan Algorithm: Basic Idea

- expand the planning graph, one action layer and one proposition layer at a time
- from the first graph for which P_g is the last proposition layer such that
  - $g \subseteq P_g$ and
  - $\neg\exists\ g_1,g_2 \in g$: $(g_1,g_2) \in \mu P_g$
- search backwards from the last (proposition) layer for a solution

Planning Graph Data Structure

- k-th planning graph G_k:
  - nodes N:
    - array of proposition layers $P_0 \dots P_k$
      - proposition layer j: set of proposition symbols
    - array of action layers $A_1 \dots A_k$
      - proposition layer j: set of action symbols
  - edges E:
    - precondition links: $pre_j \subseteq P_{j-1} \times A_j$, $j \in \{1 \dots k\}$
    - positive effect links: $e_j^+ \subseteq A_j \times P_j$, $j \in \{1 \dots k\}$
    - negative effect links: $e_j^- \subseteq A_j \times P_j$, $j \in \{1 \dots k\}$
    - proposition mutex links: $\mu A_j \subseteq A_j \times A_j$, $j \in \{1 \dots k\}$
    - action mutex links: $\mu P_j \subseteq P_j \times P_j$, $j \in \{1 \dots k\}$

Pseudo Code: expand

function expand(G_{k-1})

$A_k \leftarrow \{a \in A \mid \text{precond}(a) \subseteq P_{k-1}$ and

$\{(p_1,p_2) \mid p_1,p_2 \in \text{precond}(a)\} \cap \mu P_{k-1} = \{\}\ \}$

$\mu A_k \leftarrow \{(a_1,a_2) \mid a_1,a_2 \in A_k,\ a_1 \neq a_2,$ and $\text{mutex}(a_1,a_2,\mu P_{k-1})\ \}$

$P_k \leftarrow \{p \mid \exists a \in A_k : p \in \text{effects}^+(\text{a})\ \}$

$\mu P_k \leftarrow \{(p_1,p_2) \mid p_1,p_2 \in P_k,\ p_1 \neq p_2,$ and $\text{mutex}(p_1,p_2,\mu A_k)\ \}$

for all $a \in A_k$

$pre_k \leftarrow pre_k \cup (\{p \mid p \in P_{k-1}$ and $p \in \text{precond}(a)\} \times a)$

$e_k^+ \leftarrow e_k^+ \cup (a \times \{p \mid p \in P_k$ and $p \in \text{effects}^+(a)\})$

$e_k^- \leftarrow e_k^- \cup (a \times \{p \mid p \in P_k$ and $p \in \text{effects}^-(a)\})$

Planning Graph Complexity

- **Proposition**: The size of a planning graph up to level *k* and the time required to expand it to that level are polynomial in the size of the planning problem.
- Proof:
  - problem size: *n* propositions and *m* actions
  - $|P_j| \leq n$ and $|A_j| \leq n+m$ (incl. no-op actions)
  - algorithms for generating each layer and all link types are polynomial in size of layer

Fixed-Point Levels

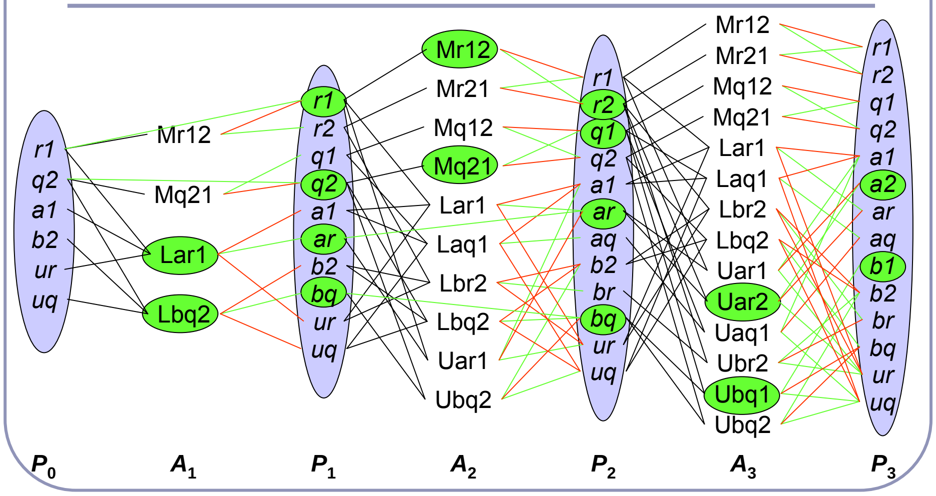
- A <u>fixed-point level</u> in a planning graph G is a level κ such that for all i, $i>\kappa$, level i of G is identical to level κ, i.e. $P_i=P_\kappa$, $\mu P_i=\mu P_\kappa$, $A_i=A_\kappa$, and $\mu A_i=\mu A_\kappa$.

- **Proposition**: Every planning graph G has a fixed-point level κ, which is the smallest k such that $|P_k|=|P_{k+1}|$ and $|\mu P_k|=|\mu P_{k+1}|$.
- Proof:
  - P_i grows monotonically and μP_i shrinks monotonically
  - A_i and P_i only depend on P_{i-1} and μP_{i-1}

Searching the Planning Graph

- general idea:
  - search backwards from the last proposition layer P_k in the current graph
  - let g be the set of goal propositions that need to be achieved at a given proposition layer P_j (initially the last layer)
  - find a set of actions $\pi_j \subseteq A_j$ such that these actions are not mutex and together achieve g
  - take the union of the preconditions of π_j as the new goal set to be achieved in proposition layer P_{j-1}

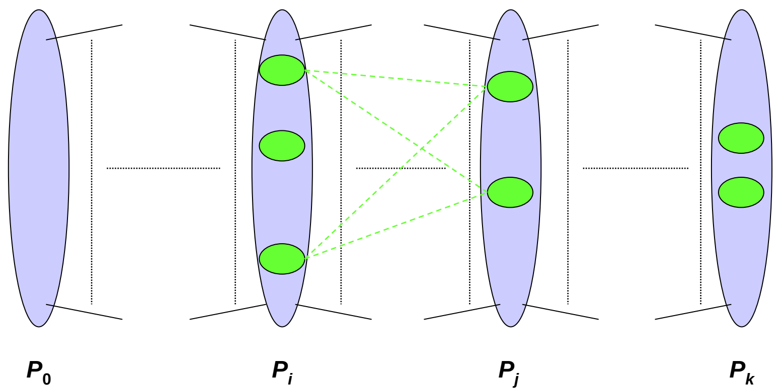
Planning Graph Search Example



Planning Graph as AND/OR-Graph

- OR-nodes:
  - nodes in proposition layers
  - links to actions that support the propositions
- AND-nodes:
  - nodes in action layers
  - *k*-connectors all preconditions of the action
- search:
  - AO* not best algorithm because it does not exploit layered structure

Repeated Sub-Goals



The *nogood* Table

- *nogood* table (denoted ∇) for planning graph up to layer *k*:
  - array of *k* sets of sets of goal propositions
    - inner set: one combination of propositions that cannot be achieved
    - outer set: all combinations that cannot be achieved (at that layer)
- before searching for set *g* in P_j:
  - check whether $g \in \nabla(j)$
- when search for set *g* in P_j has failed:
  - add *g* to $\nabla(j)$

Pseudo Code: extract

function extract(G,g,i)
  if i=0 **then return** $\langle\rangle$
  if $g\in\nabla(i)$ **then return** failure
  $\prod \leftarrow$ gpSearch(G,g,{},i)
  if $\prod\neq$failure **then return** $\prod$
  $\nabla(i) \leftarrow \nabla(i) + g$
  return failure

Pseudo Code: gpSearch

function gpSearch(G,g,π,i)
  if g={} **then**
    $\prod \leftarrow$ extract(G,$\bigcup_{a\in\pi}$precond(a),i-1)
    if $\prod$=failure **then return** failure
    return $\prod\bullet\langle\pi\rangle$
  $p \leftarrow g$.selectOne()
  resolvers $\leftarrow$ $\{a\in A_i \mid p\in\text{effects}^+(a) \text{ and } \neg\exists a'\in\pi: (a,a')\in\mu A_i\}$
  if *resolvers*={} **then return** failure
  $a \leftarrow$ *resolvers*.chooseOne()
  return gpSearch(G,g-effects$^+$(a),π+a,i)

Pseudo Code: graphplan

```
function graphplan(A,s_i,g)
    i ← 0; ∇ ← []; P_0 ← s_i; G ← (P_0,{})
    while (g⊈P_i or g²∩μP_i≠{}) and ¬fixedPoint(G) do
        i ← i+1; expand(G)
    if g⊈P_i or g²∩μP_i≠{} then return failure
    η ← fixedPoint(G) ? |∇(κ)| : 0
    ∏ ← extract(G,g,i)
    while ∏=failure do
        i ← i+1; expand(G)
        ∏ ← extract(G,g,i)
        if ∏=failure and fixedPoint(G) then
            if η=|∇(κ)| then return failure
            η ← |∇(κ)|
    return ∏
```


Graphplan Properties

- **Proposition**: The Graphplan algorithm is sound, complete, and always terminates.
  - It returns failure iff the given planning problem has no solution;
  - otherwise, it returns a layered plan ∏ that is a solution to the given planning problem.
- Graphplan is orders of magnitude faster than previous techniques!

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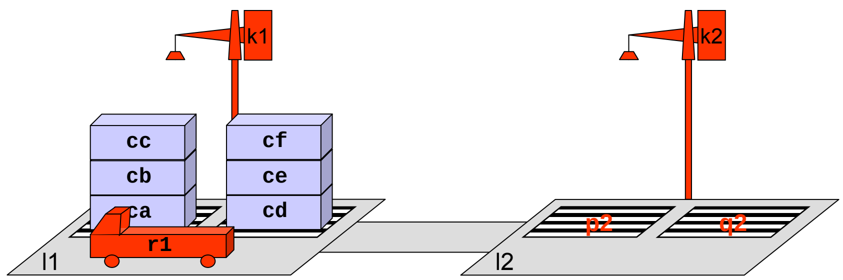
- Planning-Graph Heuristics

Forward State-Space Search

- idea: apply standard search algorithms (breadth-first, depth-first, A*, etc.) to planning problem:
  - search space is subset of state space
  - nodes correspond to world states
  - arcs correspond to state transitions
  - path in the search space corresponds to plan

DWR Example State



goal: (and
(in ca p2) (in cb q2) (in cc p2) (in cd q2) (in ce q2) (in cf q2))

Heuristics

- estimate distance to nearest goal state
  - number of unachieved goals (not admissible)
  - number of unachieved goals / max. number of positive effects per operator (admissible)
- example state (prev. slide):
  - actual goal distance: 35 actions
  - h(s) = 6
  - h(s) = 6 / 4

Finding Better Heuristics

- solve "relaxed" problem and use solution as heuristic
- planning heuristic:
  - planning problem: $P=(O,s_i,g)$
  - for $p \in g$: min-layer(p) = index of first proposition layer in planning graph that contains p
  - admissible heuristic: max($p \in g$): min-layer(p)
  - not admissible: sum($p \in g$): min-layer(p)
- no need to compute mutex relations
- no need to re-compute planning graph for ground backward search

The FF Planner (Basics)

- heuristic
  - based on planning graph without negative effects
  - backward search possible in polynomial time
- search strategy
  - enforced hill-climbing: commit to first state with better f-value

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