# **Statistical Analysis of Extreme Values**



THE UNIVERSITY of EDINBURGH

Ioannis Papastathopoulos

School of Mathematics and Maxwell Institute





- Environment: earthquakes, floods, heatwaves
- Medicine & Pharmaceutical Industry: drug toxicity
- Finance and Insurance: crashes and large claims

### **Central Limit Theorem**



 $X_1, \ldots, X_n \sim F$  i.i.d. with  $\mathbb{E}(X_i) = \mu$  and  $\mathbb{V}ar(X_i) = \sigma^2 < \infty$ . Then as *n* approaches  $\infty$ 

$$\Pr\left\{\frac{\sum_{i=1}^{n} X_{i} - n\mu}{\sigma\sqrt{n}} \le z\right\} \longrightarrow \Phi(z) \quad z \in \mathbb{R}$$

where  $\Phi$  is the standard normal distribution function.

#### **Extreme Value Theorem**



If there exist sequences of constants  $\alpha_n > 0$  and  $\beta_n$  such that, as n approaches  $\infty$ 

$$\Pr\left\{\frac{\bigvee_{i=1}^{n} X_{i} - \beta_{n}}{\alpha_{n}} \leq z\right\} \longrightarrow G(z) \quad z \in \mathbb{R}$$

for some non-degenerate distribution G, then G is given by

$$G(z) = \exp\left\{-\left(1+\xi z
ight)_{+}^{-1/\xi}
ight\}$$

# How heavy is the tail

## Illustration of tail for three values of $\xi$

- $\xi < 0$  dashed
- ξ = 0 solid
- $\xi > 0$  dotted-dashed





# Working assumption



The **fundamental premise** in all statistical extreme value modelling is that

# we can approximate the distribution of extreme values by the limiting theoretical forms



Histogram of max







- What should be the minimum height of a dam so that, *on average*, the level of the sea will not exceed it for 500 years?
- Is a proposed drug liable to incur deaths when marketed to the population due to toxicity?
- What is the probability of huge losses for a particular portfolio of stocks?

# Clinical trial data: an illustrative example



# Experimental drug given at four doses to $\simeq$ 600 patients ALT, AST, ALP and TBL



### Dose response ALT



#### Exceedances of upper limit of normal (ULN)



# **Return levels**





# ALP vs ALT



#### USA Food and Drug Administration guidance:

ALT  $\uparrow$  and TBL  $\uparrow$  while ALP moderate



Transformed residual ALT

Thank you!