Learning from Data 1, Tutorial Sheet 1 - Answers

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1. **Vector Length - Norm (Euclidean)**
   $||\vec{v}|| = \sqrt{\vec{v}.\vec{v}}$
   Solution: $||\vec{v}|| = \sqrt{6}$
2. **Angle between vectors**
   $\cos(\theta) = \frac{\vec{x}.\vec{y}}{||\vec{x}||\,||\vec{y}||}$
   Solution: $\theta = 0.2502\,[rad]$
3. **MATLAB**
   Solution: see tutorial_01.m file for commands.
4. **Matrix operations, eigenvalues and eigenvectors**
   If $A\,\vec{v} = \lambda\,\vec{v}$, then $\vec{v}$ is an *eigenvector* of A and λ is the associated *eigenvalue*
   Solution: $\vec{v}$ is an *eigenvector* of A with $\lambda = -6$
5. **Partial Derivatives**
   $f(x, y, z) = (x + 2\,y)^2\,\sin(x\,y)$
   Solution:
   $\frac{\partial f}{\partial x} = y\,(x + 2\,y)^2\,\cos(x\,y) + 2\,(x + 2\,y)\,\sin(x\,y)$
   $\frac{\partial f}{\partial y} = x\,(x + 2\,y)^2\,\cos(x\,y) + 4\,(x + 2\,y)\,\sin(x\,y)$
   $\frac{\partial f}{\partial x} = 0$
6. **Probability**
   Random variable $X \equiv$ *"number of requests satisfied"*
   Solution:
   $P(X = \text{"at least one"}) = P(X = 1) \cap P(X = 2) \cap P(X = 3) = 1 - P(X = 0)$
   $P(X = 0) = \left(\frac{3}{4}\right)^3$
   $P(X = \text{"at least one"}) = 0.5781$
7. **Distribution Functions**
   $E[X] = \int_{-\infty}^{+\infty} x\,f(x)\,dx$
   $VAR[X] = E[X^2] - E[X]^2$
   Solution: $E[X] = \frac{1}{2}$ and $VAR[X] = \frac{1}{12}$

8. **Optimization**

Notes:

If A is a positive definite symmetric matrix then the inverse matrix, A^{-1}, exists[1].

$(AB)^T = B^T A^T$;

The gradient of a scalar function ϕ (which is a vectorial quantity) is defined as:
$\nabla\phi(x_1, ..., x_n) \equiv \frac{\partial\phi}{\partial x_1}\vec{x_1} + ... + \frac{\partial\phi}{\partial x_1}\vec{x_n}$

Solutions:

a) Unconstrained minimum of $f(\vec{x})$:

$\nabla f(\vec{x}) = \vec{0}$

$\vec{x} = A^{-1}\vec{b}$

b) Minimum of $f(\vec{x})$ along $\vec{x} = \vec{a} + t\vec{v}, t \in \Re$

Substitute $\vec{x}$ by $\vec{a} + t\vec{v}$ and make $\frac{df(\vec{x})}{dt} = 0$

$t = \frac{\vec{b}^T\vec{v} - \vec{a}^T A\vec{v}}{\vec{v}^T A\vec{v}}$

c) Unconstrained minimum for given A and $\vec{b}$

$\vec{x} = (7, -3)^T$

[1] An $n \times n$ Hermitian matrix A is called positive definite if $x^T Ax > 0$ for all nonzero $x \in \Re^n$;
A real matrix is Hermitian *iff* (if and only if) it is symmetric ($A^T = A$);
The determinant of a positive definite matrix is positive;
A square matrix A has an inverse *iff* the determinant $|A| \neq 0$;
Taking these into account, we can make the statment that if A is a positive definite symmetric matrix then the matrix inverse, A^{-1}, exists.