Informatics 2D: Tutorial 3

Propositional Logic and Satisfiability*

Week 4

1 The Wumpus World

1.1 Propositional Rules

Translate the following statements into propositional logic formulae. You can use a schematic representation for the location of a square, e.g. use a proposition $W_{i,j}$ to represent that there is a wumpus in the square in the ith row and jth column (don't worry about the edges of the grid when formalising your propositions).

1. A square cannot contain the wumpus and a pit at the same time.
2. If a square is breezy then one of the (not diagonally) adjacent squares contains a pit.
3. There is a stench in the square if and only if it contains the wumpus or is (not diagonally) adjacent to the square containing the wumpus.

1.2 Entailment

Using the above rules, and the assumed facts, show the following statements are entailed by the knowledge base (either using a truth table or a diagram showing the possible models):

1. Assuming that there is a pit in square $(2, 2)$ show that the wumpus is not in square $(2, 2)$.
2. Assuming that there is a stench in square $(1, 1)$ and that there is not a wumpus in square $(1, 1)$ show that there is either a wumpus in $(1, 2)$ or a wumpus in $(2, 1)$. (Assume that the grid begins at $(1, 1)$ and ignore the off-grid squares in your rules).
3. Assuming that there is a breeze in square $(2, 2)$ and that there is not a pit in squares $(1, 2)$, $(2, 1)$ or $(3, 2)$, show that there is a pit in square $(2, 3)$.

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2 DPLL algorithm

The DPLL algorithm consists of the following steps:

- Convert proposition to CNF
- Loop through the following steps until a satisfying assignment is found or none is possible:
  - Loop through the following simplifications until the formula can't be simplified anymore:
    - Pure literal heuristic.
    - Unit Clause heuristic.
  - Select a variable and branch the search space into a formula where the variable is true and a formula where the variable is false. (This means that you try the algorithm recursively upon these new formulae, with a satisfying assignment for one of the new formula being a satisfying assignment for the original).

  Your lecture notes and R&N chapter 7 section 6 describe the steps in more detail.

  Question: Use the DPLL algorithm to show whether the following propositional formulae is satisfiable: $S_{1,1} \wedge (S_{1,1} \Leftrightarrow W_{1,2} \vee W_{1,1} \vee W_{2,1}) \wedge \neg ((W_{1,2} \wedge P_{1,2}) \vee (W_{2,1} \wedge P_{2,1})) \wedge \neg P_{1,1} \wedge \neg ((W_{1,1} \wedge W_{2,1}) \vee (W_{1,1} \wedge W_{1,2}))$

3 *More to learn[1]

- Why is the 2-SAT problem so much easier than the 3-SAT problem?

[1] Starred *problems are outside the examinable course content. Feel free to ignore them completely.