Introduction to Theoretical Computer Science

Tutorial Sheet for Week 8

First, a couple of straightforward exercises.

(1) VERTEX-COVER: given a graph, a vertex cover is a subset of vertices such that every edge has at least one end in the cover. VERTEX-COVER is, given a graph G and an integer k, does G have a vertex cover of size k?

Give reductions from CLIQUE to VERTEX-COVER, and the other way round. (*Hint:* complementary graph.)

(2) I said the following were obvious, with very brief explanations:

- PSpace $\supseteq$ PTime
- PSpace $\supseteq$ NPTime
- PSpace $\subseteq$ ExpTime

Write out explanations in enough detail to show how a proof would work.

Now, for those who like to go a bit beyond NP, and have got the oracle idea, a conceptual question:

(3) P^{NP} obviously includes all of NP and co-NP. So how does $\mathrm{NP}^{\mathrm{NP}}$ differ from it – what else is there? (Assuming, that is, that $\mathrm{P} \neq \mathrm{NP}$.)