# Inf2b - Learning
Lecture 7: Text Classification using Naive Bayes

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## Today's Schedule

1. Text classification
2. Bag-of-words models
3. Multinomial document model
4. Bernoulli document model
5. Generative models
6. Zero Probability Problem



## Identifying Spam

**Spam?**

*I got your contact information from your country's information directory during my desperate search for someone who can assist me secretly and confidentially in relocating and managing some family fortunes.*



## Identifying Spam

**Spam?**

*Dear Dr. Steve Renals, The proof for your article, Combining Spectral Representations for Large-Vocabulary Continuous Speech Recognition, is ready for your review. Please access your proof via the user ID and password provided below. Kindly log in to the website within 48 HOURS of receiving this message so that we may expedite the publication process.*



## Identifying Spam

**Spam?**

*Congratulations to you as we bring to your notice, the results of the First Category draws of THE HOLLAND CASINO LOTTO PROMO INT. We are happy to inform you that you have emerged a winner under the First Category, which is part of our promotional draws.*



## Text Classification using Bayes Theorem

- Document $\mathcal{D}$, with a fixed set of classes $C = \{1, \ldots, K\}$
- Classify $\mathcal{D}$ as the class with the highest posterior probability:

$$k_{\max} = \arg\max_k P(C_k \mid \mathcal{D}) = \arg\max_k \frac{P(\mathcal{D} \mid C_k)\, P(C_k)}{P(\mathcal{D})}$$
$$= \arg\max_k P(\mathcal{D} \mid C_k)\, P(C_k)$$

- How do we represent $\mathcal{D}$ ?
- How do we estimate $P(\mathcal{D} \mid C_k)$ and $P(C_k)$ ?



## How do we represent $\mathcal{D}$?

- A sequence of words: $\mathcal{D} = (X_1, X_2, \ldots, X_n)$
  computational very expensive, difficult to train
- A set of words (Bag-of-Words)
  - Ignore the position of the word
  - Ignore the order of the word
  - Consider the words in pre-defined vocabulary $V$ ($D = |V|$)

**Multinomial document model** a document is represented by an integer feature vector, whose elements indicate frequency of corresponding word in the document

$$\mathbf{x} = (x_1, \ldots, x_D) \qquad x_i \in \mathcal{N}_0$$

**Bernoulli document model** a document is represented by a binary feature vector, whose elements indicate absence or presence of corresponding word in the document

$$\mathbf{b} = (b_1, \ldots, b_D) \qquad b_i \in \{0, 1\}$$



## BoW models: Bernoulli vs. Multinomial

**Document $\mathcal{D}$:** "Congratulations to you as we bring to your notice, the results of the First Category draws of THE HOLLAND CASINO LOTTO PROMO INT. We are happy to inform you that you have emerged a winner under the First Category, which is part of our promotional draws."

| Term ($w_t \in V$) | Multinomial ($x_t \in \mathcal{N}_0$) $\mathbf{x} = (x_t)$ | Bernoulli ($b_t \in \{0,1\}$) $\mathbf{b} = (b_t)$ |
|---|---|---|
| bring | 1 | 1 |
| can | 0 | 0 |
| casino | 1 | 1 |
| category | 2 | 1 |
| congratulations | 1 | 1 |
| draws | 2 | 1 |
| first | 2 | 1 |
| lotto | 1 | 1 |
| the | 4 | 1 |
| true | 0 | 0 |
| winner | 1 | 1 |
| you | 3 | 1 |
| $D = 12$ | $\mathbf{x} = (1, 0, 1, 2, \ldots, 1, 3)$ | $\mathbf{b} = (1, 0, 1, 1, \ldots, 1, 1)$ |



## Notation for document model

- Training documents:

| Class | Documents |
|---|---|
| $C_1$ | $\mathcal{D}_1^{(1)} \ldots \mathcal{D}_i^{(1)} \ldots \mathcal{D}_{N_1}^{(1)}$ |
| $\vdots$ | $\vdots$ |
| $C_K$ | $\mathcal{D}_1^{(K)} \ldots \mathcal{D}_i^{(K)} \ldots \mathcal{D}_{N_K}^{(K)}$ |

- Flattened representation of training data:

| Documents | $\mathcal{D}_1$ | $\ldots$ | $\mathcal{D}_i$ | $\ldots$ | $\mathcal{D}_N$ |
|---|---|---|---|---|---|
| Class indicator | $z_{1k}$ | $\ldots$ | $z_{ik}$ | $\ldots$ | $z_{Nk}$ |

where $N = N_1 + \cdots + N_K$,

$$z_{ik} = \begin{cases} 1 & \text{if } \mathcal{D}_i \text{ belongs to class } C_k \\ 0 & \text{otherwise} \end{cases}$$

- Test document : $\mathcal{D}$

## Discrete probability distributions - review

Bernoulli distribution

Eg: Tossing a biased coin ($P(H) = p$), the probability of $k = \{0, 1\}$ 0:Tail, 1:Head is

$$P(k) = kp + (1-k)(1-p) = p^k(1-p)^{1-k}$$

Binomial distribution

Eg: Tossing a biased coin $n$ times, the probability of observing Head $k$ times is

$$P(k) = \binom{n}{k} p^k (1-p)^{n-k}. \qquad \binom{n}{k} = \frac{n!}{k!(n-k)!}$$

Multinomial distribution

Eg: Tossing a biased dice $n$ times, the probability of $\mathbf{x} = (x_1, x_2, x_3, x_4, x_5, x_6)$, where $x_i$ is the number of occurrences for face $i$, is

$$P(\mathbf{x}) = \frac{n!}{x_1! \cdots x_6!} p_1^{x_1} p_2^{x_2} p_3^{x_3} p_4^{x_4} p_5^{x_5} p_6^{x_6}.$$



## Classification with multinomial document model

Assume a test document $\mathcal{D}$ is given as a sequence of words:

$(o_1, o_2, \ldots, o_n) \qquad o_i \in V = \{w_1, \ldots, w_D\}$

Feature vector: $\mathbf{x} = (x_1, \ldots, x_D)$ $\cdots$ word frequencies, $\sum_{t=1}^{D} x_t = n$

Document likelihood with multinomial distribution:

$$P(\mathbf{x} \mid C_k) = \frac{n!}{\prod_{t=1}^{D} x_t!} \prod_{t=1}^{D} P(w_t \mid C_k)^{x_t} \qquad \text{NB: } P^0 = 1 \ (P > 0)$$

For classification, we can omit irrelevant term, so that:

$$P(\mathbf{x} \mid C_k) \propto \prod_{t=1}^{D} P(w_t \mid C_k)^{x_t} = P(o_1|C_k)\, P(o_2|C_k) \cdots P(o_n|C_k)$$

$$P(C_k \mid \mathbf{x}) \propto P(C_k) \prod_{i=1}^{n} P(o_i \mid C_k)$$



## Training of multinomial document model

**Features:** $\mathbf{x} = (x_1, \ldots, x_D)$ : *word frequencies* in a doc.

**Training data set**

| Class | Docs | Feature vectors |
|---|---|---|
| $C_1$ | $\mathcal{D}_1^{(1)}, \ldots, \mathcal{D}_{N_1}^{(1)}$ | $\begin{pmatrix} \mathbf{x}_1^{(1)} \\ \vdots \\ \mathbf{x}_{N_1}^{(1)} \end{pmatrix} = \begin{pmatrix} x_{11}^{(1)} & \ldots & x_{1D}^{(1)} \\ \vdots & & \vdots \\ x_{N_1 1}^{(1)} & \ldots & x_{N_1 D}^{(1)} \end{pmatrix}$ |
| | | $n_1(w_1), \ldots, n_1(w_D)$ |
| | $\hat{P}(C_1) = N_1/N$ | $\hat{P}(w_t\|C_1)$ : $n_1(w_1)/S_1, \ldots, n_1(w_D)/S_1$ |
| $C_k$ | $\mathcal{D}_1^{(k)}, \ldots, \mathcal{D}_{N_k}^{(k)}$ | $\begin{pmatrix} \mathbf{x}_1^{(k)} \\ \vdots \\ \mathbf{x}_{N_k}^{(k)} \end{pmatrix} = \begin{pmatrix} x_{11}^{(k)} & \ldots & x_{1D}^{(k)} \\ \vdots & & \\ x_{N_k 1}^{(k)} & \ldots & x_{N_k D}^{(k)} \end{pmatrix}$ |
| | | $n_k(w_1), \ldots, n_k(w_D)$ |
| | $\hat{P}(C_k) = N_k/N$ | $\hat{P}(w_t\|C_k)$ : $n_k(w_1)/S_k \ldots, n_k(w_D)/S_k$ |

$S_k = \sum_{t=1}^{D} n_k(w_t)$



## Multinomial doc. model – example

See Note 7!



## Classification with Bernoulli document model

A test document $\mathcal{D}$ with feature vector $\boldsymbol{b} = (b_1, \ldots, b_D)$

Document likelihood with (multivariate) Bernoulli distribution:

$$P(\boldsymbol{b} \mid C_k) = \prod_{t=1}^{D} P(b_t|C_k) = \prod_{t=1}^{D} [b_t P(w_t|C_k) + (1-b_t)(1-P(w_t|C_k))]$$

$$= \prod_{t=1}^{D} P(w_t \mid C_k)^{b_t} (1-P(w_t|C_k))^{(1-b_t)}$$

$$\hat{P}(w_t|C_k) = \frac{n_k(w_t)}{N_k}$$

(fraction of class $k$ docs with word $w_t$)

In Classification,

$$P(C_k \mid \boldsymbol{b}) \propto P(C_k)\, P(\boldsymbol{b} \mid C_k)$$



## Training of Bernoulli document model

**Features:** $\mathbf{b} = (b_1, \ldots, b_D)$ : $D = |V|$, i.e. vocabulary *binary vector* of word occurrences in a document

**Training data set**

| Class | Docs | Feature vectors |
|---|---|---|
| $C_1$ | $\mathcal{D}_1^{(1)}, \ldots, \mathcal{D}_{N_1}^{(1)}$ | $\begin{pmatrix} \mathbf{b}_1^{(1)} \\ \vdots \\ \mathbf{b}_{N_1}^{(1)} \end{pmatrix} = \begin{pmatrix} b_{11}^{(1)} & \ldots & b_{1D}^{(1)} \\ \vdots & & \vdots \\ b_{N_1 1}^{(1)} & \ldots & b_{N_1 D}^{(1)} \end{pmatrix}$ |
| | | $n_1(w_1), \ldots, n_1(w_D)$ |
| | $\hat{P}(C_1) = N_1/N$ | $\hat{P}(w_t\|C_1)$ : $n_1(w_1)/N_1, \ldots, n_1(w_D)/N_1$ |
| $C_k$ | $\mathcal{D}_1^{(k)}, \ldots, \mathcal{D}_{N_k}^{(k)}$ | $\begin{pmatrix} \mathbf{b}_1^{(k)} \\ \vdots \\ \mathbf{b}_{N_k}^{(k)} \end{pmatrix} = \begin{pmatrix} b_{11}^{(k)} & \ldots & b_{1D}^{(k)} \\ \vdots & & \\ b_{N_k 1}^{(k)} & \ldots & b_{N_k D}^{(k)} \end{pmatrix}$ |
| | | $n_k(w_1), \ldots, n_k(w_D)$ |
| | $\hat{P}(C_k) = N_k/N$ | $\hat{P}(w_t\|C_k)$ : $n_k(w_1)/N_k, \ldots, n_k(w_D)/N_k$ |



## Bernoulli doc. model – example

Classify documents as Sports ($S$) or Informatics ($I$)

**Vocabulary** $V$:

$w_1 =$ *goal*
$w_2 =$ *tutor*
$w_3 =$ *variance*
$w_4 =$ *speed*
$w_5 =$ *drink*
$w_6 =$ *defence*
$w_7 =$ *performance*
$w_8 =$ *field*

$D = |V| = 8$



## Bernoulli doc. model – example *(cont.)*

**Training data:** (rows give documents, columns word presence)

$$\mathbf{B}^{\text{Sport}} = \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \end{pmatrix}$$

$$\mathbf{B}^{\text{Inf}} = \begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \end{pmatrix}$$

**Estimating priors and likelihoods:**

$P(S) = 6/11, \quad P(I) = 5/11$

$(P(w_t|S)) = (\ 3/6 \quad 1/6 \quad 2/6 \quad 3/6 \quad 3/6 \quad 4/6 \quad 4/6 \quad 4/6\ )$

$(P(w_t|I)) = (\ 1/5 \quad 3/5 \quad 3/5 \quad 1/5 \quad 1/5 \quad 1/5 \quad 3/5 \quad 1/5\ )$



## Bernoulli doc. model – example *(cont.)*

**Test documents:** $\boldsymbol{b}_1 = [\ 1 \quad 0 \quad 0 \quad 1 \quad 1 \quad 1 \quad 0 \quad 1\ ]$

**Priors, Likelihoods:** $P(S) = 6/11, \quad P(I) = 5/11$

$(P(w_t|S)) = (\ 3/6 \quad 1/6 \quad 2/6 \quad 3/6 \quad 3/6 \quad 4/6 \quad 4/6 \quad 4/6\ )$

$(P(w_t|I)) = (\ 1/5 \quad 3/5 \quad 3/5 \quad 1/5 \quad 1/5 \quad 1/5 \quad 3/5 \quad 1/5\ )$

**Posterior probabilities:**

$$P(S \mid \boldsymbol{b}_1) \propto P(S) \prod_{t=1}^{8} [b_{1t} P(w_t \mid S) + (1-b_{1t})(1-P(w_t \mid S))]$$

$$\propto \frac{6}{11} \left( \frac{1}{2} \times \frac{5}{6} \times \frac{2}{3} \times \frac{1}{2} \times \frac{1}{2} \times \frac{2}{3} \times \frac{1}{3} \times \frac{2}{3} \right) = \frac{5}{891} = 5.6 \times 10^{-3}$$

$$P(I \mid \boldsymbol{b}_1) \propto P(I) \prod_{t=1}^{8} [b_{1t} P(w_t \mid I) + (1-b_{1t})(1-P(w_t \mid I))]$$

$$\propto \frac{5}{11} \left( \frac{1}{5} \times \frac{2}{5} \times \frac{2}{5} \times \frac{1}{5} \times \frac{1}{5} \times \frac{1}{5} \times \frac{2}{5} \times \frac{1}{5} \right) = \frac{8}{859375} = 9.3 \times 10^{-6}$$

$\Rightarrow$ Classify this document as $S$.

## Summary of the document models

| Class | Doc | Multinomial doc. model feature vectors | Bernoulli doc. model feature vectors |
|---|---|---|---|
| $C_k$ | $\mathcal{D}_1^{(k)}, \ldots, \mathcal{D}_{N_k}^{(k)}$ | $\begin{pmatrix} \mathbf{x}_1^{(k)} \\ \vdots \\ \mathbf{x}_{N_k}^{(k)} \end{pmatrix} = \begin{pmatrix} x_{11}^{(k)} & \ldots & x_{1D}^{(k)} \\ \vdots & & \vdots \\ x_{N_k 1}^{(k)} & \ldots & x_{1D}^{(k)} \end{pmatrix}$ | $\begin{pmatrix} \mathbf{b}_1^{(k)} \\ \vdots \\ \mathbf{b}_{N_k}^{(k)} \end{pmatrix} = \begin{pmatrix} b_{11}^{(k)} & \ldots & b_{1D}^{(k)} \\ \vdots & & \vdots \\ b_{N_k 1}^{(k)} & \ldots & b_{1D}^{(k)} \end{pmatrix}$ |
| $\hat{P}(C_k) = \frac{N_k}{N}$ | | $n_k(w_1), \ldots, n_k(w_D)$ | $n_k(w_1), \ldots, n_k(w_D)$ |
| $\hat{P}(w_t \mid C_k)$ : | | $\frac{n_k(w_1)}{S_k}, \ldots, \frac{n_k(w_D)}{S_k}$ | $\frac{n_k(w_1)}{N_k}, \ldots, \frac{n_k(w_D)}{N_k}$ |

$S_k = \sum_{t=1}^{D} n_k(w_t)$

$$P(\mathbf{x} \mid C_k) \propto \prod_{t=1}^{D} P(w_t \mid C_k)^{x_t} = \prod_{i=1}^{n} P(o_i \mid C_k)$$

$$P(\mathbf{b} \mid C_k) = \prod_{t=1}^{D} [b_t P(w_t \mid C_k) + (1-b_t)(1-P(w_t \mid C_k))]$$



## Question

What's the approximate value of:

$P(\text{"the"} \mid C)$

(a) in the Bernoulli model

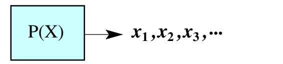
(b) in the multinomial model?

Common words, 'stop words', are often removed from feature vectors.



## Generative models

- Models that generate observable data randomly based on a distribution




- Examples
  - Coin tossing models

| Coin | Generated data sequence |
|---|---|
| Fair coin ($P(H) = P(T) = 0.5$) | $H, T, T, H, T, H, H, T, \ldots$ |
| Unfair coin ($P(H) = 0.7, P(T) = 0.3$) | $T, H, H, H, H, H, T, H, \ldots$ |

  - Dice throwing models

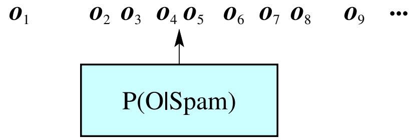
| Dice | Generated data sequence |
|---|---|
| Unbiased dice ($P(X) = 1/6,\ X \in \{1, \ldots, 6\}$) | $2, 4, 3, 5, 3, 6, 5, 5, 4, 6, \ldots$ |
| Biased dice ($P(X)) = (0.1, 0.1, 0.1, 0.1, 0.2, 0.4)$ | $6, 6, 5, 5, 6, 1, 2, 6, 6, 6, \ldots$ |



## Generative models *(cont.)*

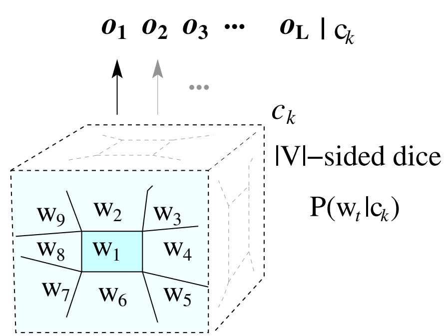
- Spam mail generator

*Congratulations to you as we bring to your notice, ...*



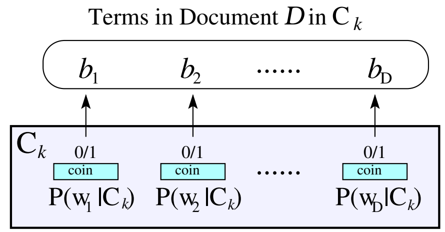



## Generative model — Multinomial document model






## Generative model — Bernoulli document model






## Word relative-frequencies of spam emails

\# of spam emails: 169

| | | | | | |
|---|---|---|---|---|---|
| to | 0.0395032 | from | 0.00664282 | http | 0.00369482 |
| the | 0.0383633 | content | 0.00644629 | money | 0.00345898 |
| you | 0.0267285 | have | 0.0059353 | by | 0.00338037 |
| of | 0.0257851 | bank | 0.0059353 | or | 0.00330176 |
| and | 0.0252349 | usd | 0.00581738 | name | 0.00322314 |
| your | 0.0222476 | on | 0.00554223 | funds | 0.00322314 |
| in | 0.0200857 | we | 0.00542432 | was | 0.00314453 |
| i | 0.0198892 | it | 0.00518848 | type | 0.00318384 |
| this | 0.0145828 | are | 0.00507056 | s | 0.00318384 |
| a | 0.0138752 | transfer | 0.00479541 | 0a | 0.00314453 |
| my | 0.0132463 | our | 0.0047561 | if | 0.00310522 |
| for | 0.0132463 | com | 0.00467749 | 1 | 0.00310522 |
| is | 0.0112024 | am | 0.00467749 | can | 0.00306592 |
| 3d | 0.0108879 | account | 0.00455957 | payment | 0.002948 |
| with | 0.00915845 | unlocked | 0.00424512 | message | 0.002948 |
| will | 0.00876538 | 20 | 0.0041665 | address | 0.00286938 |
| that | 0.00849023 | email | 0.00404858 | us | 0.00283008 |
| as | 0.00797925 | please | 0.00385205 | his | 0.00279077 |
| me | 0.00766479 | not | 0.00377344 | contact | 0.00279077 |
| be | 0.00703589 | all | 0.00377344 | has | 0.00271216 |



## Generated word sequence example

*of kin good your the part of with and atm to new from which projects has the transfer my how 3d and with united in in o beneficiary that died pathak id efforts has to studies have my as can you the 3d you your with transfer will your a your m and the your i is ve country user nokia the this for i value banking an click confirm world i it me my country is 2010 very below i and now until html of position http here of mail following there be while the by for your willing*



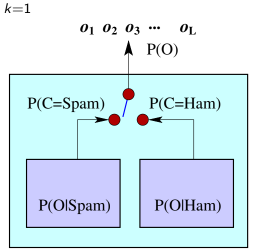
## Generative models for classification

Model for classification

$$P(C_k \mid \mathbf{x}) = \frac{P(\mathbf{x} \mid C_k)\, P(C_k)}{P(\mathbf{x})} \propto P(\mathbf{x} \mid C_k)\, P(C_k)$$

Model for observation $\cdots$ generative model

$$P(x) = \sum_{k=1}^{K} P(x \mid C_k) P(C_k)$$



## Smoothing in multinomial document model

- Zero probability problem

$$P(\mathbf{x} \mid C_k) \propto \prod_{t=1}^{D} P(w_t \mid C_k)^{x_t} \;=\; 0 \text{ if } \exists j : P(w_j|C_k) = 0$$

$$P(w_t \mid C_k) = \frac{\sum_{i=1}^{N} x_{it} z_{ik}}{\sum_{t'=1}^{|V|} \sum_{i=1}^{N} x_{it'} z_{ik}} = \frac{n_k(w_t)}{\sum_{t'=1}^{D} n_k(w_{t'})}$$

- Smoothing – a 'trick' to avoid zero counts:

$$P(w_t \mid C_k) = \frac{1 + \sum_{i=1}^{N} x_{it} z_{ik}}{|V| + \sum_{t'=1}^{|V|} \sum_{i=1}^{N} x_{it'} z_{ik}} = \frac{1 + n_k(w_t)}{D + \sum_{t'=1}^{D} n_k(w_{t'})}$$

Known as *Laplace's rule of succession* or *add one smoothing*.



## Multinomial vs Bernoulli doc. models

| | Multinomial | Bernoulli |
|---|---|---|
| Generative model | draw a words from a multinomial distribution | draw a document from a multi-dimensional Bernoulli distribution |
| Document representation | Vector of frequencies | Binary vector |
| Multiple occurrences | Taken into account | Ignored |
| Document length | Longer docs OK | Best for short docs |
| Feature vector dimension | Longer OK | Shorter |
| Behaviour with "the" | $P(\text{"the"}\,\vert C_k) \approx 0.05$ | $P(\text{"the"}\,\vert C_k) \approx 1.0$ |
| Non-occurring words in test doc | do not affect likelihood | affect likelihood |



## Multinomial vs Bernoulli doc. models *(cont.)*

Fig. 1 in A. McCallum and K.Nigam, "A Comparison of Event Models for Naive Bayes Text Classification", AAAI Workshop on Learning for Text Categorization, 1998



## Document pre-processing

- Stop-word removal
  Remove pre-defined common words that are not specific or discriminatory to the different classes.
- Stemming
  Reduce different forms of the same word into a single word (base/root form)
- Feature selection
  e.g. choose words based on the mutual information



## Exercise 1

Use the Bernoulli model and the Naive Bayes assumption for the following.
Consider the vocabulary $V = \{\texttt{apple}, \texttt{banana}, \texttt{computer}\}$. We have two classes of documents $F$ (fruit) and $E$ (electronics). There are four training documents in class $F$; they are listed below in terms of the number of occurrences of each word from $V$ in each document:

- `apple(2); banana(1); computer(0)`
- `apple(0); banana(1); computer(0)`
- `apple(3); banana(2); computer(1)`
- `apple(1); banana(0); computer(0)`

There are also four training documents in class $E$:

- `apple(2); banana(0); computer(0)`
- `apple(0); banana(0); computer(1)`
- `apple(3); banana(1); computer(2)`
- `apple(0); banana(0); computer(1)`



## Exercise 1 *(cont.)*

1. Write the training data as a matrix for each class, where each row corresponds to a training document.
2. Estimate the prior probabilities from the training data
3. For each class ($F$ and $E$) and for each word (apple, banana and computer) estimate the likelihood of the word given the class.
4. Consider two test documents:
   - `apple(1); banana(0); computer(0)`
   - `apple(1); banana(1); computer(0)`

   For each test document, estimate the posterior probabilities of each class given the document, and hence classify the document.



## Exercise 2

Use the Multinomial model and the Naive Bayes assumption for the following.
Consider the vocabulary $V = \{\texttt{fish}, \texttt{chip}, \texttt{circuit}\}$. We have two classes of documents $F$ (food) and $E$ (electronics). There are four training documents in class $F$; they are listed below;

- `fish chip fish`
- `chip`
- `circuit fish chip`
- `fish fish`

There are also four training documents in class $E$:

- `circuit circuit`
- `chip circuit`
- `chip chip`
- `circuit`



## Exercise 2 *(cont.)*

1. Estimate the parameters of a multinomial model for the two document classes, using add-one smoothing.
2. Consider two test documents:
   - `fish chip`
   - `chip circuit chip circuit fish chip circuit`

   Classify each of the test documents by (approximately) estimating the posterior probability of each class
3. With reference to the test documents in the previous question, explain why a process such as add-one smoothing is used when estimating the parameters of a multinomial model.



## Exercise 3

Consider two writers, Baker and Clark, who were twins, and who published four and six children's books, respectively. The following table shows the frequencies of four words, **wizard**, **river**, **star**, and **warp**, with respect to the first page of each book, and the information whether the book was a bestseller or not.

| | Words | | | | |
|---|---|---|---|---|---|
| Author | wizard | river | star | warp | Bestseller |
| Baker | 1 | 1 | 1 | 0 | No |
| Baker | 1 | 1 | 0 | 1 | No |
| Baker | 1 | 1 | 1 | 1 | yes |
| Baker | 1 | 1 | 0 | 0 | No |
| Clark | 0 | 1 | 0 | 1 | No |
| Clark | 0 | 0 | 2 | 1 | No |
| Clark | 0 | 2 | 1 | 2 | Yes |
| Clark | 1 | 1 | 1 | 2 | No |
| Clark | 0 | 1 | 2 | 2 | Yes |
| Clark | 0 | 1 | 2 | 1 | Yes |

Two unpublished book drafts, Doc 1 and Doc 2, were found after the death of the writers, but it's not clear which of them wrote the documents.

# Exercise 3 *(cont.)*

1. Without having any information about Doc 1 and Doc 2, decide the most probable author of each document in terms of minimum classification error, and justify your decision.
2. The same analysis of word frequencies was carried out for Doc 1 and Doc 2, whose result is shown below. Using the Naive Bayes classification with the multinomial document model without smoothing, find the author of each document.

| | wizard | river | start | warp |
|---|---|---|---|---|
| Doc 1 | 2 | 1 | 1 | 0 |
| Doc 2 | 1 | 1 | 2 | 1 |

3. In addition to modifications to the vocabulary, discuss two possible methods for improving the classification performance.
4. Another document, Doc 3, was found later, and a publisher is considering its publication. Assuming the Naive Bayes classification with the multinomial document model with no smoothing, without identifying the author, predict whether Doc 3 is likely to be a bestseller or not based on the word frequency table for Doc 3 shown below.

| | wizard | river | start | warp |
|---|---|---|---|---|
| Doc 3 | 0 | 1 | 1 | 2 |

5. Using the same situations as in part (d) except that we now know the author of Doc 3 was Baker, predict whether Doc 3 is likely to be a bestseller or not.



# Summary

- Our first 'real' application of Naive Bayes
- Two BoW models for documents: Multinomial and Bernoulli
- Generative models
- Smoothing (Add-one/Laplace smoothing)
- Good reference:
  C. Manning, P. Raghavan and H. Schütze, Introduction to Information Retrieval, University Press. 2008.
  See Chapter 13 Text classification & Naive Bayes
- **As always:**
  be able to implement, describe, compare and contrast (see Lecture Note)