Convolutional Networks

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Machine Learning Practical — MLP Lecture 7

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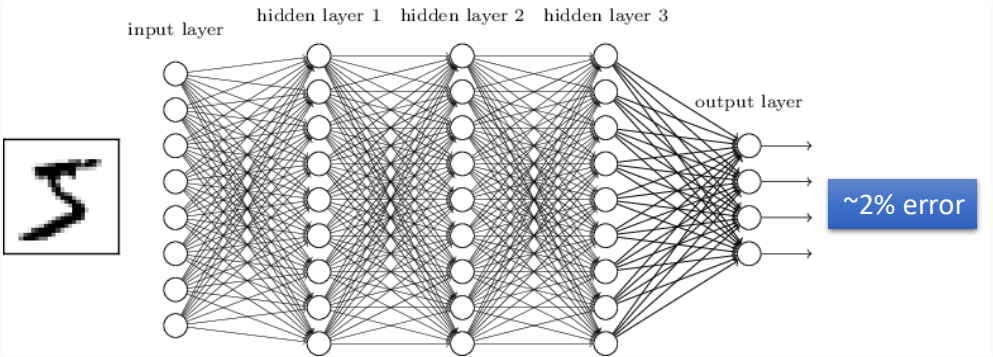
Recap: Fully-connected network for MNIST

On MNIST, we can get about 2% error (or even better) using these kind of networks



(image from: Michael Nielsen, *Neural Networks and Deep Learning*,
http://neuralnetworksanddeeplearning.com/chap6.html)

How about more complex images?

image credit: COCO dataset

Generic object classification

- Large variations in appearance and pose
- Objects are not always centered
- Objects are in different scales
- High resolution images
- Background and clutter
- Occlusions

Fully-connected network in high dimension

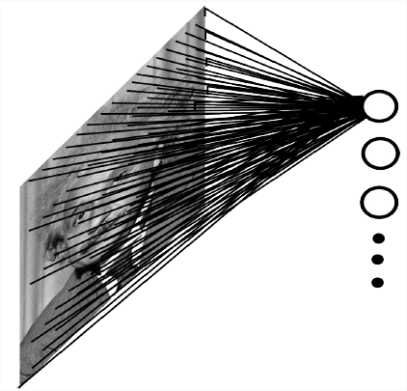


image credit: Lecun & Ranzato

For an input size of 200×200, a deep network with one hidden layer (1000 hidden units) has

- # input units is 40,000
- # connections is 40,000,000
- # parameters is 40,000,000

Fully-connected network in high dimension

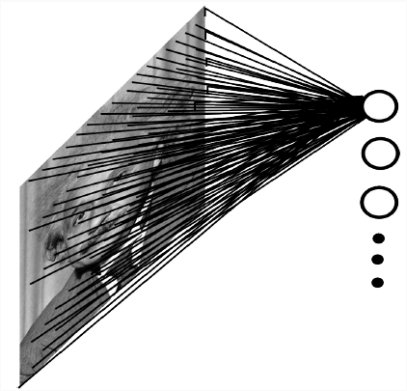


image credit: Lecun & Ranzato

For an input size of 200×200, a deep network with one hidden layer (1000 hidden units) has

- # input units is 40,000
- # connections is 40,000,000
- # parameters is 40,000,000

Observations:

- Too many parameters to learn!
- Spatial (2-D) structure of input image is ignored
- No easy way of learning the same feature at different coordinates

Convolution operation

- Convolution is an operation on two functions of a real- valued argument

$$s(t) = \int x(a)w(t-a)da$$

- Denoted with asterisk $(*)$

$$s(t) = (x * w)(t)$$

- Discrete convolution

$$s(t) = \sum_{a=-\infty}^{a=\infty} x(a)w(t-a)da$$

- Convolution in multiple dimensions

$$s(i,j) = (\mathbf{x} * \mathbf{w})(i,j) = \sum_{k=0}^{m-1} \sum_{l=0}^{n-1} x_{k,l} w_{i-k,j-l}$$

Convolution operation

- Multiple dimensions

$$s(i,j) = (\mathbf{x} * \mathbf{w})(i,j) = \sum_{k=0}^{m-1} \sum_{l=0}^{n-1} x_{k,l} w_{i-k,j-l}$$

- Convolution is commutative

$$s(i,j) = (\mathbf{w} * \mathbf{x})(i,j) = \sum_{k=0}^{m-1} \sum_{l=0}^{n-1} x_{i-k,j-l} w_{k,l}$$

Convolution operation

- Multiple dimensions

$$s(i,j) = (\mathbf{x} * \mathbf{w})(i,j) = \sum_{k=0}^{m-1} \sum_{l=0}^{n-1} x_{k,l} w_{i-k,j-l}$$

- Convolution is commutative

$$s(i,j) = (\mathbf{w} * \mathbf{x})(i,j) = \sum_{k=0}^{m-1} \sum_{l=0}^{n-1} x_{i-k,j-l} w_{k,l}$$

- Convolutions in deep learning libraries are actually **cross-correlation** ($\otimes$)

$$s(i,j) = (\mathbf{w} \otimes \mathbf{x})(i,j) = \sum_{k=0}^{m-1} \sum_{l=0}^{n-1} x_{i+k,j+l} w_{k,l}$$

- If we "flip" (reflect horizontally and vertically) $\mathbf{w}$ (in cross-correlation) then we obtain $\mathbf{w}$ (in convolution)

Convolution vs Cross-correlation

- In machine learning the network learns the kernel appropriate to its orientation – so if convolution is implemented with a flipped kernel, it will learn that it is a flipped implementation
- So it is OK to use an efficient (flipped) implementation of convolution for convolutional layers
- In convolutional networks, we also use a bias term:

$$s(i,j) = (\mathbf{w} \otimes \mathbf{x})(i,j) + b = \sum_{k=0}^{m-1} \sum_{l=0}^{n-1} x_{i+k,j+l} w_{k,l} + b$$

Example: convolution operation

- $\mathbf{h} = f(\underbrace{\mathbf{x} \otimes \mathbf{w}}_{\mathbf{z}})$ where f is an activation function
- $\mathbf{h}$ is called **feature map**

$$\underbrace{\begin{pmatrix} z_{11} & z_{12} & z_{13} & z_{14} & z_{15} \\ z_{21} & z_{22} & z_{23} & z_{24} & z_{25} \\ z_{31} & z_{32} & z_{33} & z_{34} & z_{35} \\ z_{41} & z_{42} & z_{43} & z_{44} & z_{45} \\ z_{51} & z_{52} & z_{53} & z_{54} & z_{55} \end{pmatrix}}_{\mathbf{z}} = \underbrace{\begin{pmatrix} x_{11} & x_{12} & x_{13} & x_{14} & x_{15} & x_{16} & x_{17} \\ x_{21} & x_{22} & x_{23} & x_{24} & x_{25} & x_{26} & x_{27} \\ x_{31} & x_{32} & x_{33} & x_{34} & x_{35} & x_{36} & x_{37} \\ x_{41} & x_{42} & x_{43} & x_{44} & x_{45} & x_{46} & x_{47} \\ x_{51} & x_{52} & x_{53} & x_{54} & x_{55} & x_{56} & x_{57} \\ x_{61} & x_{62} & x_{63} & x_{64} & x_{65} & x_{66} & x_{67} \\ x_{71} & x_{72} & x_{73} & x_{74} & x_{75} & x_{76} & x_{77} \end{pmatrix}}_{\mathbf{x}\text{: Input}} \otimes \underbrace{\begin{pmatrix} w_{11} & w_{12} & w_{13} \\ w_{21} & w_{22} & w_{23} \\ w_{31} & w_{32} & w_{33} \end{pmatrix}}_{\mathbf{w}\text{: Kernel}}$$

Example: convolution operation

$$\begin{pmatrix} w_{00} & w_{01} & w_{02} \\ w_{10} & w_{11} & w_{12} \\ w_{20} & w_{21} & w_{22} \end{pmatrix}$$

$$\boxed{\otimes}$$

$$\begin{pmatrix} z_{00} & z_{01} & z_{02} & z_{03} & z_{04} \\ z_{10} & z_{11} & z_{12} & z_{13} & z_{14} \\ z_{20} & z_{21} & z_{22} & z_{23} & z_{24} \\ z_{30} & z_{31} & z_{32} & z_{33} & z_{34} \\ z_{40} & z_{41} & z_{42} & z_{43} & z_{44} \end{pmatrix} \boxed{=} \begin{pmatrix} x_{00} & x_{01} & x_{02} & x_{03} & x_{04} & x_{05} & x_{06} \\ x_{10} & x_{11} & x_{12} & x_{13} & x_{14} & x_{15} & x_{16} \\ x_{20} & x_{21} & x_{22} & x_{23} & x_{24} & x_{25} & x_{26} \\ x_{30} & x_{31} & x_{32} & x_{33} & x_{34} & x_{35} & x_{36} \\ x_{40} & x_{41} & x_{42} & x_{43} & x_{44} & x_{45} & x_{46} \\ x_{50} & x_{51} & x_{52} & x_{53} & x_{54} & x_{55} & x_{56} \\ x_{60} & x_{61} & x_{62} & x_{63} & x_{64} & x_{65} & x_{66} \end{pmatrix}$$

Example: convolution operation

$$z_{00} = \sum_{k=0}^{2} \sum_{l=0}^{2} x_{k,l} w_{kl} + b$$

$$\begin{pmatrix} z_{00} & z_{01} & z_{02} & z_{03} & z_{04} \\ z_{10} & z_{11} & z_{12} & z_{13} & z_{14} \\ z_{20} & z_{21} & z_{22} & z_{23} & z_{24} \\ z_{30} & z_{31} & z_{32} & z_{33} & z_{34} \\ z_{40} & z_{41} & z_{42} & z_{43} & z_{44} \end{pmatrix} = \begin{pmatrix} x_{00} & x_{01} & x_{02} & x_{03} & x_{04} & x_{05} & x_{06} \\ x_{10} & x_{11} & x_{12} & x_{13} & x_{14} & x_{15} & x_{16} \\ x_{20} & x_{21} & x_{22} & x_{23} & x_{24} & x_{25} & x_{26} \\ x_{30} & x_{31} & x_{32} & x_{33} & x_{34} & x_{35} & x_{36} \\ x_{40} & x_{41} & x_{42} & x_{43} & x_{44} & x_{45} & x_{46} \\ x_{50} & x_{51} & x_{52} & x_{53} & x_{54} & x_{55} & x_{56} \\ x_{60} & x_{61} & x_{62} & x_{63} & x_{64} & x_{65} & x_{66} \end{pmatrix} \otimes \begin{pmatrix} w_{00} & w_{01} & w_{02} \\ w_{10} & w_{11} & w_{12} \\ w_{20} & w_{21} & w_{22} \end{pmatrix}$$

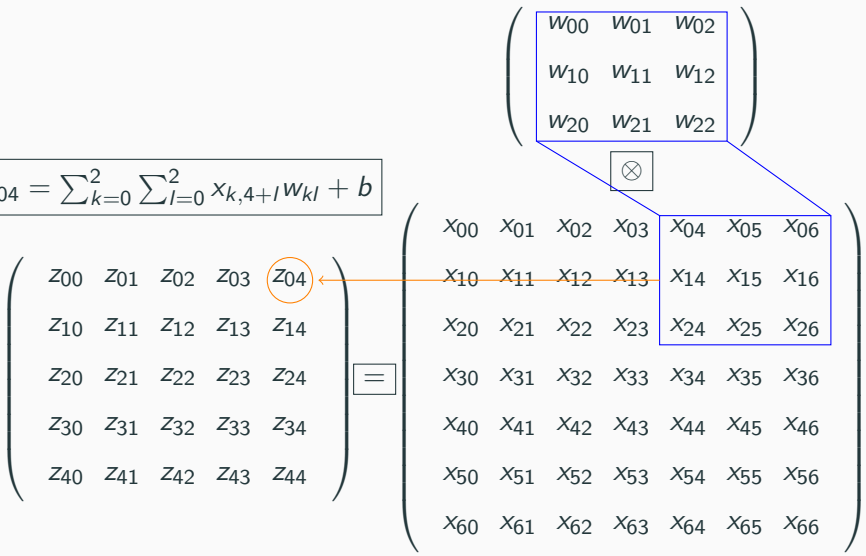
Example: convolution operation

$$z_{01} = \sum_{k=0}^{2} \sum_{l=0}^{2} x_{k,1+l} w_{kl} + b$$

$$\begin{pmatrix} z_{00} & z_{01} & z_{02} & z_{03} & z_{04} \\ z_{10} & z_{11} & z_{12} & z_{13} & z_{14} \\ z_{20} & z_{21} & z_{22} & z_{23} & z_{24} \\ z_{30} & z_{31} & z_{32} & z_{33} & z_{34} \\ z_{40} & z_{41} & z_{42} & z_{43} & z_{44} \end{pmatrix} = \begin{pmatrix} x_{00} & x_{01} & x_{02} & x_{03} & x_{04} & x_{05} & x_{06} \\ x_{10} & x_{11} & x_{12} & x_{13} & x_{14} & x_{15} & x_{16} \\ x_{20} & x_{21} & x_{22} & x_{23} & x_{24} & x_{25} & x_{26} \\ x_{30} & x_{31} & x_{32} & x_{33} & x_{34} & x_{35} & x_{36} \\ x_{40} & x_{41} & x_{42} & x_{43} & x_{44} & x_{45} & x_{46} \\ x_{50} & x_{51} & x_{52} & x_{53} & x_{54} & x_{55} & x_{56} \\ x_{60} & x_{61} & x_{62} & x_{63} & x_{64} & x_{65} & x_{66} \end{pmatrix} \otimes \begin{pmatrix} w_{00} & w_{01} & w_{02} \\ w_{10} & w_{11} & w_{12} \\ w_{20} & w_{21} & w_{22} \end{pmatrix}$$

Example: convolution operation

$$z_{04} = \sum_{k=0}^{2} \sum_{l=0}^{2} x_{k,4+l} w_{kl} + b$$



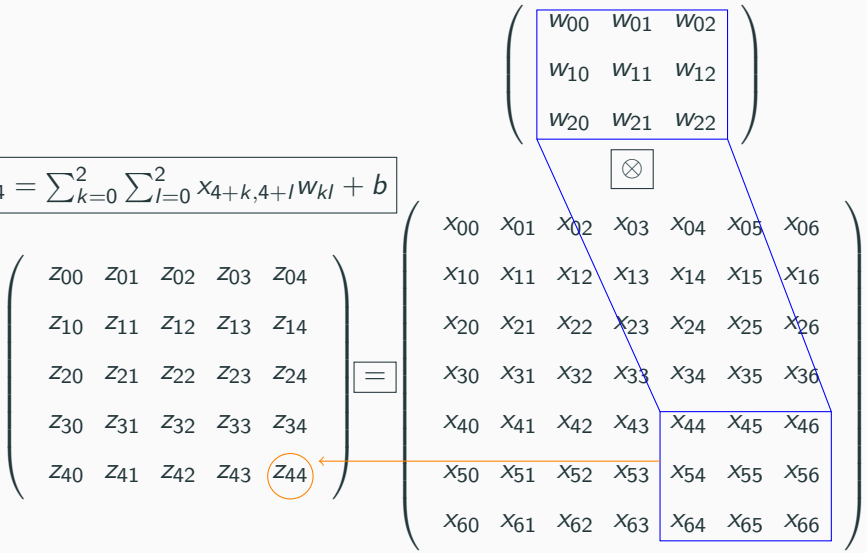
Example: convolution operation

$$z_{10} = \sum_{k=0}^{2} \sum_{l=0}^{2} x_{1+k,l} w_{kl} + b$$

$$\begin{pmatrix} z_{00} & z_{01} & z_{02} & z_{03} & z_{04} \\ z_{10} & z_{11} & z_{12} & z_{13} & z_{14} \\ z_{20} & z_{21} & z_{22} & z_{23} & z_{24} \\ z_{30} & z_{31} & z_{32} & z_{33} & z_{34} \\ z_{40} & z_{41} & z_{42} & z_{43} & z_{44} \end{pmatrix} = \begin{pmatrix} x_{00} & x_{01} & x_{02} & x_{03} & x_{04} & x_{05} & x_{06} \\ x_{10} & x_{11} & x_{12} & x_{13} & x_{14} & x_{15} & x_{16} \\ x_{20} & x_{21} & x_{22} & x_{23} & x_{24} & x_{25} & x_{26} \\ x_{30} & x_{31} & x_{32} & x_{33} & x_{34} & x_{35} & x_{36} \\ x_{40} & x_{41} & x_{42} & x_{43} & x_{44} & x_{45} & x_{46} \\ x_{50} & x_{51} & x_{52} & x_{53} & x_{54} & x_{55} & x_{56} \\ x_{60} & x_{61} & x_{62} & x_{63} & x_{64} & x_{65} & x_{66} \end{pmatrix} \otimes \begin{pmatrix} w_{00} & w_{01} & w_{02} \\ w_{10} & w_{11} & w_{12} \\ w_{20} & w_{21} & w_{22} \end{pmatrix}$$

Example: convolution operation

$$z_{44} = \sum_{k=0}^{2} \sum_{l=0}^{2} x_{4+k,4+l} w_{kl} + b$$



Example: convolution operation

Number of ...

- input units are 7×7
- weights is $3 \times 3 + 1$ (9 for kernel + 1 for bias)
- hidden units is 5×5
- connections is $5 \times 5 \times 3 \times 3$

Example: convolution operation

Number of ...

- input units are 7×7
- weights is $3 \times 3 + 1$ (9 for kernel + 1 for bias)
- hidden units is 5×5
- connections is $5 \times 5 \times 3 \times 3$

Properties

- Weights (kernel) are shared across all hidden units
- Spatial correspondence between pixels and hidden units
  ("2D matrix of hidden units" = "feature map")
- Translation invariance: extract same features irrespective of where an image patch is located in the input

Question

If $\mathbf{x}$ is 7×7 and $\mathbf{w}$ is 3×3, then the feature map $\mathbf{h}$ is 5×5.

If $\mathbf{x}$ is 7×7 and $\mathbf{w}$ is 5×5, what will be the dimensions of $\mathbf{h}$?

(a) 3×3, (b) 5×5, (c) 7×7

Question

If $\mathbf{x}$ is 7×7 and $\mathbf{w}$ is 5×5, what will be the dimensions of $\mathbf{h}$?

$$\begin{pmatrix} w_{00} & w_{01} & w_{02} & w_{03} & w_{04} \\ w_{10} & w_{11} & w_{12} & w_{13} & w_{14} \\ w_{20} & w_{21} & w_{22} & w_{23} & w_{24} \\ w_{30} & w_{31} & w_{32} & w_{33} & w_{34} \\ w_{40} & w_{41} & w_{42} & w_{43} & w_{44} \end{pmatrix}$$

$$\boxed{\otimes}$$

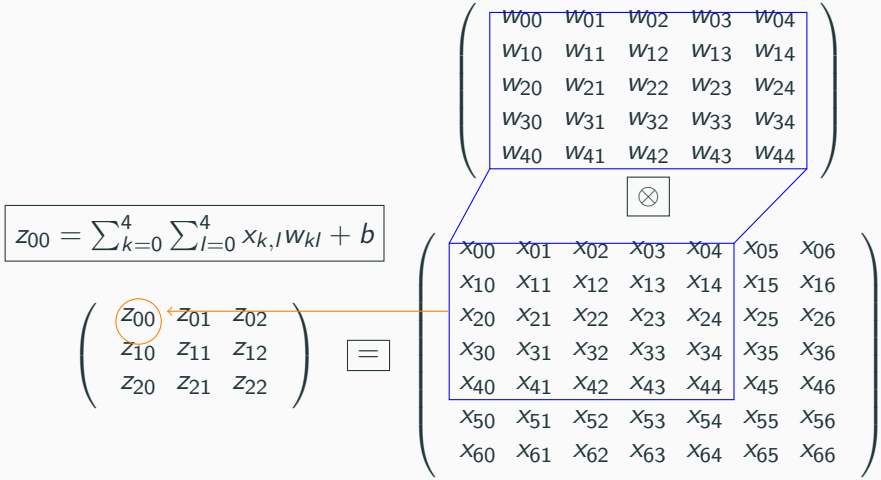
$$\begin{pmatrix} x_{00} & x_{01} & x_{02} & x_{03} & x_{04} & x_{05} & x_{06} \\ x_{10} & x_{11} & x_{12} & x_{13} & x_{14} & x_{15} & x_{16} \\ x_{20} & x_{21} & x_{22} & x_{23} & x_{24} & x_{25} & x_{26} \\ x_{30} & x_{31} & x_{32} & x_{33} & x_{34} & x_{35} & x_{36} \\ x_{40} & x_{41} & x_{42} & x_{43} & x_{44} & x_{45} & x_{46} \\ x_{50} & x_{51} & x_{52} & x_{53} & x_{54} & x_{55} & x_{56} \\ x_{60} & x_{61} & x_{62} & x_{63} & x_{64} & x_{65} & x_{66} \end{pmatrix}$$

$$\boxed{=}$$

$$\begin{pmatrix} z_{00} & z_{01} & z_{02} \\ z_{10} & z_{11} & z_{12} \\ z_{20} & z_{21} & z_{22} \end{pmatrix}$$

Question

If $\mathbf{x}$ is 7×7 and $\mathbf{w}$ is 5×5, what will be the dimensions of $\mathbf{h}$?



Question

If $\mathbf{x}$ is 7×7 and $\mathbf{w}$ is 5×5, what will be the dimensions of $\mathbf{h}$?

$$z_{01} = \sum_{k=0}^{4} \sum_{l=0}^{4} x_{k,1+l} w_{kl} + b$$

$$\begin{pmatrix} z_{00} & z_{01} & z_{02} \\ z_{10} & z_{11} & z_{12} \\ z_{20} & z_{21} & z_{22} \end{pmatrix} = \begin{pmatrix} x_{00} & x_{01} & x_{02} & x_{03} & x_{04} & x_{05} & x_{06} \\ x_{10} & x_{11} & x_{12} & x_{13} & x_{14} & x_{15} & x_{16} \\ x_{20} & x_{21} & x_{22} & x_{23} & x_{24} & x_{25} & x_{26} \\ x_{30} & x_{31} & x_{32} & x_{33} & x_{34} & x_{35} & x_{36} \\ x_{40} & x_{41} & x_{42} & x_{43} & x_{44} & x_{45} & x_{46} \\ x_{50} & x_{51} & x_{52} & x_{53} & x_{54} & x_{55} & x_{56} \\ x_{60} & x_{61} & x_{62} & x_{63} & x_{64} & x_{65} & x_{66} \end{pmatrix} \otimes \begin{pmatrix} w_{00} & w_{01} & w_{02} & w_{03} & w_{04} \\ w_{10} & w_{11} & w_{12} & w_{13} & w_{14} \\ w_{20} & w_{21} & w_{22} & w_{23} & w_{24} \\ w_{30} & w_{31} & w_{32} & w_{33} & w_{34} \\ w_{40} & w_{41} & w_{42} & w_{43} & w_{44} \end{pmatrix}$$

Question

If $\mathbf{x}$ is 7×7 and $\mathbf{w}$ is 5×5, what will be the dimensions of $\mathbf{h}$?

$$z_{02} = \sum_{k=0}^{4} \sum_{l=0}^{4} x_{k,2+l} w_{kl} + b$$

$$\begin{pmatrix} z_{00} & z_{01} & z_{02} \\ z_{10} & z_{11} & z_{12} \\ z_{20} & z_{21} & z_{22} \end{pmatrix} = \begin{pmatrix} x_{00} & x_{01} & x_{02} & x_{03} & x_{04} & x_{05} & x_{06} \\ x_{10} & x_{11} & x_{12} & x_{13} & x_{14} & x_{15} & x_{16} \\ x_{20} & x_{21} & x_{22} & x_{23} & x_{24} & x_{25} & x_{26} \\ x_{30} & x_{31} & x_{32} & x_{33} & x_{34} & x_{35} & x_{36} \\ x_{40} & x_{41} & x_{42} & x_{43} & x_{44} & x_{45} & x_{46} \\ x_{50} & x_{51} & x_{52} & x_{53} & x_{54} & x_{55} & x_{56} \\ x_{60} & x_{61} & x_{62} & x_{63} & x_{64} & x_{65} & x_{66} \end{pmatrix} \otimes \begin{pmatrix} w_{00} & w_{01} & w_{02} & w_{03} & w_{04} \\ w_{10} & w_{11} & w_{12} & w_{13} & w_{14} \\ w_{20} & w_{21} & w_{22} & w_{23} & w_{24} \\ w_{30} & w_{31} & w_{32} & w_{33} & w_{34} \\ w_{40} & w_{41} & w_{42} & w_{43} & w_{44} \end{pmatrix}$$

Question

If $\mathbf{x}$ is 7×7 and $\mathbf{w}$ is 5×5, what will be the dimensions of $\mathbf{h}$?

$$z_{22} = \sum_{k=0}^{4} \sum_{l=0}^{4} x_{2+k,2+l} w_{kl} + b$$

$$\begin{pmatrix} z_{00} & z_{01} & z_{02} \\ z_{10} & z_{11} & z_{12} \\ z_{20} & z_{21} & z_{22} \end{pmatrix} = \begin{pmatrix} x_{00} & x_{01} & x_{02} & x_{03} & x_{04} & x_{05} & x_{06} \\ x_{10} & x_{11} & x_{12} & x_{13} & x_{14} & x_{15} & x_{16} \\ x_{20} & x_{21} & x_{22} & x_{23} & x_{24} & x_{25} & x_{26} \\ x_{30} & x_{31} & x_{32} & x_{33} & x_{34} & x_{35} & x_{36} \\ x_{40} & x_{41} & x_{42} & x_{43} & x_{44} & x_{45} & x_{46} \\ x_{50} & x_{51} & x_{52} & x_{53} & x_{54} & x_{55} & x_{56} \\ x_{60} & x_{61} & x_{62} & x_{63} & x_{64} & x_{65} & x_{66} \end{pmatrix} \otimes \begin{pmatrix} w_{00} & w_{01} & w_{02} & w_{03} & w_{04} \\ w_{10} & w_{11} & w_{12} & w_{13} & w_{14} \\ w_{20} & w_{21} & w_{22} & w_{23} & w_{24} \\ w_{30} & w_{31} & w_{32} & w_{33} & w_{34} \\ w_{40} & w_{41} & w_{42} & w_{43} & w_{44} \end{pmatrix}$$

Calculating the feature map size

Q. If $\mathbf{x}$ is $q \times r$ and $\mathbf{w}$ is $m \times n$ dimensional, what is the output dimensionality?

A. $(q - m + 1) \times (r - n + 1)$

Padding

Q. What can we do to get 7×7 feature map size?

$$\begin{pmatrix} z_{00} & z_{01} & z_{02} & z_{03} & z_{04} & z_{05} & z_{06} \\ z_{10} & z_{11} & z_{12} & z_{13} & z_{14} & z_{15} & z_{16} \\ z_{20} & z_{21} & z_{22} & z_{23} & z_{24} & z_{25} & z_{26} \\ z_{30} & z_{31} & z_{32} & z_{33} & z_{34} & z_{35} & z_{36} \\ z_{40} & z_{41} & z_{42} & z_{43} & z_{44} & z_{45} & z_{46} \\ z_{50} & z_{51} & z_{52} & z_{53} & z_{54} & z_{55} & z_{56} \\ z_{60} & z_{61} & z_{62} & z_{63} & z_{64} & z_{65} & z_{66} \end{pmatrix} \boxed{=} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & x_{00} & x_{01} & x_{02} & x_{03} & x_{04} & x_{05} & x_{06} & 0 \\ 0 & x_{10} & x_{11} & x_{12} & x_{13} & x_{14} & x_{15} & x_{16} & 0 \\ 0 & x_{20} & x_{21} & x_{22} & x_{23} & x_{24} & x_{25} & x_{26} & 0 \\ 0 & x_{30} & x_{31} & x_{32} & x_{33} & x_{34} & x_{35} & x_{36} & 0 \\ 0 & x_{40} & x_{41} & x_{42} & x_{43} & x_{44} & x_{45} & x_{46} & 0 \\ 0 & x_{50} & x_{51} & x_{52} & x_{53} & x_{54} & x_{55} & x_{56} & 0 \\ 0 & x_{60} & x_{61} & x_{62} & x_{63} & x_{64} & x_{65} & x_{66} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} w_{00} & w_{01} & w_{02} \\ w_{10} & w_{11} & w_{12} \\ w_{20} & w_{21} & w_{22} \end{pmatrix}$$

$$\boxed{\otimes}$$

Padding

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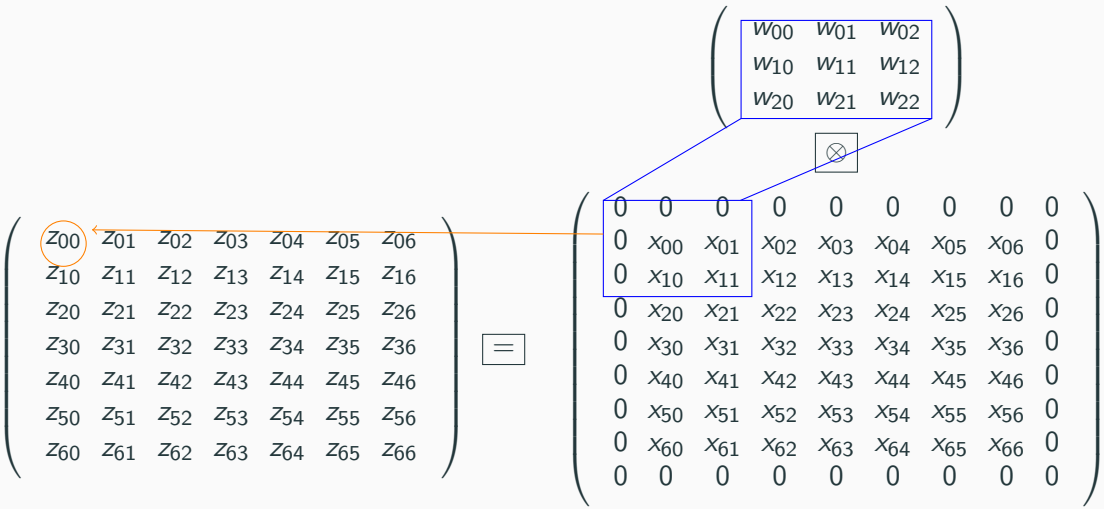
$$\begin{pmatrix} z_{00} & z_{01} & z_{02} & z_{03} & z_{04} & z_{05} & z_{06} \\ z_{10} & z_{11} & z_{12} & z_{13} & z_{14} & z_{15} & z_{16} \\ z_{20} & z_{21} & z_{22} & z_{23} & z_{24} & z_{25} & z_{26} \\ z_{30} & z_{31} & z_{32} & z_{33} & z_{34} & z_{35} & z_{36} \\ z_{40} & z_{41} & z_{42} & z_{43} & z_{44} & z_{45} & z_{46} \\ z_{50} & z_{51} & z_{52} & z_{53} & z_{54} & z_{55} & z_{56} \\ z_{60} & z_{61} & z_{62} & z_{63} & z_{64} & z_{65} & z_{66} \end{pmatrix}$$

$$\boxed{=}$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & x_{00} & x_{01} & x_{02} & x_{03} & x_{04} & x_{05} & x_{06} & 0 \\ 0 & x_{10} & x_{11} & x_{12} & x_{13} & x_{14} & x_{15} & x_{16} & 0 \\ 0 & x_{20} & x_{21} & x_{22} & x_{23} & x_{24} & x_{25} & x_{26} & 0 \\ 0 & x_{30} & x_{31} & x_{32} & x_{33} & x_{34} & x_{35} & x_{36} & 0 \\ 0 & x_{40} & x_{41} & x_{42} & x_{43} & x_{44} & x_{45} & x_{46} & 0 \\ 0 & x_{50} & x_{51} & x_{52} & x_{53} & x_{54} & x_{55} & x_{56} & 0 \\ 0 & x_{60} & x_{61} & x_{62} & x_{63} & x_{64} & x_{65} & x_{66} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Padding

Q. What can we do to get 7×7 feature map size?



Stride $s = 2$

$$\begin{pmatrix} w_{00} & w_{01} & w_{02} \\ w_{10} & w_{11} & w_{12} \\ w_{20} & w_{21} & w_{22} \end{pmatrix}$$

$$\boxed{\otimes}$$

$$\begin{pmatrix} z_{00} & z_{01} & z_{02} \\ z_{10} & z_{11} & z_{12} \\ z_{20} & z_{21} & z_{22} \end{pmatrix} \quad \boxed{=} \quad \begin{pmatrix} x_{00} & x_{01} & x_{02} & x_{03} & x_{04} & x_{05} & x_{06} \\ x_{10} & x_{11} & x_{12} & x_{13} & x_{14} & x_{15} & x_{16} \\ x_{20} & x_{21} & x_{22} & x_{23} & x_{24} & x_{25} & x_{26} \\ x_{30} & x_{31} & x_{32} & x_{33} & x_{34} & x_{35} & x_{36} \\ x_{40} & x_{41} & x_{42} & x_{43} & x_{44} & x_{45} & x_{46} \\ x_{50} & x_{51} & x_{52} & x_{53} & x_{54} & x_{55} & x_{56} \\ x_{60} & x_{61} & x_{62} & x_{63} & x_{64} & x_{65} & x_{66} \end{pmatrix}$$

Stride $s = 2$

$$z_{00} = \sum_{k=0}^{2} \sum_{l=0}^{2} x_{k,l} w_{kl} + b$$

$$\begin{pmatrix} w_{00} & w_{01} & w_{02} \\ w_{10} & w_{11} & w_{12} \\ w_{20} & w_{21} & w_{22} \end{pmatrix}$$

$$\otimes$$

$$\begin{pmatrix} x_{00} & x_{01} & x_{02} & x_{03} & x_{04} & x_{05} & x_{06} \\ x_{10} & x_{11} & x_{12} & x_{13} & x_{14} & x_{15} & x_{16} \\ x_{20} & x_{21} & x_{22} & x_{23} & x_{24} & x_{25} & x_{26} \\ x_{30} & x_{31} & x_{32} & x_{33} & x_{34} & x_{35} & x_{36} \\ x_{40} & x_{41} & x_{42} & x_{43} & x_{44} & x_{45} & x_{46} \\ x_{50} & x_{51} & x_{52} & x_{53} & x_{54} & x_{55} & x_{56} \\ x_{60} & x_{61} & x_{62} & x_{63} & x_{64} & x_{65} & x_{66} \end{pmatrix}$$

$$=$$

$$\begin{pmatrix} z_{00} & z_{01} & z_{02} \\ z_{10} & z_{11} & z_{12} \\ z_{20} & z_{21} & z_{22} \end{pmatrix}$$

Stride $s = 2$

$$z_{01} = \sum_{k=0}^{2} \sum_{l=0}^{2} x_{k,s+l} w_{kl} + b$$

$$\begin{pmatrix} z_{00} & z_{01} & z_{02} \\ z_{10} & z_{11} & z_{12} \\ z_{20} & z_{21} & z_{22} \end{pmatrix} = \begin{pmatrix} x_{00} & x_{01} & x_{02} & x_{03} & x_{04} & x_{05} & x_{06} \\ x_{10} & x_{11} & x_{12} & x_{13} & x_{14} & x_{15} & x_{16} \\ x_{20} & x_{21} & x_{22} & x_{23} & x_{24} & x_{25} & x_{26} \\ x_{30} & x_{31} & x_{32} & x_{33} & x_{34} & x_{35} & x_{36} \\ x_{40} & x_{41} & x_{42} & x_{43} & x_{44} & x_{45} & x_{46} \\ x_{50} & x_{51} & x_{52} & x_{53} & x_{54} & x_{55} & x_{56} \\ x_{60} & x_{61} & x_{62} & x_{63} & x_{64} & x_{65} & x_{66} \end{pmatrix} \otimes \begin{pmatrix} w_{00} & w_{01} & w_{02} \\ w_{10} & w_{11} & w_{12} \\ w_{20} & w_{21} & w_{22} \end{pmatrix}$$

Stride $s = 2$

$$z_{02} = \sum_{k=0}^{2} \sum_{l=0}^{2} x_{k,2s+l} w_{kl} + b$$

$$\begin{pmatrix} z_{00} & z_{01} & z_{02} \\ z_{10} & z_{11} & z_{12} \\ z_{20} & z_{21} & z_{22} \end{pmatrix} = \begin{pmatrix} x_{00} & x_{01} & x_{02} & x_{03} & x_{04} & x_{05} & x_{06} \\ x_{10} & x_{11} & x_{12} & x_{13} & x_{14} & x_{15} & x_{16} \\ x_{20} & x_{21} & x_{22} & x_{23} & x_{24} & x_{25} & x_{26} \\ x_{30} & x_{31} & x_{32} & x_{33} & x_{34} & x_{35} & x_{36} \\ x_{40} & x_{41} & x_{42} & x_{43} & x_{44} & x_{45} & x_{46} \\ x_{50} & x_{51} & x_{52} & x_{53} & x_{54} & x_{55} & x_{56} \\ x_{60} & x_{61} & x_{62} & x_{63} & x_{64} & x_{65} & x_{66} \end{pmatrix} \otimes \begin{pmatrix} w_{00} & w_{01} & w_{02} \\ w_{10} & w_{11} & w_{12} \\ w_{20} & w_{21} & w_{22} \end{pmatrix}$$

Stride $s = 2$

$$z_{10} = \sum_{k=0}^{2} \sum_{l=0}^{2} x_{s+k,l} w_{kl} + b$$

$$\begin{pmatrix} z_{00} & z_{01} & z_{02} \\ z_{10} & z_{11} & z_{12} \\ z_{20} & z_{21} & z_{22} \end{pmatrix} = \begin{pmatrix} x_{00} & x_{01} & x_{02} & x_{03} & x_{04} & x_{05} & x_{06} \\ x_{10} & x_{11} & x_{12} & x_{13} & x_{14} & x_{15} & x_{16} \\ x_{20} & x_{21} & x_{22} & x_{23} & x_{24} & x_{25} & x_{26} \\ x_{30} & x_{31} & x_{32} & x_{33} & x_{34} & x_{35} & x_{36} \\ x_{40} & x_{41} & x_{42} & x_{43} & x_{44} & x_{45} & x_{46} \\ x_{50} & x_{51} & x_{52} & x_{53} & x_{54} & x_{55} & x_{56} \\ x_{60} & x_{61} & x_{62} & x_{63} & x_{64} & x_{65} & x_{66} \end{pmatrix} \otimes \begin{pmatrix} w_{00} & w_{01} & w_{02} \\ w_{10} & w_{11} & w_{12} \\ w_{20} & w_{21} & w_{22} \end{pmatrix}$$

Take-home exercise

What is the feature map size when $\mathbf{x}$ is $q \times r$ and $\mathbf{w}$ is $m \times n$ dimensional, for padding p and stride s?

Receptive field

- Biology: The **receptive field** of an individual sensory neuron is the particular region of the sensory space,
- Convolutional networks: The region in the input space that a hidden unit is looking at.

$$\begin{pmatrix} h_{00} & h_{01} & h_{02} & h_{03} & h_{04} \\ h_{10} & h_{11} & h_{12} & h_{13} & h_{14} \\ h_{20} & h_{21} & h_{22} & h_{23} & h_{24} \\ h_{30} & h_{31} & h_{32} & h_{33} & h_{34} \\ h_{40} & h_{41} & h_{42} & h_{43} & h_{44} \end{pmatrix}$$

$$\begin{pmatrix} x_{00} & x_{01} & x_{02} & x_{03} & x_{04} & x_{05} & x_{06} \\ x_{10} & x_{11} & x_{12} & x_{13} & x_{14} & x_{15} & x_{16} \\ x_{20} & x_{21} & x_{22} & x_{23} & x_{24} & x_{25} & x_{26} \\ x_{30} & x_{31} & x_{32} & x_{33} & x_{34} & x_{35} & x_{36} \\ x_{40} & x_{41} & x_{42} & x_{43} & x_{44} & x_{45} & x_{46} \\ x_{50} & x_{51} & x_{52} & x_{53} & x_{54} & x_{55} & x_{56} \\ x_{60} & x_{61} & x_{62} & x_{63} & x_{64} & x_{65} & x_{66} \end{pmatrix}$$

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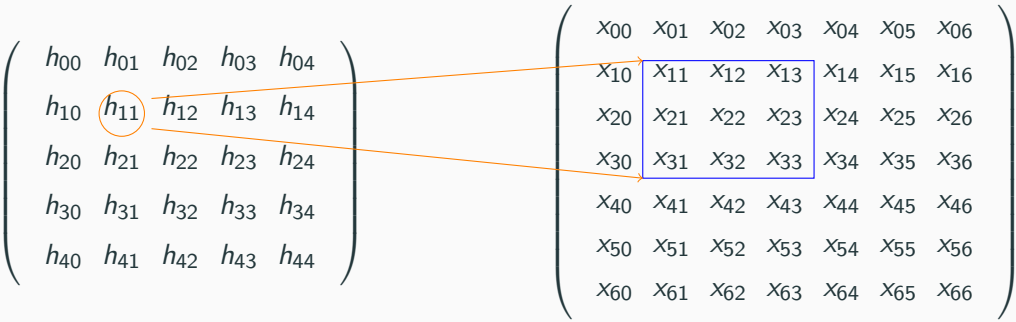

Receptive field

$$\mathbf{h}^{(2)} = f(\mathbf{w}^2 \otimes f(\mathbf{w}^1 \otimes \mathbf{x}))$$

Assume that $\mathbf{w}^1$ and $\mathbf{w}^2$ are 3×3 kernels and $\mathbf{x}$ is 7×7.

Q. What size is the receptive field of a hidden unit in second convolutional layer?

A. 3, 5, 6, 7?

$$\begin{pmatrix} h^{(2)}_{00} & h^{(2)}_{01} & h^{(2)}_{02} \\ h^{(2)}_{10} & h^{(2)}_{11} & h^{(2)}_{12} \\ h^{(2)}_{20} & h^{(2)}_{21} & h^{(2)}_{22} \end{pmatrix} \begin{pmatrix} h^{(1)}_{00} & h^{(1)}_{01} & h^{(1)}_{02} & h^{(1)}_{03} & h^{(1)}_{04} \\ h^{(1)}_{10} & h^{(1)}_{11} & h^{(1)}_{12} & h^{(1)}_{13} & h^{(1)}_{14} \\ h^{(1)}_{20} & h^{(1)}_{21} & h^{(1)}_{22} & h^{(1)}_{23} & h^{(1)}_{24} \\ h^{(1)}_{30} & h^{(1)}_{31} & h^{(1)}_{32} & h^{(1)}_{33} & h^{(1)}_{34} \\ h^{(1)}_{40} & h^{(1)}_{41} & h^{(1)}_{42} & h^{(1)}_{43} & h^{(1)}_{44} \end{pmatrix} \begin{pmatrix} x_{00} & x_{01} & x_{02} & x_{03} & x_{04} & x_{05} & x_{06} \\ x_{10} & x_{11} & x_{12} & x_{13} & x_{14} & x_{15} & x_{16} \\ x_{20} & x_{21} & x_{22} & x_{23} & x_{24} & x_{25} & x_{26} \\ x_{30} & x_{31} & x_{32} & x_{33} & x_{34} & x_{35} & x_{36} \\ x_{40} & x_{41} & x_{42} & x_{43} & x_{44} & x_{45} & x_{46} \\ x_{50} & x_{51} & x_{52} & x_{53} & x_{54} & x_{55} & x_{56} \\ x_{60} & x_{61} & x_{62} & x_{63} & x_{64} & x_{65} & x_{66} \end{pmatrix}$$

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Assume that $\mathbf{w}^1$ and $\mathbf{w}^2$ are 3×3 kernels and $\mathbf{x}$ is 7×7.

Q. What size is the receptive field of a hidden unit in second convolutional layer?

A. 3, 5, 6, 7?

$$\begin{pmatrix} h^{(2)}_{00} & h^{(2)}_{01} & h^{(2)}_{02} \\ h^{(2)}_{10} & h^{(2)}_{11} & h^{(2)}_{12} \\ h^{(2)}_{20} & h^{(2)}_{21} & h^{(2)}_{22} \end{pmatrix} \begin{pmatrix} h^{(1)}_{00} & h^{(1)}_{01} & h^{(1)}_{02} & h^{(1)}_{03} & h^{(1)}_{04} \\ h^{(1)}_{10} & h^{(1)}_{11} & h^{(1)}_{12} & h^{(1)}_{13} & h^{(1)}_{14} \\ h^{(1)}_{20} & h^{(1)}_{21} & h^{(1)}_{22} & h^{(1)}_{23} & h^{(1)}_{24} \\ h^{(1)}_{30} & h^{(1)}_{31} & h^{(1)}_{32} & h^{(1)}_{33} & h^{(1)}_{34} \\ h^{(1)}_{40} & h^{(1)}_{41} & h^{(1)}_{42} & h^{(1)}_{43} & h^{(1)}_{44} \end{pmatrix} \begin{pmatrix} x_{00} & x_{01} & x_{02} & x_{03} & x_{04} & x_{05} & x_{06} \\ x_{10} & x_{11} & x_{12} & x_{13} & x_{14} & x_{15} & x_{16} \\ x_{20} & x_{21} & x_{22} & x_{23} & x_{24} & x_{25} & x_{26} \\ x_{30} & x_{31} & x_{32} & x_{33} & x_{34} & x_{35} & x_{36} \\ x_{40} & x_{41} & x_{42} & x_{43} & x_{44} & x_{45} & x_{46} \\ x_{50} & x_{51} & x_{52} & x_{53} & x_{54} & x_{55} & x_{56} \\ x_{60} & x_{61} & x_{62} & x_{63} & x_{64} & x_{65} & x_{66} \end{pmatrix}$$

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$$\begin{pmatrix} h^{(2)}_{00} & h^{(2)}_{01} & h^{(2)}_{02} \\ h^{(2)}_{10} & h^{(2)}_{11} & h^{(2)}_{12} \\ h^{(2)}_{20} & h^{(2)}_{21} & h^{(2)}_{22} \end{pmatrix} \begin{pmatrix} h^{(1)}_{00} & h^{(1)}_{01} & h^{(1)}_{02} & h^{(1)}_{03} & h^{(1)}_{04} \\ h^{(1)}_{10} & h^{(1)}_{11} & h^{(1)}_{12} & h^{(1)}_{13} & h^{(1)}_{14} \\ h^{(1)}_{20} & h^{(1)}_{21} & h^{(1)}_{22} & h^{(1)}_{23} & h^{(1)}_{24} \\ h^{(1)}_{30} & h^{(1)}_{31} & h^{(1)}_{32} & h^{(1)}_{33} & h^{(1)}_{34} \\ h^{(1)}_{40} & h^{(1)}_{41} & h^{(1)}_{42} & h^{(1)}_{43} & h^{(1)}_{44} \end{pmatrix} \begin{pmatrix} x_{00} & x_{01} & x_{02} & x_{03} & x_{04} & x_{05} & x_{06} \\ x_{10} & x_{11} & x_{12} & x_{13} & x_{14} & x_{15} & x_{16} \\ x_{20} & x_{21} & x_{22} & x_{23} & x_{24} & x_{25} & x_{26} \\ x_{30} & x_{31} & x_{32} & x_{33} & x_{34} & x_{35} & x_{36} \\ x_{40} & x_{41} & x_{42} & x_{43} & x_{44} & x_{45} & x_{46} \\ x_{50} & x_{51} & x_{52} & x_{53} & x_{54} & x_{55} & x_{56} \\ x_{60} & x_{61} & x_{62} & x_{63} & x_{64} & x_{65} & x_{66} \end{pmatrix}$$

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Q. What size is the receptive field of a hidden unit in second convolutional layer?

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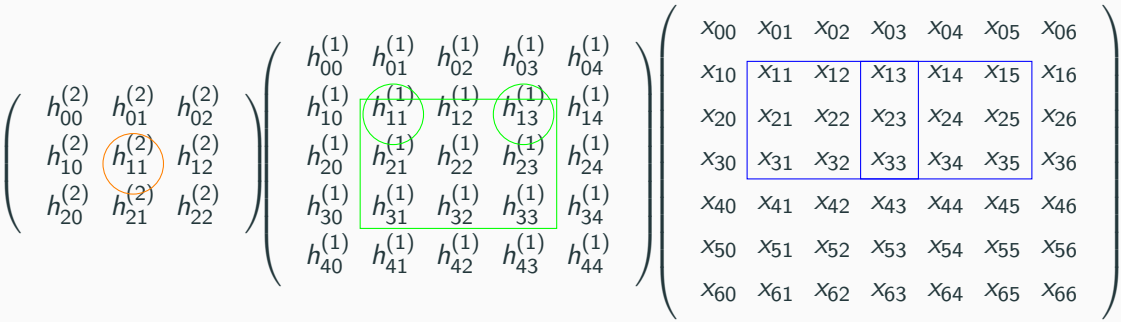

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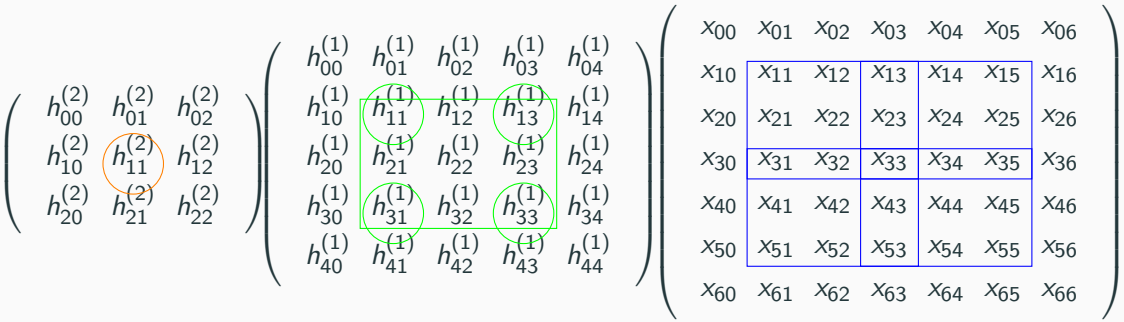


Feature maps

- Constrain each hidden unit h_{ij} to extract the same feature by **sharing weights** across the receptive fields
- Local receptive fields with shared weights result in a **feature map** which is a map showing where the feature corresponding to the shared weight matrix (kernel) occurs in the image
- Feature map encodes **translation invariance**: extract the same features irrespective of where an image is located in the input

Multiple output feature maps

$$\begin{pmatrix} h_{000} & h_{001} & \dots & h_{004} \\ h_{010} & h_{011} & \dots & h_{014} \\ \vdots & \vdots & \ddots & \vdots \\ h_{040} & h_{041} & \dots & h_{044} \end{pmatrix} = \begin{pmatrix} x_{00} & x_{01} & \dots & x_{06} \\ x_{10} & x_{11} & \dots & x_{16} \\ \vdots & \vdots & \ddots & \vdots \\ x_{60} & x_{61} & \dots & x_{66} \end{pmatrix} \otimes \begin{pmatrix} w_{000} & w_{001} & w_{002} \\ w_{010} & w_{011} & w_{012} \\ w_{020} & w_{021} & w_{022} \end{pmatrix}$$

$$\begin{pmatrix} h_{100} & h_{101} & \dots & h_{104} \\ h_{110} & h_{111} & \dots & h_{114} \\ \vdots & \vdots & \ddots & \vdots \\ h_{140} & h_{141} & \dots & h_{144} \end{pmatrix} = \begin{pmatrix} x_{00} & x_{01} & \dots & x_{06} \\ x_{10} & x_{11} & \dots & x_{16} \\ \vdots & \vdots & \ddots & \vdots \\ x_{60} & x_{61} & \dots & x_{66} \end{pmatrix} \otimes \begin{pmatrix} w_{100} & w_{101} & w_{102} \\ w_{110} & w_{111} & w_{112} \\ w_{120} & w_{121} & w_{122} \end{pmatrix}$$

h_{ijk}: i-th feature-map, j-th row, k-th column

Multiple output feature maps

- # input units is 7×7
- # feature maps is F_{out}
- # hidden units is $F_{\text{out}} \times (5 \times 5)$
- # of parameters is $F_{\text{out}} \times (3 \times 3 + 1)$
- # of connections is $F_{\text{out}} \times (5 \times 5 \times 3 \times 3)$

Multiple input feature maps (or input channels)

$$\begin{pmatrix} h_{00} & h_{01} & \dots & h_{04} \\ h_{10} & h_{11} & \dots & h_{14} \\ \vdots & \vdots & \ddots & \vdots \\ h_{40} & h_{41} & \dots & h_{44} \end{pmatrix} = \begin{pmatrix} x_{000} & x_{001} & \dots & x_{006} \\ x_{010} & x_{011} & \dots & x_{016} \\ \vdots & \vdots & \ddots & \vdots \\ x_{060} & x_{061} & \dots & x_{066} \end{pmatrix} \otimes \begin{pmatrix} w_{000} & w_{001} & w_{002} \\ w_{010} & w_{011} & w_{012} \\ w_{020} & w_{021} & w_{022} \end{pmatrix} + \begin{pmatrix} x_{100} & x_{101} & \dots & x_{106} \\ x_{110} & x_{111} & \dots & x_{116} \\ \vdots & \vdots & \ddots & \vdots \\ x_{160} & x_{161} & \dots & x_{166} \end{pmatrix} \otimes \begin{pmatrix} w_{100} & w_{101} & w_{102} \\ w_{110} & w_{111} & w_{112} \\ w_{120} & w_{121} & w_{122} \end{pmatrix}$$

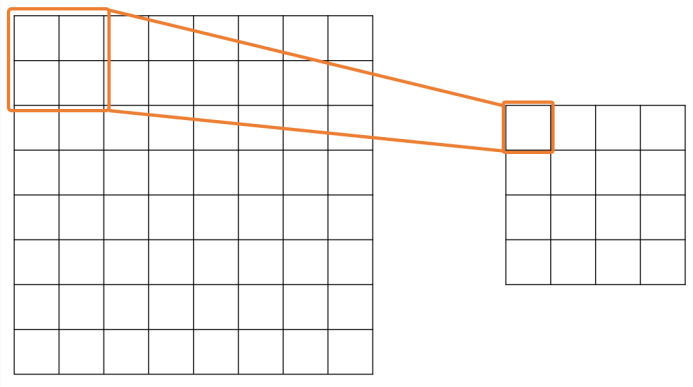
Multiple input feature maps (or input channels)

- # input channels is F_{in}
- # input units is $F_{\text{in}} \times 7 \times 7$
- # hidden units is 5×5
- # parameters is $F_{\text{in}} \times 3 \times 3 + 1$ for bias
- # connections is $F_{\text{in}} \times 5 \times 5 \times 3 \times 3$
- Typically we do not tie weights across feature maps
- Local receptive fields across multiple input images

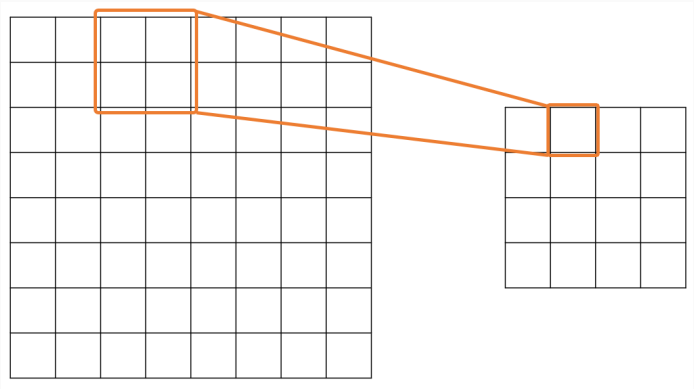
Multiple input and output feature maps

- # input channels is F_{in}
- # feature maps is F_{out}
- # input units is $F_{\text{in}} \times 7 \times 7$
- # hidden units is $F_{\text{out}} \times 5 \times 5$
- # parameters is $F_{\text{in}} \times F_{\text{out}} \times 3 \times 3 + F_{\text{out}}$ for bias
- # connections is $F_{\text{in}} \times F_{\text{out}} \times 5 \times 5 \times 3 \times 3$

Pooling (subsampling)



Pooling (subsampling)



- Similar to convolution, slides over input pixels but no learnable parameters
- Has local receptive field too
- Typical stride $s > 1$

Pooling

- Pooling or subsampling takes a feature map and reduces it in size – e.g. by transforming a set of 2x2 regions to a single unit
- Reduces computation time and memory use
- Pooling functions
  - Max-pooling – takes the maximum value of the units in the region
  - L_p-pooling – take the L_p norm of the units in the region:

$$h' = \left(\sum_{i \in \text{region}} h_i^p \right)^{1/p}$$

  - Average- / Sum-pooling – takes the average / sum value of the pool
- Information reduction – pooling removes precise location information for a feature
- Apply pooling to each feature map separately

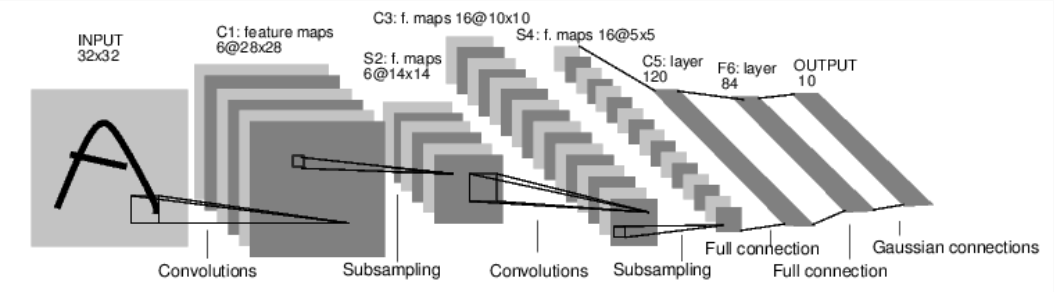
Learning image kernels

Identity	$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	
Edge detection	$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 1 \end{bmatrix}$	
	$\begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$	
	$\begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix}$	
Sharpen	$\begin{bmatrix} 0 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 0 \end{bmatrix}$	

https://en.wikipedia.org/wiki/Kernel_(image_processing)

- Image kernels have been **hand designed** and used for feature extraction in image processing (e.g. edge detection)
- Pros: No need for data and training
- Cons 1: Learning filters can be more optimal (minimising network cost function)
- Cons 2: Difficult to design filters for complex tasks (e.g. recognising a cat)
- Automating feature engineering

Example: LeNet5 (LeCun et al, 1997)



MNIST Results (1997)

ImageNet Classification ("AlexNet")

Krizhevsky, Sutskever and Hinton, "ImageNet Classification with Deep Convolutional Neural Networks", NIPS'12.

http://papers.nips.cc/paper/4824-imagenet-classification-with-deep-convolutional-neural-networks.pdf

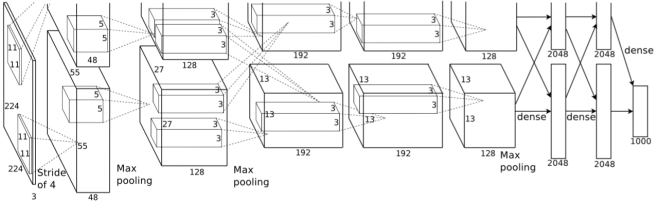
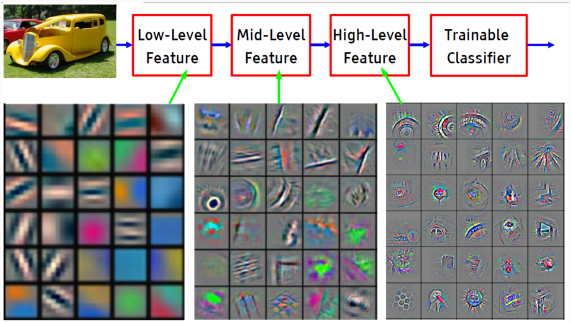


Figure 2: An illustration of the architecture of our CNN, explicitly showing the delineation of responsibilities between the two GPUs. One GPU runs the layer-parts at the top of the figure while the other runs the layer-parts at the bottom. The GPUs communicate only at certain layers. The network's input is 150,528-dimensional, and the number of neurons in the network's remaining layers is given by 253,440–186,624–64,896–64,896–43,264–4096–4096–1000.

Model	Top-1	Top-5
Sparse coding [2]	*47.1%*	*28.2%*
SIFT + FVs [24]	*45.7%*	*25.7%*
CNN	**37.5%**	**17.0%**

Hierarchical Representations

Pixel → edge → texton → motif → part → object



Zeiler & Fergus, "Visualizing and Understanding Convolutional Networks", ECCV'14.

https://cs.nyu.edu/~fergus/papers/zeilerECCV2014.pdf

Slide credits: Lecun & Ranzato

Training Convolutional Networks

- Train convolutional networks with a straightforward but careful application of backprop / SGD
- Exercise: prior to the next lecture, write down the gradients for the weights and biases of the feature maps in a convolutional network. Remember to take account of weight sharing.
- Next lecture: implementing convolutional networks: how to deal with local receptive fields and tied weights, computing the required gradients...

Summary

Convolutional networks include local receptive fields, weight sharing, and pooling leading to:

- Modelling the spatial structure
- Translation invariance
- Local feature detection

Reading:

Michael Nielsen, *Neural Networks and Deep Learning* (ch 6)

http://neuralnetworksanddeeplearning.com/chap6.html

Yann LeCun et al, "Gradient-Based Learning Applied to Document Recognition", *Proc IEEE*, 1998. http://dx.doi.org/10.1109/5.726791

Ian Goodfellow, Yoshua Bengio & Aaron Courville, *Deep Learning* (ch 9)

http://www.deeplearningbook.org/contents/convnets.html