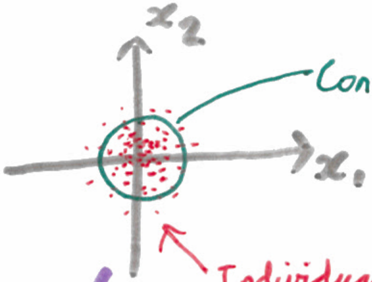
Multivariate Gaussians

$$x_d \sim \mathcal{N}(0,1) \Rightarrow p(\mathbf{x}) = \frac{1}{(2\pi)^{D/2}} e^{-\mathbf{x}^{T}\mathbf{x}/2}$$

constant radius



Contours of $p(\mathbf{x})$ circular, (or spherical)

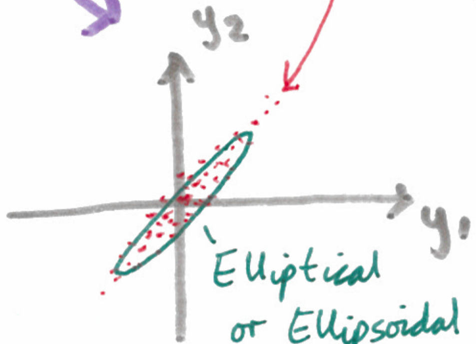
Individual samples

$$\mathrm{cov}[\mathbf{x}] = \mathbb{I} = \begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}$$

$$\mathbf{y} = A\mathbf{x}$$

$$\mathrm{cov}[\mathbf{y}] = \mathbb{E}[\mathbf{y}\mathbf{y}^{T}] - \mathbb{E}[\mathbf{y}]\mathbb{E}[\mathbf{y}]^{T}$$

$$= AA^{T} = \Sigma$$



Elliptical or Ellipsoidal contours

$$p(\mathbf{y}) = \frac{1}{|\Sigma|^{1/2}(2\pi)^{D/2}} e^{-\frac{1}{2}\mathbf{y}^{T}\Sigma^{-1}\mathbf{y}}$$

General Gaussian

$$\underline{z} = \underbrace{A\underline{x}}_{\underline{y}} + \underline{\mu}, \qquad \underline{y} = \underline{z} - \underline{\mu}$$

$$p(\underline{z}) = \frac{1}{|\Sigma|^{1/2}(2\pi)^{D/2}} e^{-\frac{1}{2}(\underline{z}-\underline{\mu})^T \Sigma^{-1}(\underline{z}-\underline{\mu})}$$

$$= N(\underline{z};\underline{\mu},\Sigma)$$

$\uparrow AA^T$

Variances are +ve, Covariances are usually +ve definite

Iff $\Sigma = AA^T$, Σ is positive semi-definite

Positive Definite: $\underline{z}^T \Sigma \underline{z} > 0$ for all $\underline{z}$

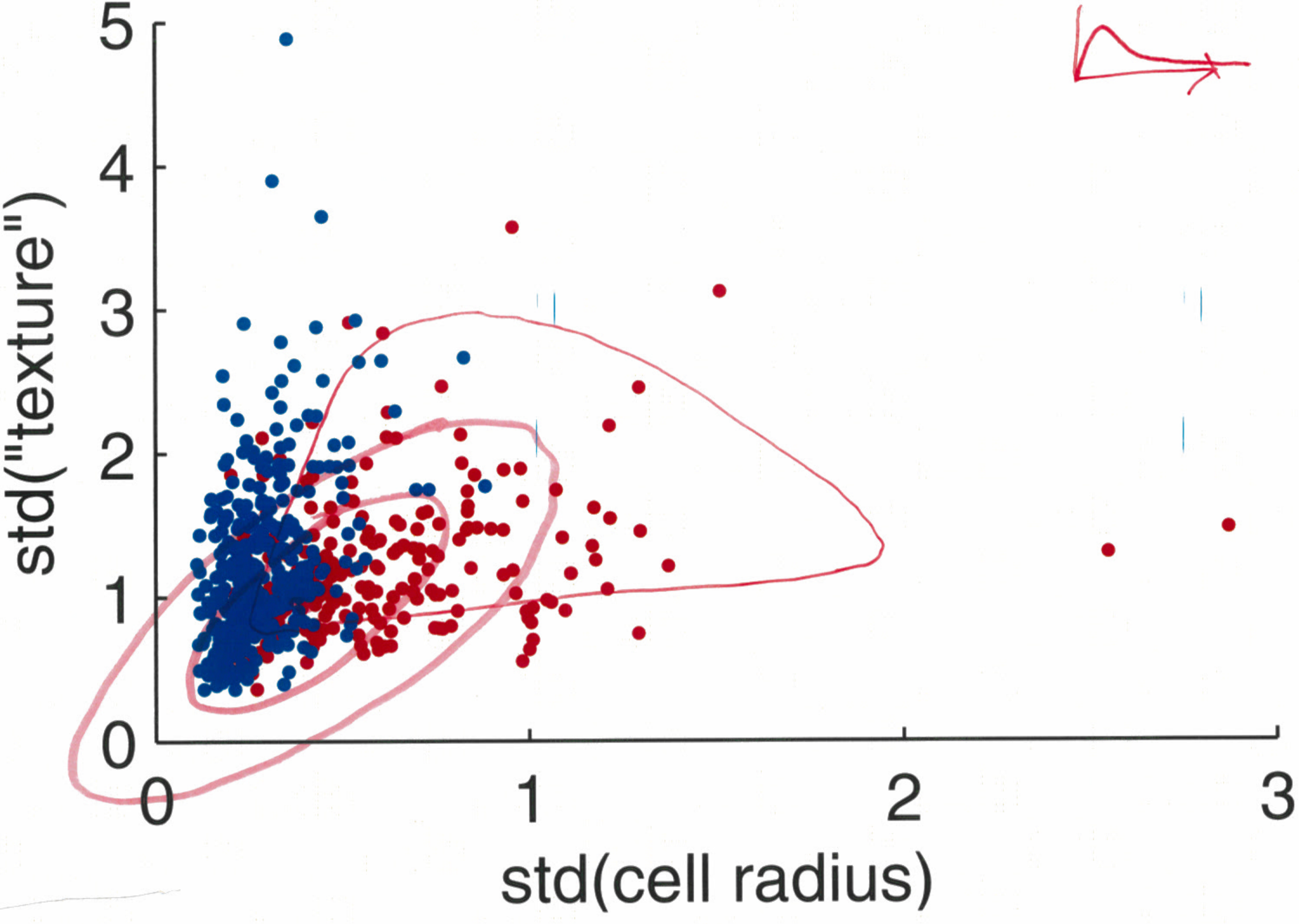
also $\underline{z}^T \Sigma^{-1} \underline{z} > 0$ " " "

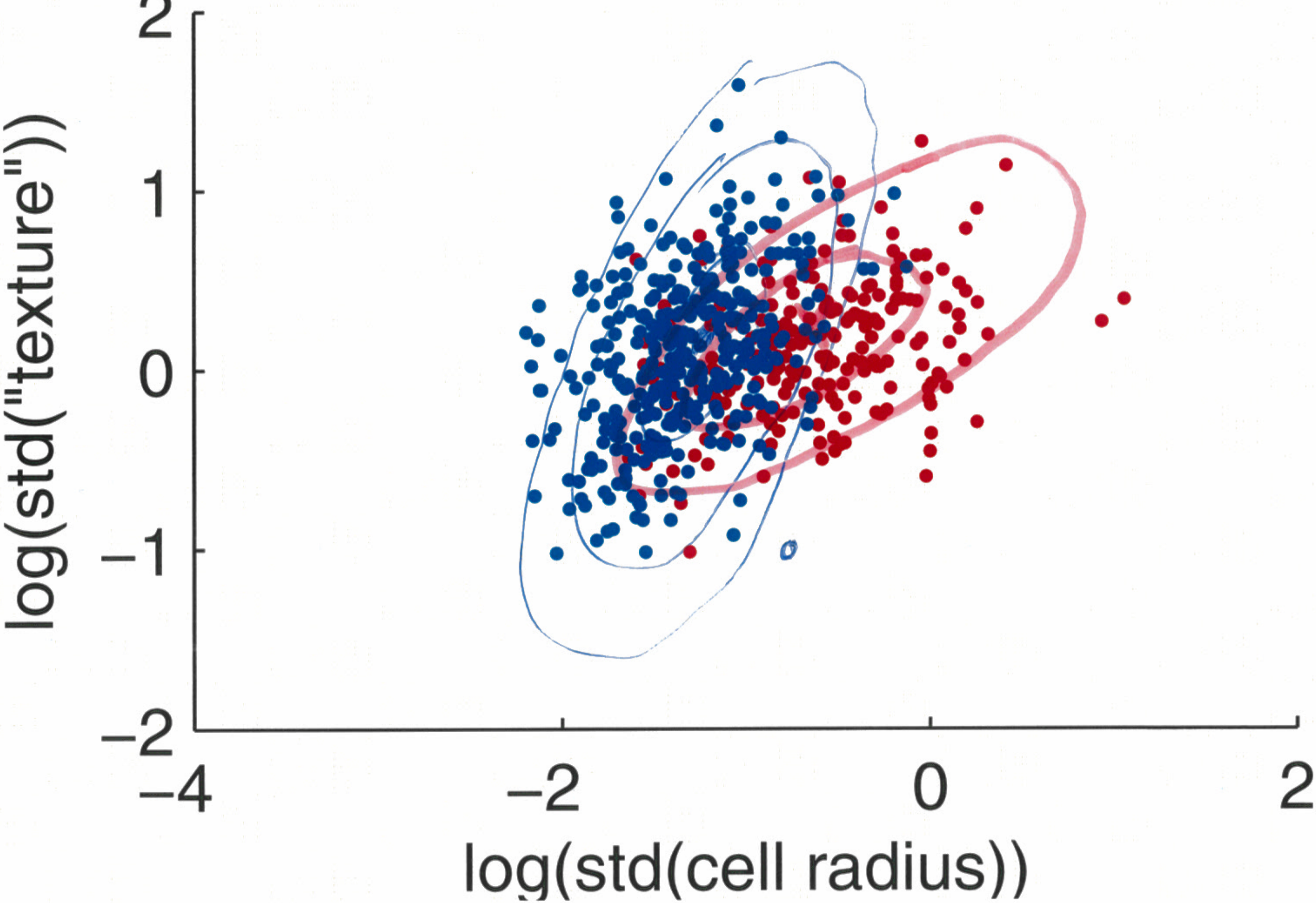
+ve Semi Definite if $\underline{z}\Sigma\underline{z} \geq 0$ for all $\underline{z}$

Eg semi definite matrix $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

If Σ is +ve definite: $L = \mathrm{chol}(\Sigma)$, $\Sigma = LL^T$

(More in tutorial 2.)


std("texture")
0
1
2
3
4
5
std(cell radius)
0
1
2
3


log(std("texture"))
2
1
0
−1
−2
−4
−2
0
2
log(std(cell radius))

Bayes Classifiers

At training time create model:

Features: $p(\underline{x} \mid y=k)$ e.g. $N(\underline{x}; \underline{\mu}^{(k)}, \Sigma^{(k)})$

or discrete dist.

Labels: $p(y=k) = \pi_k$ $\left(\sum_k \pi_k = 1\right)$

k class $k = 1 \dots K$

K ← # classes.

$$\pi_k \approx \frac{\text{\# points in class } k}{N}$$

$\underline{\mu}^{(k)}, \Sigma^{(k)}$ set mean & cov. of the points in class k.

At test time use Bayes' rule

$$p(y \mid \underline{x}) \propto p(y, \underline{x}) \propto p(y)\, p(\underline{x} \mid y)$$

$$p(y=k \mid \underline{x}) = \frac{p(y=k)\, p(\underline{x} \mid y)}{\sum_{k'} p(y=k')\, p(\underline{x} \mid y=k')}$$

quadric decision boundary

contour

$N(\underline{x};\underbrace{\mu^{(0)},\Sigma^{(0)}}_{\text{fit}})$

contour

$N(\underline{x};\mu^{(1)},\Sigma^{(1)})$

Along this line

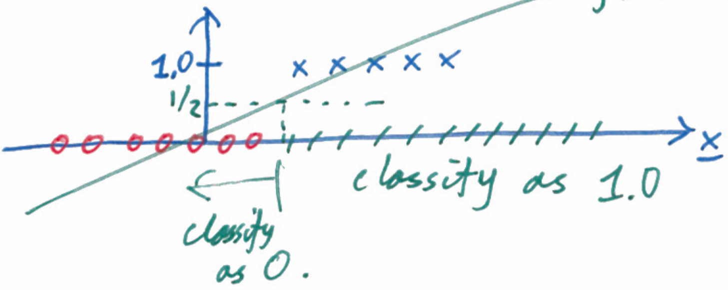
To right class 1 more probable

On line

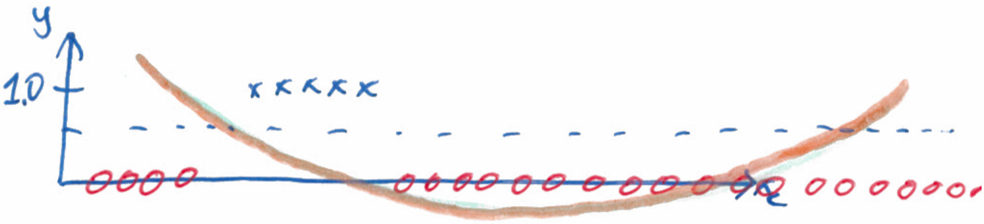
$$\pi_1 N(\underline{x};\mu^{(1)},\Sigma^{(1)}) = \pi_0 N(\underline{x};\mu^{(0)},\Sigma^{(0)})$$

Regression to labels

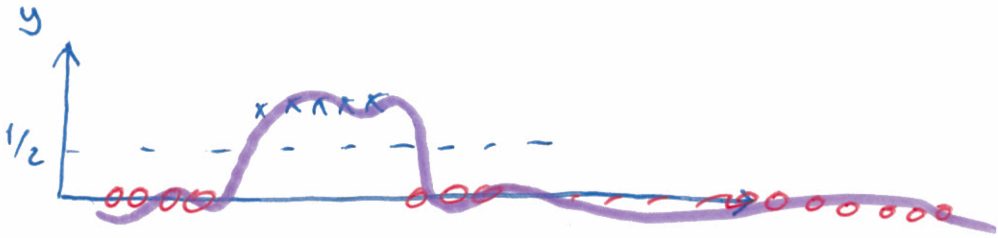
$f(\underline{x}) = \underline{w}^T\underline{x} + b$



If $f(\underline{x}) > 1/2$, guess $y=1$



$$f(x) = w_1 + w_2 x + w_3 x^2$$



Fit with RBFs