Convolutional Networks (part 2)

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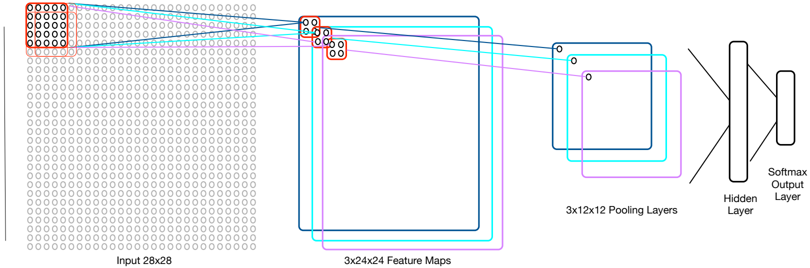
Machine Learning Practical — MLP Lecture 8
9 November 2016

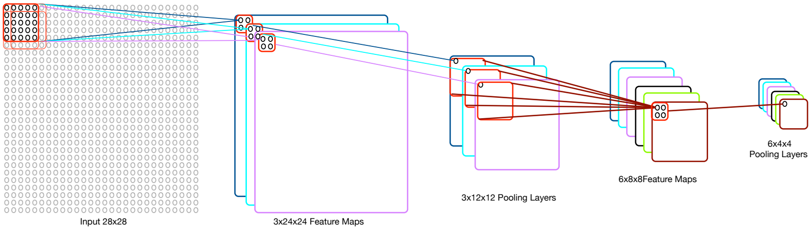
Recap: Convolutional Network



Simple ConvNet:

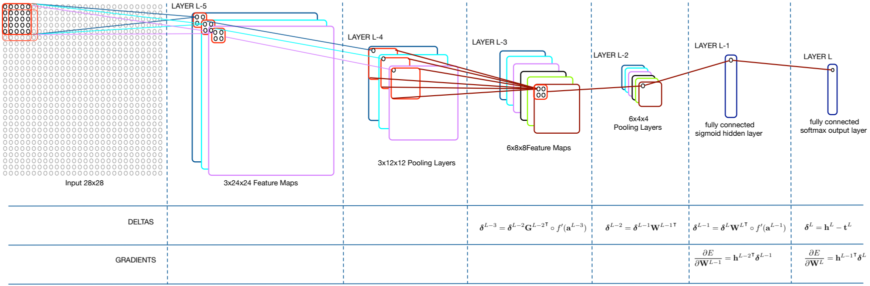
- One convolutional layer with max-pooling
- Final fully connected hidden layer (no sharing weight)
- Softmax output layer

Recap: Stacking convolutional layers



- Local receptive fields
- Weight sharing
- Pooling/subsampling

Training Convolutional Networks – Pooling Layer



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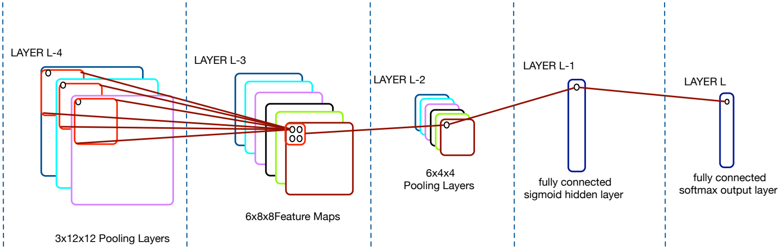
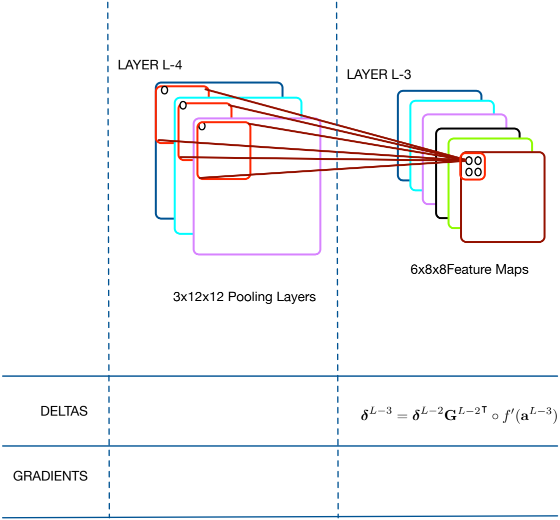


	LAYER L-4	LAYER L-3	LAYER L-2	LAYER L-1	LAYER L
DELTAS		$\boldsymbol{\delta}^{L-3} = \boldsymbol{\delta}^{L-2}\mathbf{G}^{L-2\intercal} \circ f'(\mathbf{a}^{L-3})$	$\boldsymbol{\delta}^{L-2} = \boldsymbol{\delta}^{L-1}\mathbf{W}^{L-1\intercal}$	$\boldsymbol{\delta}^{L-1} = \boldsymbol{\delta}^{L}\mathbf{W}^{L\intercal} \circ f'(\mathbf{a}^{L-1})$	$\boldsymbol{\delta}^{L} = \mathbf{h}^{L} - \mathbf{t}^{L}$
GRADIENTS				$\frac{\partial E}{\partial \mathbf{W}^{L-1}} = \mathbf{h}^{L-2\intercal}\boldsymbol{\delta}^{L-1}$	$\frac{\partial E}{\partial \mathbf{W}^{L}} = \mathbf{h}^{L-1\intercal}\boldsymbol{\delta}^{L}$

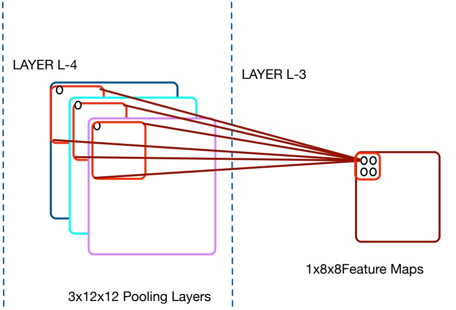
- **G** is a "pseudo-weight matrix" for max-pooling which is set during the forward propagation: $G_{ba} = 1$ if feature map unit b is contained in max-pool a and is the maximum value for the current input. Note that **G** is different for each item in the minibatch.

Training Convolutional Networks – Convolutional Layer



Training the convolutional layer is more complicated

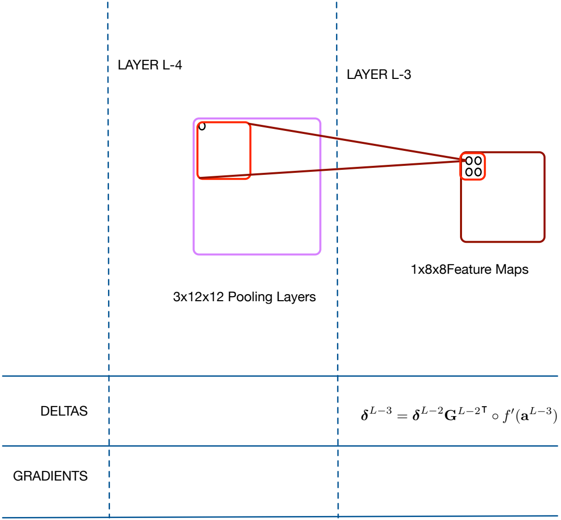
Training Convolutional Networks – Convolutional Layer



DELTAS		$\delta^{L-3} = \delta^{L-2} \mathbf{G}^{L-2\mathsf{T}} \circ f'(\mathbf{a}^{L-3})$
GRADIENTS		

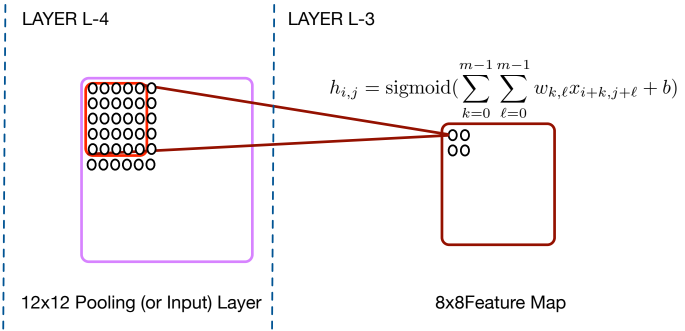
Only need to consider one pooling layer

Training Convolutional Networks – Convolutional Layer



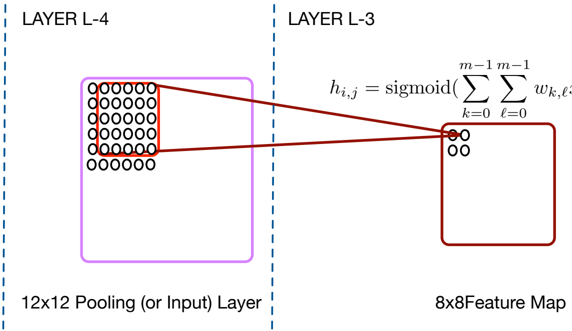
Simplify by only considering one feature map

Convolutional Layer – Forward Prop



In the forward propagation, each hidden unit is connected to a region of input units (the receptive field)

Convolutional Layer – Forward Prop



$$h_{i,j} = \mathrm{sigmoid}\left(\sum_{k=0}^{m-1} \sum_{\ell=0}^{m-1} w_{k,\ell} x_{i+k,j+\ell} + b\right)$$

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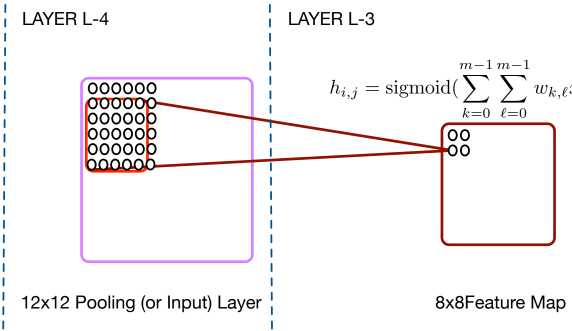
Convolutional Layer – Forward Prop



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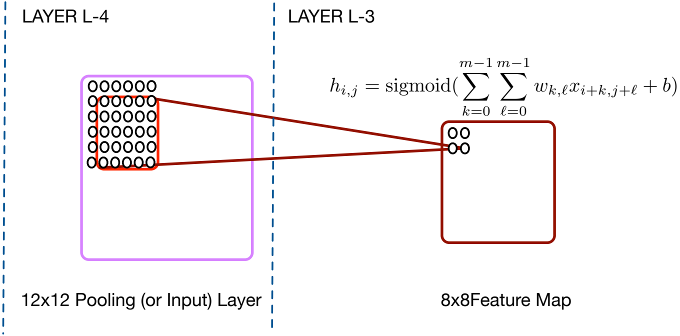

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Convolutional Layer – Back Prop

For backprop we need to consider the region of hidden units connected to each input unit.

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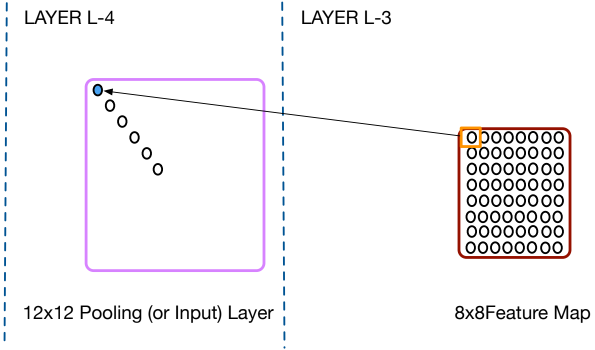
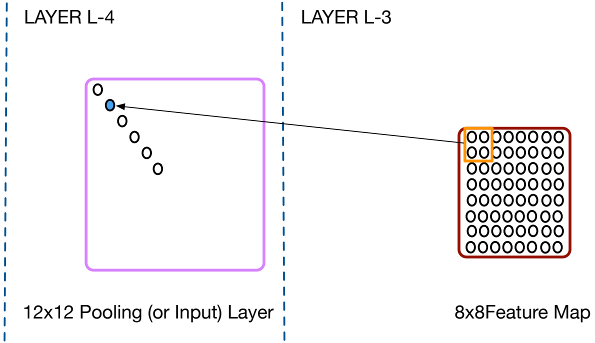


The top-left input unit (1,1) is connected to just one hidden unit

Convolutional Layer – Back Prop

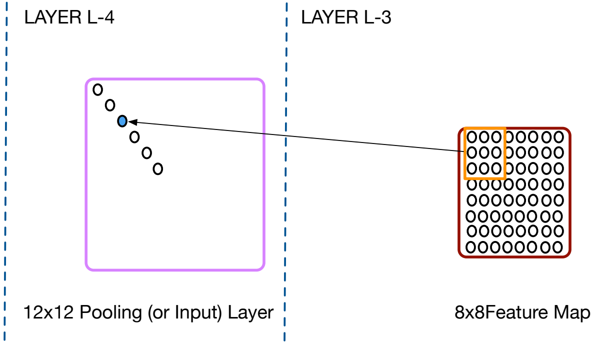
For backprop we need to consider the region of hidden units connected to each input unit.



Input unit (2,2) is in the receptive fields of $2 \times 2 = 4$ hidden units

Convolutional Layer – Back Prop

For backprop we need to consider the region of hidden units connected to each input unit.



(3,3) is in the receptive fields of $3 \times 3 = 9$ hidden units

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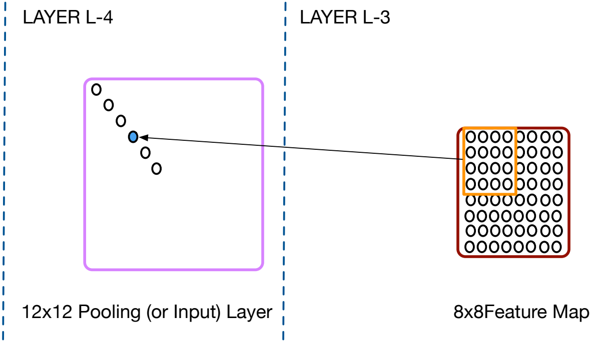
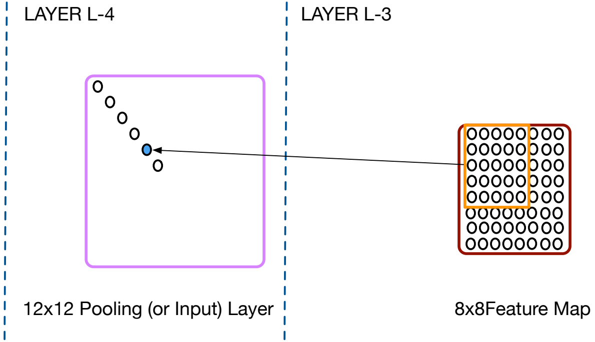


(4,4) is in the receptive fields of $4 \times 4 = 16$ hidden units

Convolutional Layer – Back Prop

For backprop we need to consider the region of hidden units connected to each input unit.



(5,5) and all units away from the edge are in the receptive fields of $5 \times 5 = 25$ hidden units

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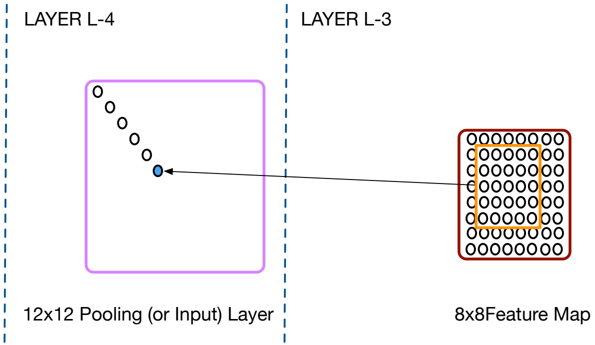


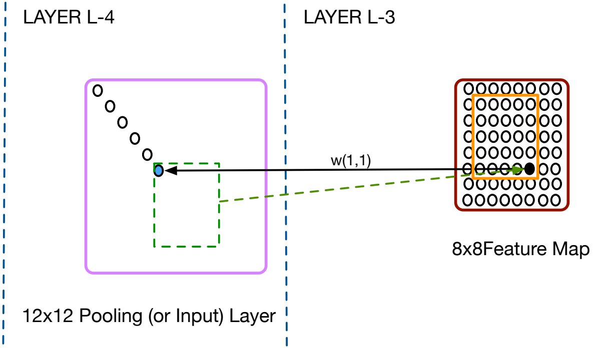
(5,5) and all units away from the edge are in the receptive fields of $5 \times 5 = 25$ hidden units

Convolutional Layer – Back Prop

As usual we want to back-propagate the δ values:

$$\delta_s^{L-4} = \sum_{j \in \text{connected to } s} w_{js} \delta_j^{L-3} f'(a_s)$$

Look at the shared weights used for back prop:



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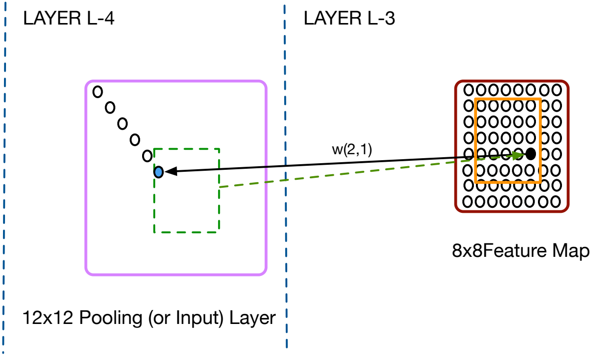

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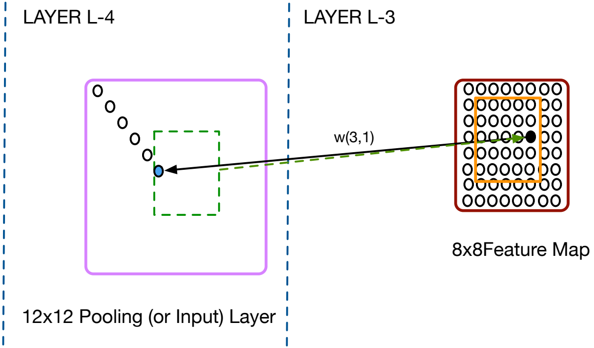

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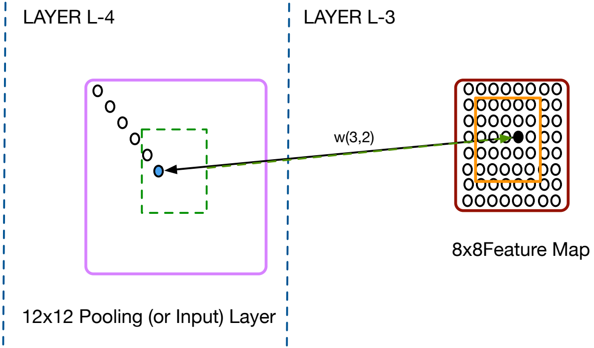

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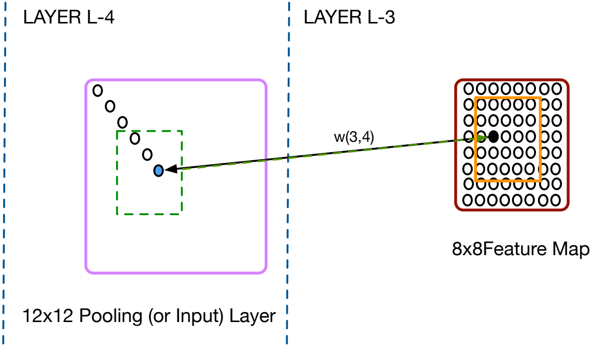

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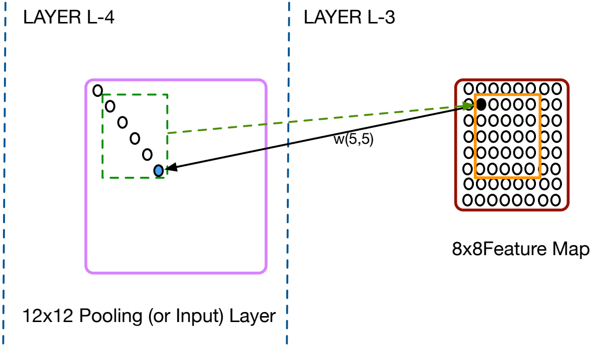
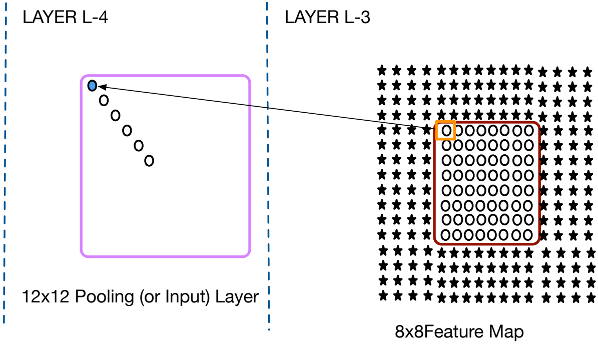


Backprop as convolution

If we have an $m \times m$ kernel size, we can pad the feature map with $(m-1)$ rows and columns of 0s top and bottom, left and right.



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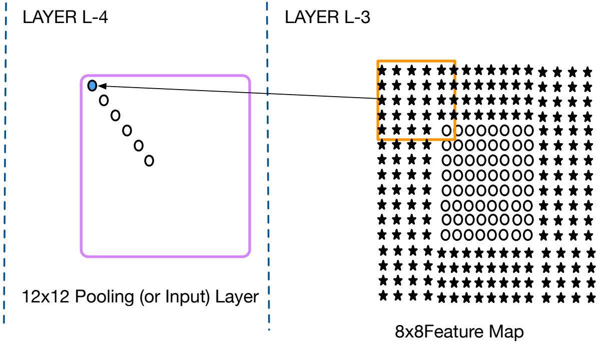

Back prop can then be carried out as a convolution using the weight matrix to scan the padded feature map... BUT the *weight matrix is rotated by 180°* as shown before

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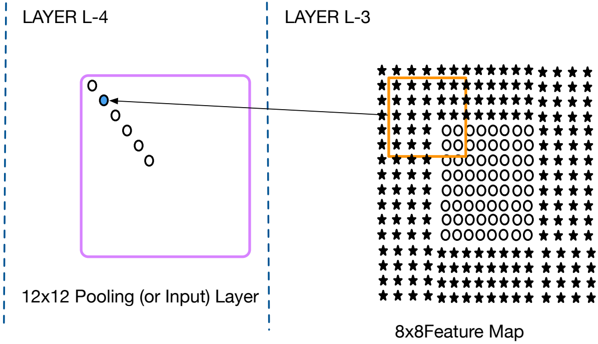

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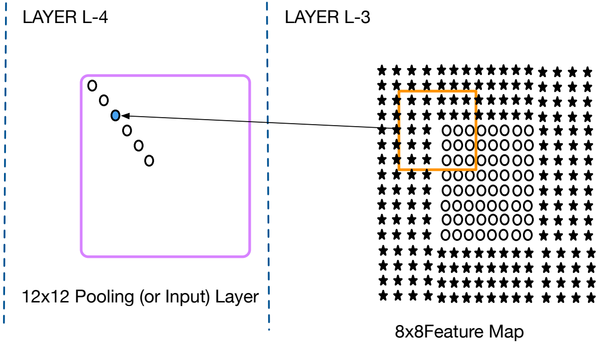

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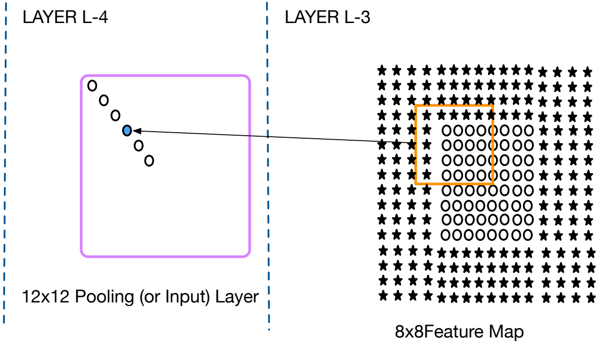

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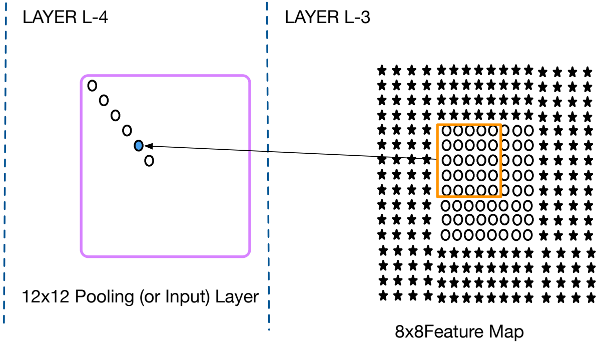

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Convolutional Layer – Back Prop

Back-propagation in the convolution layer, is also a convolution!
But we have to *rotate* the weight matrix $\mathbf{W}$ by 180°, $\mathbf{W}^R$
Using the convolution operator we saw we can write the forward prop as:

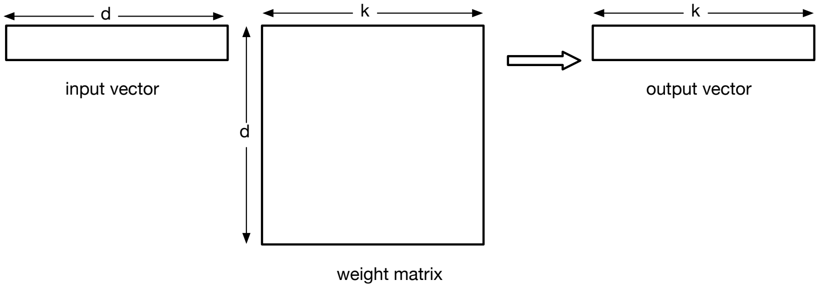
$$\mathbf{h}^{L-3} = \text{sigmoid}(\mathbf{W}^{L-3} * \mathbf{h}^{L-4} + \mathbf{b}^{L-3})$$

And we can write the back-prop as:

$$\boldsymbol{\delta}^{L-4} = {\mathbf{W}^{L-3}}^R * \boldsymbol{\delta}^{L-3} \circ f'(\mathbf{a}^{\mathbf{L-4}})$$

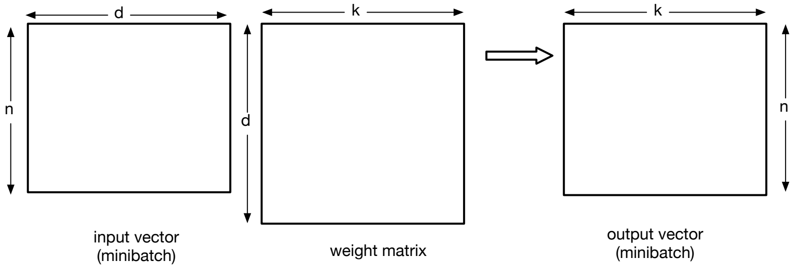
Implementing multilayer networks

Example at a time:



Implementing multilayer networks

Minibatch:



Implementing multilayer networks

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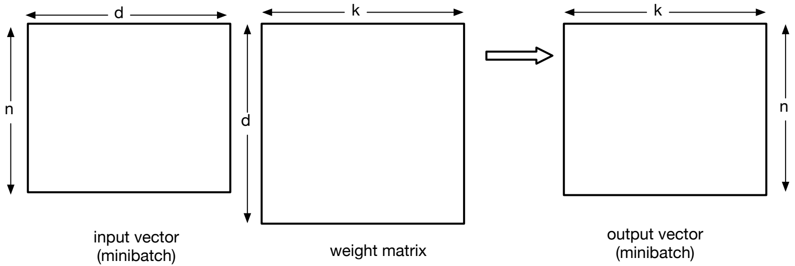
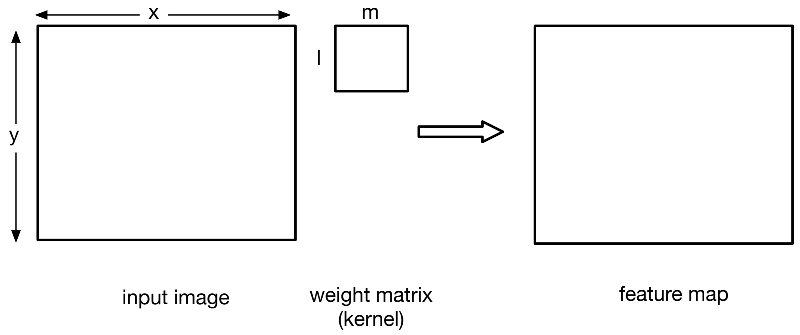


input dimension x minibatch: Represent each layer as a 2-dimension matrix, where each row corresponds to a training example, and the number of minibatch examples is the number of rows

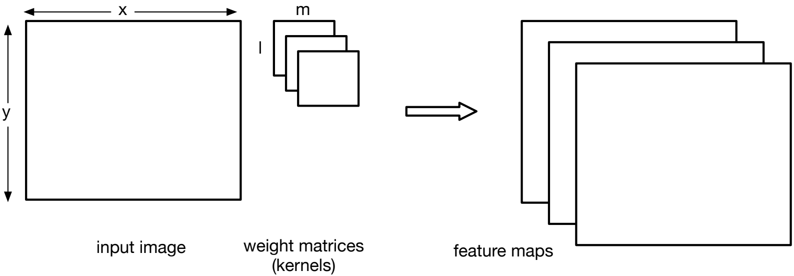
Implementing Convolutional Networks

Example at a time, single input image, single feature map:



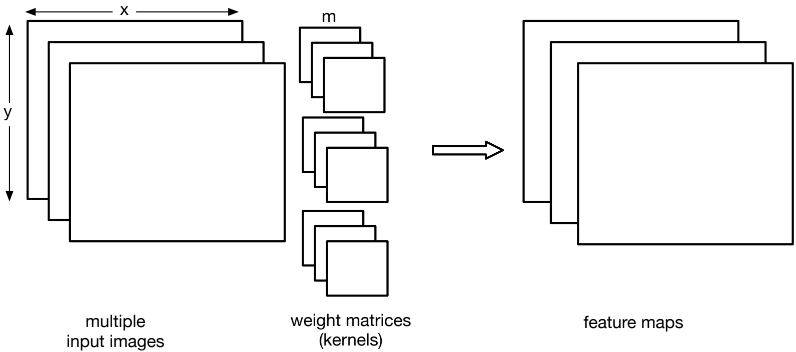
Implementing Convolutional Networks

Example at a time, single input image, multiple feature map:



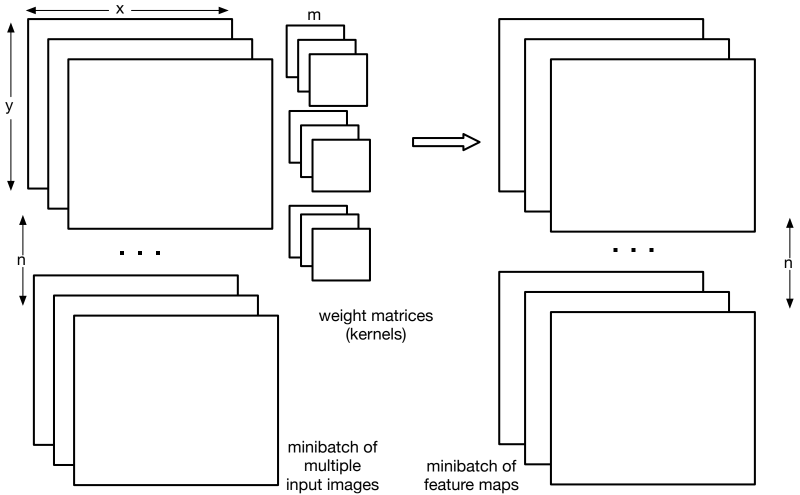
Implementing Convolutional Networks

Example at a time, multiple input images, multiple feature map:



Implementing Convolutional Networks

Minibatch, multiple input images, multiple feature map:



Implementing Convolutional Networks

- Inputs / layer values:
  - Each input image (and convlutional and pooling layer) is 2-dimensions (x,y)
  - If we have multiple feature maps, then that is a third dimension
  - And the minibatch adds a fourth dimension
  - Thus we represent each input (layer values) using a 4-dimension *tensor* (array): (minibatch-size, num-fmaps, x, y)
- Weight matrices (kernels)
  - Each weight matrix used to scan across an image has 2 spatial dimensions (x,y)
  - If there are multiple feature maps to be computed, then that is a third dimension
  - Multiple input feature maps adds a fourth dimension
  - Thus the weight matrices are also represented using a 4-dimension tensor: (num-fmaps-in, num-fmaps-out, x, y)

4D tensors in numpy

Both forward and back prop thus involves multiplying 4D tensors. There are various ways to do this:

- Explicitly loop over the dimensions: this results in simpler code, but can be inefficient. Although using cython to compile the loops as C can speed things up
- Serialisation: By replicating input patches and weight matrices, it is possible to convert the required 4D tensor multiplications into a large dot product. Requires careful manipulation of indices!
- Convolutions: use explicit convolution functions for forward and back prop, rotating for the backprop

Recent advances using convolutional networks

ImageNet Classification ("AlexNet")

Krizhevsky, Sutskever and Hinton, "ImageNet Classification with Deep Convolutional Neural Networks", NIPS-2012. http://papers.nips.cc/paper/4824-imagenet-classification-with-deep-convolutional-neural-networks.pdf

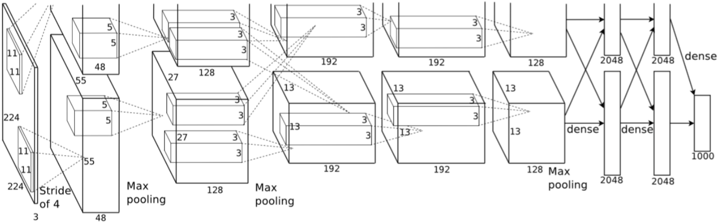


Figure 2: An illustration of the architecture of our CNN, explicitly showing the delineation of responsibilities between the two GPUs. One GPU runs the layer-parts at the top of the figure while the other runs the layer-parts at the bottom. The GPUs communicate only at certain layers. The network's input is 150,528-dimensional, and the number of neurons in the network's remaining layers is given by 253,440–186,624–64,896–64,896–43,264–4096–4096–1000.

Model	Top-1	Top-5
Sparse coding [2]	*47.1%*	*28.2%*
SIFT + FVs [24]	*45.7%*	*25.7%*
CNN	**37.5%**	**17.0%**

ImageNet Classification ("VGGNet")

Simonyan and Zisserman, "Very Deep Convolutional Networks for Large-Scale Visual Recognition", ILSVRC-2014.

http://www.robots.ox.ac.uk/~vgg/research/very_deep/

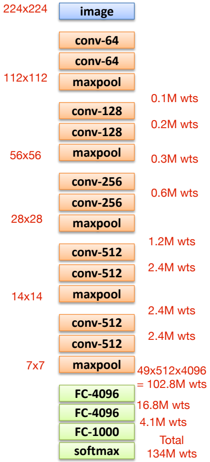
Network Design

Key design choices:

- 3x3 conv. kernels – very small
- conv. stride 1 – no loss of information

Other details:

- Rectification (ReLU) non-linearity
- 5 max-pool layers (x2 reduction)
- no normalisation
- 3 fully-connected (FC) layers



Deep Residual Learning ("ResNets")

He et al, "Deep Residual Learning for Image Recognition", CVPR-2016.

http://arxiv.org/abs/1512.03385

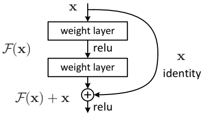
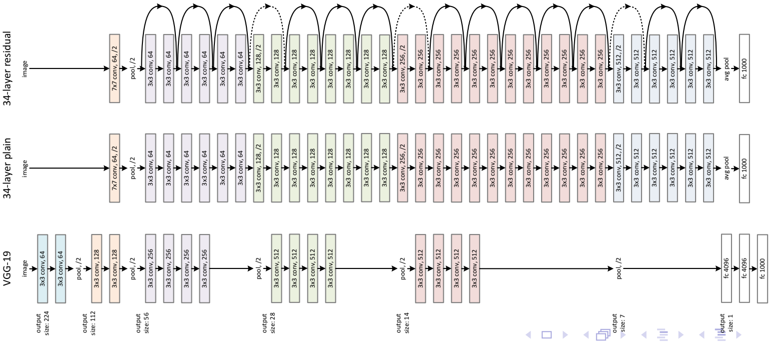


Figure 2. Residual learning: a building block.

method	top-1 err.	top-5 err.
VGG [41] (ILSVRC'14)	-	8.43†
GoogLeNet [44] (ILSVRC'14)	-	7.89
VGG [41] (v5)	24.4	7.1
PReLU-net [13]	21.59	5.71
BN-inception [16]	21.99	5.81
ResNet-34 B	21.84	5.71
ResNet-34 C	21.53	5.60
ResNet-50	20.74	5.25
ResNet-101	19.87	4.60
ResNet-152	**19.38**	**4.49**



Summary

- Convolutional networks include local receptive fields, weight sharing, and pooling leading
- Backprop training can also be implemented as a "reverse" convolutional layer (with the weight matrix rotated)
- Implement using 4D tensors:
  - Inputs / Layer values: minibatch-size, number-fmaps, x, y
  - Weights: number-fmaps-in, number-fmaps-out, x, y
- Reading:
  Yoshua Bengio et al, *Deep Learning* (ch 9)
  http://goodfeli.github.io/dlbook/contents/convnets.html