Learning from Data 1, Tutorial Sheet 1

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1. Calculate the length of the vector $(1, -1, 2)^T$.

2. Calculate the angle between the vectors $(1, 2, 3, 4)^T$ and $(5, 6, 7, 8)^T$

3. Please download the MATLAB file `http://anc.ed.ac.uk/~dbarber/lfd1/tut2002w2.m` (You can also do this by clicking on the link in the tutorials section of the LFD1 homepage). Open the file in an editor (you can invoke one by typing `edit` within the MATLAB command window. Read the instructions in the file for the question.

4. Let A and $\boldsymbol{v}$ be defined as

$$A = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 1 & -3 \\ 3 & -3 & -3 \end{pmatrix} \qquad \boldsymbol{v} = \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix}$$

Calculate $A\boldsymbol{v}$. Is $\boldsymbol{v}$ an eigenvector of A and, if so, what is the corresponding eigenvalue?

5. Partial derivatives. Find the partial derivatives of the function $f(x, y, z) = (x + 2y)^2 \sin(xy)$.

6. Probability. Lois knows that on average radio station RANDOM-FM plays 1 out of 4 of her requests. If she makes 3 requests, what is the probability that at least one request is played?

7. Let X be distributed according to a uniform distribution, i.e. $f(x) = 1$ for $0 \leq x \leq 1$, and 0 otherwise. Show that X has mean $\frac{1}{2}$ and variance $\frac{1}{12}$.

8. (Just in case you've nothing better to do!) Find the (unconstrained) minimum of the function

$$f(\boldsymbol{x}) = \frac{1}{2}\boldsymbol{x}^T A \boldsymbol{x} - \boldsymbol{b}^T \boldsymbol{x}$$

where A is a positive definite symmetric matrix.

Find the minimum of $f(\boldsymbol{x})$ along the line $\boldsymbol{x} = \boldsymbol{a} + t\boldsymbol{v}$ where t is a real parameter.

Calculate explicitly the unconstrained minimum in the case that

$$A = \begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix} \qquad \boldsymbol{b} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$