Logic Programming: Terms, unification and proof search

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Today

- Compound terms
- Equality and unification
- How Prolog searches for answers

Terms

So far we have seen ...

- Atoms: `homer marge 'Mr. Burns'`
- Variables: `X Y Z MR_BURNS`

We also have ...

- Numbers: `1 2 3 42 -0.12435`
- **Complex terms**
- Additional **constants** and **infix operators**

Complex terms

- A complex term is of the form

$$\mathtt{f}(\mathtt{t}_1, \ldots, \mathtt{t}_\mathtt{n})$$

- where $\mathtt{f}$ is an atom and $\mathtt{t}_1, \ldots, \mathtt{t}_\mathtt{n}$ are (maybe complex) terms

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- where f is an atom and $\mathtt{t}_1, \ldots, \mathtt{t}_\mathtt{n}$ are (maybe complex) terms

Examples:

```
f(1,2)    node(leaf,leaf)    cons(42,cons(43,nil))
household(homer, marge, bart, lisa, maggie)
```

More about lists

Lists are built-in (and very useful) data structures.

Syntax:

```
[1,2,3,4]
[a,[1,2,3],42,'forty-two']
[a,b,c|Xs]
```

Lots more on this next week . . .

Prolog has built-in **constants** and **infix operators**.

Examples:

- Equality: `t = u` (or `=(t,u)`)
- Pairing: `(t,u)` (or `,(t,u)`)
- Empty list: `[]`
- *cons*: list given by first element and rest: `[X|Y]` (or `.(X,Y)`)

You can also define your own infix operators!

The equation `t = u` is a basic goal
with a special meaning

What happens if we ask:

```
?- X = c.
?- f(X,g(Y,Z)) = f(c,g(X,Y)).
?- f(X,g(Y,f(X))) = f(c,g(X,Y)).
```

And how does it do that?

Unification II

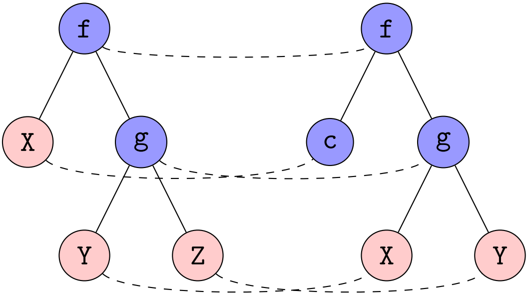
```
?- X = c.
X=c
yes

?- f(X,g(Y,Z)) = f(c,g(X,Y)).
X=c
Y=c
Z=c
yes

?- f(X,g(Y,f(X))) = f(c,g(X,Y)).
no
```

Unification III

- A **substitution** is a mapping from variables to terms
  - $X_1 = t_1, \ldots, X_n = t_n$
- Given two terms t and u
  - with free variables $X_1, \ldots, X_n$,
- a unifier is a substitution that makes t and u identical when applied to t and u.

Example I

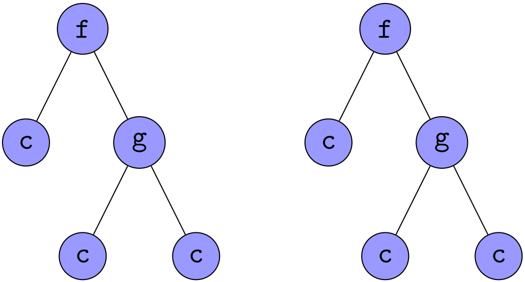
$$f(X,g(Y,Z)) = f(c,g(X,Y))$$



X=c
Y=X
Z=Y

Example I: apply the substitution

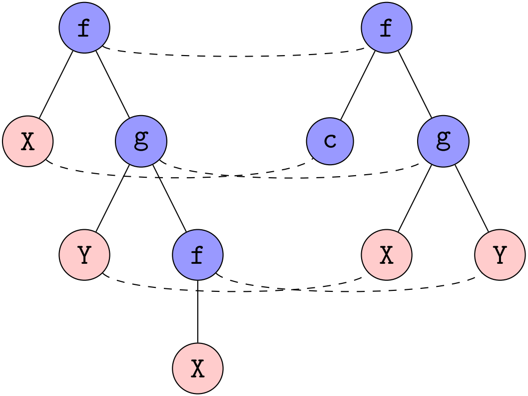
`f(X,g(Y,Z)) = f(c,g(X,Y))`



`X=c`
`Y=c`
`Z=c`

Example II

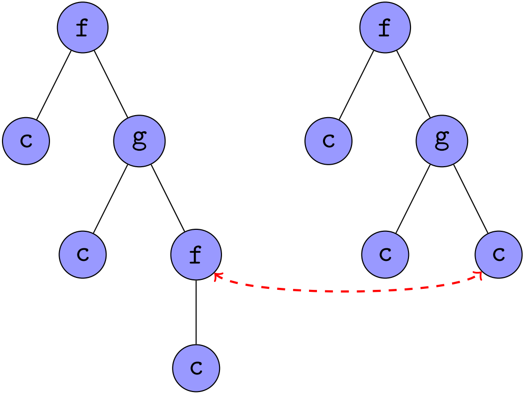
```
f(X,g(Y,f(X))) = f(c,g(X,Y))
```



X=c
Y=X

Example II: apply partial substitution

`f(X,g(Y,f(X))) = f(c,g(X,Y))`



`X=c`

`Y=c`

`Y=f(X)`

`f(X)=c???`

- Consider a general unification problem

$$\mathtt{t}_1 = \mathtt{u}_1, \quad \mathtt{t}_2 = \mathtt{u}_2, \quad \ldots, \quad \mathtt{t}_\mathtt{n} = \mathtt{u}_\mathtt{n}$$

Robinson's algorithm

- Consider a general unification problem

$$\mathtt{t}_1 = \mathtt{u}_1, \quad \mathtt{t}_2 = \mathtt{u}_2, \quad \ldots, \quad \mathtt{t}_\mathtt{n} = \mathtt{u}_\mathtt{n}$$

- Reduce the problem by decomposing each equation into one or more "smaller" equations
- Succeed if we reduce to a "solved form", otherwise fail.

Robinson's algorithm ctd

- Two function applications unify if the head symbols are equal, and the corresponding arguments unify:

$$\begin{array}{c} f(t_1, \ldots, t_n) = f(u_1, \ldots, u_n), \quad P \quad \Rightarrow \\ t_1 = u_1, \ \ldots, \ t_n = u_n, \quad P \end{array}$$

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- Must have same name, and equal number of arguments:

$$\begin{array}{rcl} f(\ldots) = c, \quad P & \Rightarrow & \mathit{fail} \\ f(\ldots) = g(\ldots), \quad P & \Rightarrow & \mathit{fail} \end{array}$$

- Otherwise, a variable X unifies with a term t, provided X does not occur in t:
- proceed by substituting t for X in P:

$$X = t, \quad P \quad \Rightarrow \quad P[t/X]$$

occurs check: provided X does not occur in t

Occurs check

- What happens if we try to unify X with something that *contains* X?

```
?- X = f(X).
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Occurs check

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```

- Logically this should fail
  there is no (finite) unifier!
- Most Prolog implementations skip this check for efficiency reasons
  - can use `unify_with_occurs_check/2`

Execution model

The query is run by trying to find a solution to the goal using the clauses:

- Unification is used to match goals and clauses
- There may be zero, one, or many solutions
- Execution may backtrack

The formal model is called **SLD** resolution, which you'll see in the theory lectures

Depth-first search

Basic Idea:

To solve atomic goal A:

- **If** B is a fact in the program, and there is a substitution θ such that $\theta(A) = \theta(B)$, then return answer θ;
- **else**,

  if B :- $G_1, ..., G_n$ is a clause in the program,
  and θ unifies A with B,
  then solve $\theta(G_1), \ldots, \theta(G_n)$
- **else** give up on this goal:
  - **backtrack** to last choice point

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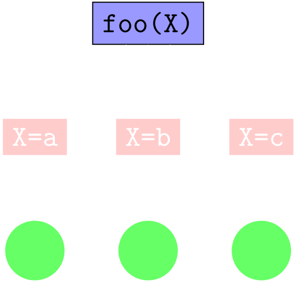
- Clauses are tried in **declaration order**
- Compound goals are tried in **left-right order**

Depth-first search

Prolog tries clauses in order of appearance in the program.
We look at a couple of **search trees** for query execution.
Assume: `foo(a).  foo(b).  foo(c).`
then:

```
?- foo(X).
```



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?- foo(X).
X=a
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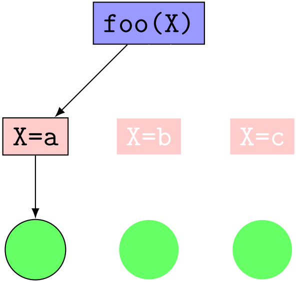

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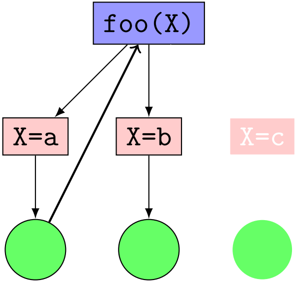
X=a ;
X=b
```



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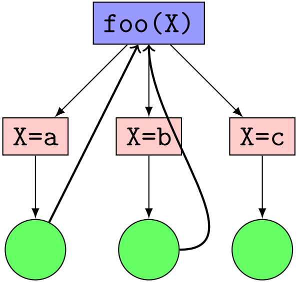

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Assume: `foo(a).` `foo(b).` `foo(c).`
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?- foo(X).
X=a ;
X=b ;
X=c ;
no
```

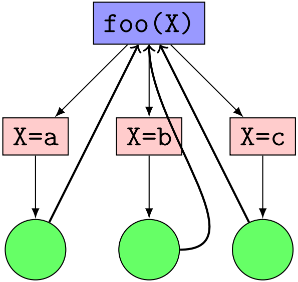

Depth first search ctd

Prolog *backtracks* to the last choice point if a sub-goal fails.
Assume: `bar(b).` `bar(c).` `baz(c).` then:

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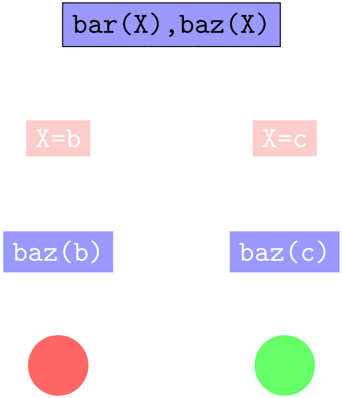

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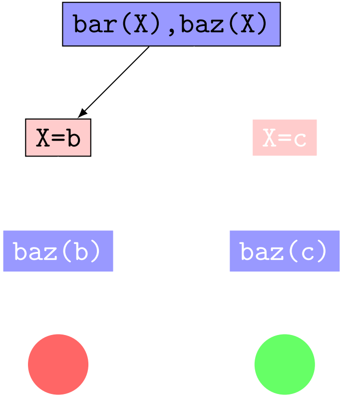

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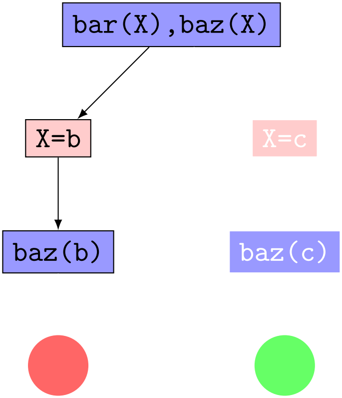

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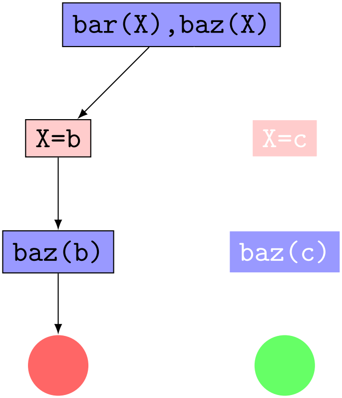

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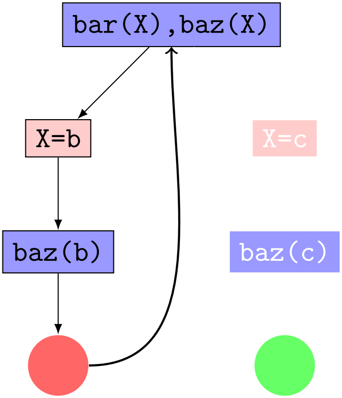

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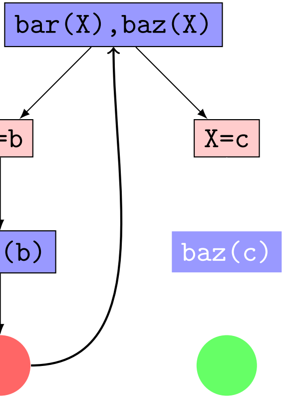

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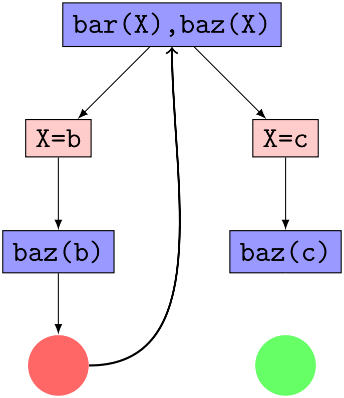

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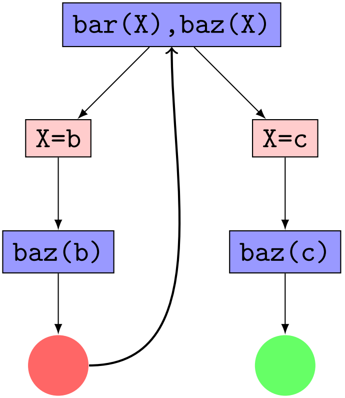
```
?- bar(X),baz(X).
X = c
```



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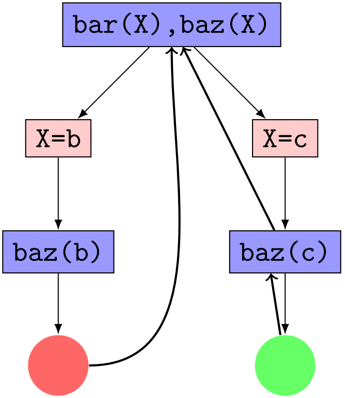
no
```



"Generate and test"

- Common Prolog programming idiom:

  ```
  find(X) :- generate(X), test(X).
  ```

  where:
  - `generate(X)` produces candidates on backtracking
  - `test(X)` succeeds or fails on candidates

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```

  where:
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- Use this to constrain (maybe infinite) search spaces;
- Can use different generators to get different search strategies besides depth-first.

Coming Attractions

- Recursion
- Lists
- Trees, data structures

For further reading, see LPN ch. 2.