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  Examples: Logic programming (Prolog), database query languages (SQL), functional programming (Haskell)

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  Examples: Imperative programming (C), object-oriented programming (Java)

Logic programming — idealistically

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  One declarative aspect is that the user does not specify the method by which the system determines whether or not the query is a consequence of the program.

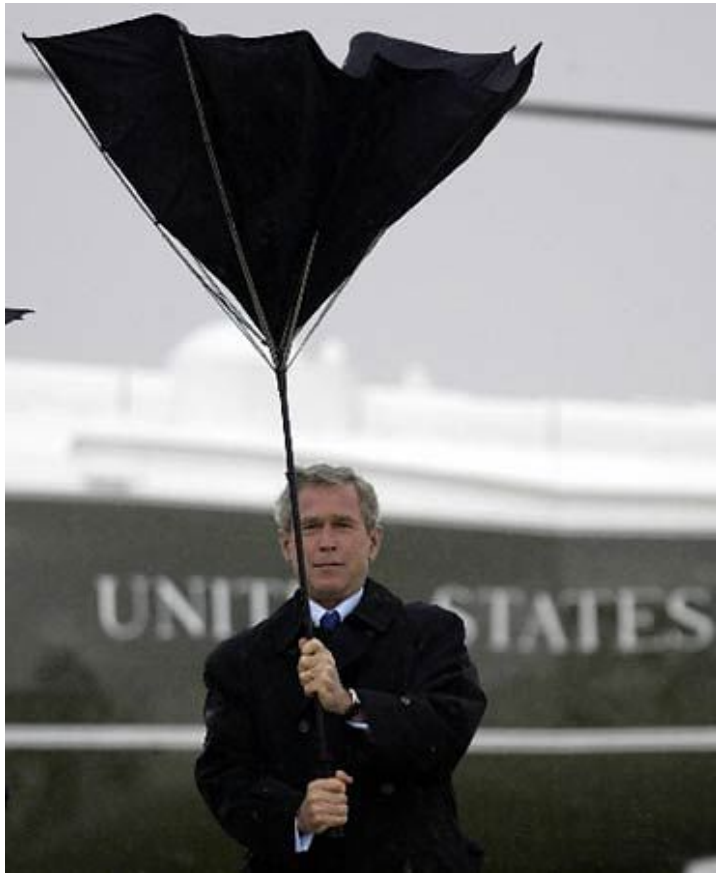
  Another is that whether or not the query is indeed a consequence is independent of the method chosen by the system.

Example logic program

$$
\begin{aligned}
\texttt{chicago} &\rightarrow \texttt{windy} \\
\texttt{edinburgh} &\rightarrow \texttt{windy} \\
\texttt{edinburgh} &\rightarrow \texttt{scotland} \\
\texttt{scotland} &\rightarrow \texttt{rainy} \\
\texttt{windy} \wedge \texttt{rainy} &\rightarrow \texttt{insideOutUmbrella} \\
&\quad\ \texttt{edinburgh}
\end{aligned}
$$

Example query

Does `insideOutUmbrella` hold?



Let's try this in Sicstus Prolog

Program:

```
windy :- chicago.
windy :- edinburgh.
scotland :- edinburgh.
rainy :- scotland.
insideOutUmbrella :- windy, rainy.
edinburgh.
```

Query:

```
| ?- insideOutUmbrella.
```

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edinburgh.
```

Query:

```
| ?- insideOutUmbrella.
!  Existence error in user:chicago/0
!  procedure user:chicago/0 does not exist
!  goal:  user:chicago/0
```

Slightly modified Sicstus Prolog code

Program:

```
windy :- chicago.
windy :- edinburgh.
scotland :- edinburgh.
rainy :- scotland.
insideOutUmbrella :- windy, rainy.
edinburgh.
chicago :- false.
```

Query:

```
| ?- insideOutUmbrella.
yes
```

We avoid this quirk, by working with idealized Prolog

Program:

```
windy :- chicago.
windy :- edinburgh.
scotland :- edinburgh.
rainy :- scotland.
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| ?- insideOutUmbrella.
yes
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Key points to address

- Why is this the correct answer?
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  In Lectures 1–2 we restrict attention to *propositional logic*.
  From Lecture 3 we shall look at *predicate logic*.

Propositional logic (recap)

(Recall notes on logic from Inf 1 - Computation and Logic. Alternatively use on-line references (e.g., Wikipedia).)

Grammar of formulas:

$$\mathit{form} \ ::= \ \mathit{atom} \mid \neg \mathit{form} \mid \mathit{form} \wedge \mathit{form} \mid \mathit{form} \vee \mathit{form} \mid \mathit{form} \rightarrow \mathit{form}$$

The formulas in our example logic program all have very simple structure. (We shall see later this is no coincidence.)

An example of a more complex propositional formula:

$$\texttt{scotland} \wedge \neg \texttt{windy} \ \rightarrow \ \texttt{glasgow} \vee \texttt{perth}$$

Interpretations

An *interpretation* is a function assigning truth values **true**, **false** to atoms.

For example, if the atoms are $\texttt{poor}, \texttt{happy}$ then

$$\{\texttt{poor} \mapsto \textbf{false}, \texttt{happy} \mapsto \textbf{true}\}$$

is an interpretation.

A formula F is *true* under an interpretation $\mathcal{I}$, notation

$$\mathcal{I} \models F \ ,$$

iff the truth value of the formula comes out as true using the standard truth tables.

For example,

$$\{\texttt{poor} \mapsto \textbf{false}, \texttt{happy} \mapsto \textbf{true}\} \models \texttt{poor} \rightarrow \texttt{happy}$$

Logical consequence

A formula G is said to be a *logical consequence* of formulas $F_1, F_2, \ldots, F_n$, notation

$$F_1, \ldots, F_n \models G \ ,$$

iff, for all interpretations $\mathcal{I}$,

$$\text{if } \mathcal{I} \models F_1 \text{ and } \ldots \text{ and } \mathcal{I} \models F_n \text{ then } \mathcal{I} \models G \ .$$

Don't get confused! The symbol $\models$ is used in two different ways:

$$\mathcal{I} \models F$$
$$F_1, \ldots, F_n \models G$$

In the first the left-hand-side is an interpretation, in the second it is a sequence (or set) of formulas.

Examples

A logical consequence:

$$\texttt{poor} \rightarrow \texttt{happy}, \neg\texttt{happy} \models \neg\texttt{poor}$$

We look at all (four!) interpretations. Every time both formulas on the left are true, so is the formula on the right.

A non consequence:

$$\texttt{poor} \rightarrow \texttt{happy}, \neg\texttt{poor} \not\models \neg\texttt{happy}$$

The interpretation that assigns 'false' to `poor` and 'true' to `happy` makes both formulas on the left true, but $\neg$`happy` is false.

Idea of a (propositional) logic program

A logic program is given by a list of (propositional) formulas

$$F_1, F_2, \ldots, F_n \ .$$

A goal is given by another formula

$$G \ .$$

The task of the system is to determine whether

$$F_1, \ldots, F_n \models G \ .$$

If so, the system returns '`yes`'. Otherwise the system returns '`no`'.

Checking logical consequence semantically

In principle, the system can check the logical consequence by constructing a truth table, with one row for every possible interpretation.

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Equivalently, does $P = NP$ hold?

(Clay Mathematics Institute — Millenium Prize Problems)

Route to feasibility

- Restrict formulas in logic programs to *definite clauses*
- Use *proof search* to determine logical consequence.

  (That is, use syntactic methods rather than the semantic method of checking truth tables.)

This methodology is effective and flexible.

In later lectures we shall see that it extends to predicate (i.e., first-order) logic (and beyond!)

(Indeed Prolog implements predicate logic, not only propositional logic.)

Propositional definite clauses

A *definite clause* is a formula of one of the two shapes below

$$q \qquad \text{(a Prolog } \textit{fact}\ q\,.\ \text{)}$$

$$p_1 \wedge \cdots \wedge p_k \ \rightarrow q \qquad \text{(a Prolog } \textit{rule}\ q\ \texttt{:-}\ p_1, \ldots, p_k.\text{)}$$

where $p_1, \ldots, p_k, q$ are all *atoms* (that is, atomic statements).

A *logic program* is a list $F_1, \ldots, F_n$ of definite clauses

A *goal* is a list $g_1, \ldots, g_m$ of atoms.

The job of the system is to ascertain whether the logical consequence below holds.

$$F_1, \ldots, F_n \models g_1 \wedge \cdots \wedge g_m \ .$$

Main points today

propositional logic

logical consequence

definite clauses

Next time

inference systems

propositional Prolog proof search procedure

properties of Prolog proof search