CS3 Language Semantics and Implementation
Graded Coursework 2 2013

The deadline for this exercise is 4pm on Monday 25th March 2013. Please submit your solutions directly to the ITO.

1. This question is about the dynamic and static semantics of LFP^+.

   Let M be the following LFP^+ program.

   $$\begin{array}{lcl} M & \stackrel{\text{def}}{=} & \mathbf{rec}\, g.(\lambda x.\lambda y.\mathbf{if}\ !\,x \neq !\,y\ \mathbf{then}\ C\ \mathbf{else}\ \mathbf{skip} \\ C & \stackrel{\text{def}}{=} & (\mathbf{if}\ !\,x > !\,y\ \mathbf{then}\ x := !\,x - !\,y\ \mathbf{else}\ y := !\,y - !\,x); (g x) y \end{array}$$

   (a) Assume that s is the state $\{l \mapsto 15, l' \mapsto 10\}$. Write down a **full derivation** of the transition $\langle (M\, l)\, l', s\rangle \Downarrow \langle \mathbf{skip}, s'\rangle$, using the axioms and rules for LFP^+ evaluation assuming call-by-name application. What is the memory state s'? [30 marks]

   (b) Justify that the program M is typable according to the static semantics of LFP^+ by writing down a **full typing derivation** with conclusion $\emptyset \vdash M : \tau$. What is the type τ? [30 marks]

2. This question concerns a sublanguage LLE of LLC defined as follows.

   $$E ::= n \mid !\,l \mid x \mid E\ iop\ E \mid \mathbf{let}\ x\ =\ E\ \mathbf{in}\ E$$

   Here n ranges over integers, l over locations, x over identifiers and iop over binary arithmetic operators.

   (a) Give the axioms and rules for call-by-value evaluation for the language LLE: the evaluation relation has the form $\langle E, s\rangle \Downarrow \langle n, s'\rangle$ where s is a memory state. [10 marks]

   (b) Prove by induction the following property: for any LLE program E and memory s, if E is closed (does not contain free identifiers) and every location mentioned in E is in $\mathrm{dom}(s)$ then for some n, $\langle E, s\rangle \Downarrow \langle n, s\rangle$. State which kind of induction you are using. [30 marks]