LP/p2

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9th Nov 2015

Logic Programming (Theory)

Assessed Coursework

This coursework is due at **3:00 pm, Monday 23rd November**, 2015.
You should hand in your solutions on paper to the ITO; they may be hand-written, formatted as you wish, or some combination of these.

1. (a) Consider the following Prolog program and query.

```
d( f(X), f(Y) ) :- d( X, Y ).
d( f(X), Y ) :- d( X, Y ).
d( g(X), g(Y) ) :- d( X, Y ).
d( a, a ).

?-d( f(g(a)), Z ), d( g(f(a)), Z ).
```

Rewrite the program and query in the language of first-order logic, writing all quantifiers explicitly.

[6 marks]

(b) Draw the full search tree for the above program and query.

[10 marks]

(c) Consider the three terms below:

```
f(g(Y),h(Y))    f(g(Y),h(Z))    f(g(Y),Y)
```

For each pair of thee terms, say if the two terms are unifiable, and do one of the following:

i. if they are unifiable, say what the most general unifier is, and give another unifier which is not most general;

ii. if the two terms are not unifiable, then explain why not.

[9 marks]

2. (a) Suppose that f is a function from a power set of a set X to itself:

$$f : \mathcal{P}(X) \to \mathcal{P}(X)$$

What does it mean to say:

i. f is monotone?

ii. Y is a fixed point of f?

[4 marks]

(b) Consider the following propositional logic program.

```
a :- b, d.
b :- b.
b :- d.
c :- e.
d.
```

The meaning of the program is definied by the least fixed point of a function $f : \mathcal{P}(\{\, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{d}, \mathtt{e} \,\}) \to \mathcal{P}(\{\, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{d}, \mathtt{e} \,\})$. Explain what the function f is, and calculate sufficiently many iterated applications of f to the empty set to find the least fixed point of f.

[10 marks]

(c) How many models does the above program have? (i.e. how many interpretations of the atoms $\{\, \mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{d}, \mathtt{e} \,\}$ are there which make the program statements all true?)

[5 marks]

(d) Explain what is means by a *decision procedure* for definite clause propositional logic. Describe one possible decision procedure in detail.

[6 marks]