Neural Information Processing: 2011-2012
Assignment 1

School of Informatics, University of Edinburgh

Instructor: Mark van Rossum

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```
submit
msc nip 1 nipasst1.pdf
```

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Population codes and multiplicative noise

In this assignment we analyse the Fisher Information for an alternative noise model, the multiplicative Gaussian noise model.
At each trial the firing rate of neuron i, denoted r_i, is modelled as

$$r_i = f_i(s) + \sigma_i(s)\eta_i$$

where tuning curve $f_i(s)$ is the mean response of the neuron to stimulus s. Furthermore, η_i are independent Gaussian random variables with standard deviation 1. To mimic realistic neural variability, the amplitude of the noise $\sigma(s)$ is dependent on the average response rate and thus in general stimulus dependent. The response probability distribution is thus

$$P(r_i|s) = \frac{1}{\sqrt{2\pi}\sigma(s)} \exp\left[-\frac{(r_i - f_i(s))^2}{2\sigma^2(s)}\right]$$

In particular we set $\sigma(s) = \sigma_0 f^\gamma(s)$ with $(\gamma \geq 0)$.

Question 1: What value should γ and σ_0 take so that the variance in the responses would equal the variance of Poisson neurons?

Question 2: First consider the case that $\gamma = 0$, so that the noise is additive and stimulus independent. Show that the Fisher Information of a single neuron equals

$$I_F^0(s) = \left(\frac{f'(s)}{\sigma_0}\right)^2$$

Question 3: Now consider the case of general γ. Show that

$$I_F^\gamma(s) = \left(\frac{f'(s)}{\sigma(s)}\right)^2 + 2\gamma^2\left(\frac{f'(s)}{f(s)}\right)^2$$

Question 4: Consider the case of Gaussian shape tuning curves, i.e.

$$f_i(s) = A\exp\left[-\frac{(s_i - s)^2}{2w^2}\right]$$

where the preferred stimulus of neuron i is s_i. Consider a dense homogeneous population of neurons encoding a stimulus. Is there something awkward with the noise model? How can it be fixed?