Categorial grammar

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A set of **terminals** {we, helped, Hans, paint, the house}

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A set of **atomic categories** (nonterminals) {NP, S, VP}

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The complete set of **categories**: if A and B are categories, then A/B and $A\backslash B$ are also categories.

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The complete set of **categories**: if A and B are categories, then A/B and $A\backslash B$ are also categories.

A **lexicon**: a subset of terminals × categories × lambda terms

$$\begin{aligned}
\text{we} &\vdash \mathsf{NP} : we' \\
\text{helped} &\vdash \mathsf{S\backslash NP/VP/NP} : \lambda x.\lambda f.\lambda y.helped'\, f\, x\, y \\
\text{Hans} &\vdash \mathsf{NP} : Hans' \\
\text{paint} &\vdash \mathsf{VP/NP} : \lambda x.paint'\, x \\
\text{the house} &\vdash \mathsf{NP} : house'
\end{aligned}$$

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A **lexicon**: a subset of terminals × categories × lambda terms

functional category

we $\vdash$ NP : we'

helped $\vdash$ S\NP/VP/NP : $\lambda x.\lambda f.\lambda y.helped'\, f\, x\, y$

Hans $\vdash$ NP : $Hans'$

paint $\vdash$ VP/NP : $\lambda x.paint'\, x$

the house $\vdash$ NP : $house'$

Categorial grammar

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The complete set of **categories**: if A and B are categories, then A/B and $A\backslash B$ are also categories.

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target category

$$
\begin{aligned}
\text{we} &\vdash \mathsf{NP} : we' \\
\text{helped} &\vdash \mathsf{S}\backslash\mathsf{NP}/\mathsf{VP}/\mathsf{NP} : \lambda x.\lambda f.\lambda y.helped'\, f\, x\, y \\
\text{Hans} &\vdash \mathsf{NP} : Hans' \\
\text{paint} &\vdash \mathsf{VP}/\mathsf{NP} : \lambda x.paint'\, x \\
\text{the house} &\vdash \mathsf{NP} : house'
\end{aligned}
$$

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A set of **atomic categories** (nonterminals) {NP, S, VP}

The complete set of **categories**: if A and B are categories, then A/B and $A\backslash B$ are also categories.

A **lexicon**: a subset of terminals × categories × lambda terms

argument categories

we $\vdash$ NP : we'

helped $\vdash$ S\NP/VP/NP : $\lambda x.\lambda f.\lambda y.helped'\, f\, x\, y$

Hans $\vdash$ NP : $Hans'$

paint $\vdash$ VP/NP : $\lambda x.paint'\, x$

the house $\vdash$ NP : $house'$

Categorial grammar

we helped Hans paint the house

$$\begin{aligned}
\text{we} &\vdash \mathsf{NP} : we' \\
\text{helped} &\vdash \mathsf{S}\backslash\mathsf{NP}/\mathsf{VP}/\mathsf{NP} : \lambda x.\lambda f.\lambda y.helped' f x y \\
\text{Hans} &\vdash \mathsf{NP} : Hans' \\
\text{paint} &\vdash \mathsf{VP}/\mathsf{NP} : \lambda x.paint' x \\
\text{the house} &\vdash \mathsf{NP} : house'
\end{aligned}$$

Categorial grammar

we	helped	Hans	paint	the house
$\mathsf{NP} : we'$	$\mathsf{S}\backslash\mathsf{NP}/\mathsf{VP}/\mathsf{NP} : \lambda x.\lambda f.\lambda y.helped'\,f\,x\,y$	$\mathsf{NP} : Hans'$	$\mathsf{VP}/\mathsf{NP} : \lambda x.paint'\,x$	$\mathsf{NP} : house'$

$$\begin{aligned}
\text{we} &\vdash \mathsf{NP} : we' \\
\text{helped} &\vdash \mathsf{S}\backslash\mathsf{NP}/\mathsf{VP}/\mathsf{NP} : \lambda x.\lambda f.\lambda y.helped'\,f\,x\,y \\
\text{Hans} &\vdash \mathsf{NP} : Hans' \\
\text{paint} &\vdash \mathsf{VP}/\mathsf{NP} : \lambda x.paint'\,x \\
\text{the house} &\vdash \mathsf{NP} : house'
\end{aligned}$$

Categorial grammar

forward application

$$A/B : f \quad B : g \Rightarrow A : fg$$

backward application

$$B : g \quad A\backslash B : f \Rightarrow A : fg$$

$$\frac{\text{we}}{\mathsf{NP} : we'} \quad \frac{\text{helped}}{\mathsf{S}\backslash\mathsf{NP}/\mathsf{VP}/\mathsf{NP} : \lambda x.\lambda f.\lambda y.helped' f x y} \quad \frac{\text{Hans}}{\mathsf{NP} : Hans'} \quad \frac{\text{paint}}{\mathsf{VP}/\mathsf{NP} : \lambda x.paint' x} \quad \frac{\text{the house}}{\mathsf{NP} : house'}$$

$$\begin{aligned}
\text{we} &\vdash \mathsf{NP} : we' \\
\text{helped} &\vdash \mathsf{S}\backslash\mathsf{NP}/\mathsf{VP}/\mathsf{NP} : \lambda x.\lambda f.\lambda y.helped' f x y \\
\text{Hans} &\vdash \mathsf{NP} : Hans' \\
\text{paint} &\vdash \mathsf{VP}/\mathsf{NP} : \lambda x.paint' x \\
\text{the house} &\vdash \mathsf{NP} : house'
\end{aligned}$$

Categorial grammar

forward application

$$A/B : f \quad B : g \Rightarrow A : fg$$

backward application

$$B : g \quad A\backslash B : f \Rightarrow A : fg$$

we	helped	Hans	paint	the house
$\mathsf{NP} : we'$	$\mathsf{S}\backslash\mathsf{NP}/\mathsf{VP}/\mathsf{NP} : \lambda x.\lambda f.\lambda y.helped'\,f\,x\,y$	$\mathsf{NP} : Hans'$	$\mathsf{VP}/\mathsf{NP} : \lambda x.paint'\,x$	$\mathsf{NP} : house'$

$$\begin{aligned}
\text{we} &\vdash \mathsf{NP} : we' \\
\text{helped} &\vdash \mathsf{S}\backslash\mathsf{NP}/\mathsf{VP}/\mathsf{NP} : \lambda x.\lambda f.\lambda y.helped'\,f\,x\,y \\
\text{Hans} &\vdash \mathsf{NP} : Hans' \\
\text{paint} &\vdash \mathsf{VP}/\mathsf{NP} : \lambda x.paint'\,x \\
\text{the house} &\vdash \mathsf{NP} : house'
\end{aligned}$$

Categorial grammar

forward application

$$A/B : f \quad B : g \Rightarrow A : fg$$

backward application

$$B : g \quad A\backslash B : f \Rightarrow A : fg$$

primary premise

$$\frac{\text{we}}{\mathsf{NP} : we'} \quad \frac{\text{helped}}{\mathsf{S}\backslash\mathsf{NP}/\mathsf{VP}/\mathsf{NP} : \lambda x.\lambda f.\lambda y.helped'\,f\,x\,y} \quad \frac{\text{Hans}}{\mathsf{NP} : Hans'} \quad \frac{\text{paint}}{\mathsf{VP}/\mathsf{NP} : \lambda x.paint'\,x} \quad \frac{\text{the house}}{\mathsf{NP} : house'}$$

$$\begin{aligned}
\text{we} &\vdash \mathsf{NP} : we' \\
\text{helped} &\vdash \mathsf{S}\backslash\mathsf{NP}/\mathsf{VP}/\mathsf{NP} : \lambda x.\lambda f.\lambda y.helped'\,f\,x\,y \\
\text{Hans} &\vdash \mathsf{NP} : Hans' \\
\text{paint} &\vdash \mathsf{VP}/\mathsf{NP} : \lambda x.paint'\,x \\
\text{the house} &\vdash \mathsf{NP} : house'
\end{aligned}$$

Categorial grammar

forward application

$$A/B : f \quad B : g \Rightarrow A : fg$$

backward application

$$B : g \quad A\backslash B : f \Rightarrow A : fg$$

secondary premise

$$\frac{\text{we}}{\mathsf{NP} : we'} \quad \frac{\text{helped}}{\mathsf{S}\backslash\mathsf{NP}/\mathsf{VP}/\mathsf{NP} : \lambda x.\lambda f.\lambda y.helped'\,f\,x\,y} \quad \frac{\text{Hans}}{\mathsf{NP} : Hans'} \quad \frac{\text{paint}}{\mathsf{VP}/\mathsf{NP} : \lambda x.paint'\,x} \quad \frac{\text{the house}}{\mathsf{NP} : house'}$$

$$\begin{aligned}
\text{we} &\vdash \mathsf{NP} : we' \\
\text{helped} &\vdash \mathsf{S}\backslash\mathsf{NP}/\mathsf{VP}/\mathsf{NP} : \lambda x.\lambda f.\lambda y.helped'\,f\,x\,y \\
\text{Hans} &\vdash \mathsf{NP} : Hans' \\
\text{paint} &\vdash \mathsf{VP}/\mathsf{NP} : \lambda x.paint'\,x \\
\text{the house} &\vdash \mathsf{NP} : house'
\end{aligned}$$

Categorial grammar

forward application

$$A/B : f \quad B : g \Rightarrow A : fg$$

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we	helped	Hans	paint	the house
$\mathsf{NP} : we'$	$\mathsf{S}\backslash\mathsf{NP}/\mathsf{VP}/\mathsf{NP} : \lambda x.\lambda f.\lambda y.helped'\,f\,x\,y$	$\mathsf{NP} : Hans'$	$\mathsf{VP}/\mathsf{NP} : \lambda x.paint'\,x$	$\mathsf{NP} : house'$

$$\begin{aligned}
\text{we} &\vdash \mathsf{NP} : we' \\
\text{helped} &\vdash \mathsf{S}\backslash\mathsf{NP}/\mathsf{VP}/\mathsf{NP} : \lambda x.\lambda f.\lambda y.helped'\,f\,x\,y \\
\text{Hans} &\vdash \mathsf{NP} : Hans' \\
\text{paint} &\vdash \mathsf{VP}/\mathsf{NP} : \lambda x.paint'\,x \\
\text{the house} &\vdash \mathsf{NP} : house'
\end{aligned}$$

Categorial grammar

forward application

$$A/B : f \quad B : g \Rightarrow A : fg$$

backward application

$$B : g \quad A\backslash B : f \Rightarrow A : fg$$

we	helped	Hans	paint	the house
NP : we'	S\NP/VP/NP : $\lambda x.\lambda f.\lambda y.helped'\,fxy$	NP : $Hans'$	VP/NP : $\lambda x.paint'\,x$	NP : $house'$
			VP : $paint'\,house'$ (>)	

$$\begin{aligned}
\text{we} &\vdash \mathsf{NP} : we' \\
\text{helped} &\vdash \mathsf{S\backslash NP/VP/NP} : \lambda x.\lambda f.\lambda y.helped'\,fxy \\
\text{Hans} &\vdash \mathsf{NP} : Hans' \\
\text{paint} &\vdash \mathsf{VP/NP} : \lambda x.paint'\,x \\
\text{the house} &\vdash \mathsf{NP} : house'
\end{aligned}$$

Categorial grammar

forward application

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$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\text{we}}{\mathsf{NP} : we'} \quad \dfrac{\text{helped}}{\mathsf{S\backslash NP/VP/NP} : \lambda x.\lambda f.\lambda y.helped'\,f\,x\,y} \quad \dfrac{\text{Hans}}{\mathsf{NP} : Hans'}}{\mathsf{S\backslash NP/VP} : \lambda f.\lambda y.helped'\,f\,hans'\,y} \quad \dfrac{\text{paint}}{\mathsf{VP/NP} : \lambda x.paint'\,x}}{\mathsf{S\backslash NP/NP} : \lambda y.helped'\,paint'\,hans'\,y} \quad \dfrac{\text{the house}}{\mathsf{NP} : house'}}{\mathsf{S\backslash NP} : helped'\,paint'\,hans'\,house'}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\text{we}}{\mathsf{NP} : we'} \quad \dfrac{\text{helped}}{\mathsf{S\backslash NP/VP/NP} : \lambda x.\lambda f.\lambda y.helped'\,f\,x\,y} \quad \dfrac{\text{Hans}}{\mathsf{NP} : Hans'}}{\mathsf{S\backslash NP/VP} : \lambda f.\lambda y.helped'\,f\,hans'\,y} \quad \dfrac{\text{paint}}{\mathsf{VP/NP} : \lambda x.paint'\,x}}{\mathsf{S\backslash NP/NP} : \lambda y.helped'\,paint'\,hans'\,y} \quad \dfrac{\text{the house}}{\mathsf{NP} : house'}}{\mathsf{S\backslash NP} : helped'\,paint'\,hans'\,house'} >$$

$$\begin{aligned}
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\text{Hans} &\vdash \mathsf{NP} : Hans' \\
\text{paint} &\vdash \mathsf{VP/NP} : \lambda x.paint'\,x \\
\text{the house} &\vdash \mathsf{NP} : house'
\end{aligned}$$

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we	helped	Hans	paint	the house
$\mathsf{NP} : we'$	$\mathsf{S}\backslash\mathsf{NP}/\mathsf{VP}/\mathsf{NP} : \lambda x.\lambda f.\lambda y.helped'\,f\,x\,y$	$\mathsf{NP} : Hans'$	$\mathsf{VP}/\mathsf{NP} : \lambda x.paint'\,x$	$\mathsf{NP} : house'$
				$\xrightarrow{>}\ \mathsf{VP} : paint'\,house'$

$$\begin{aligned}
\text{we} &\vdash \mathsf{NP} : we' \\
\text{helped} &\vdash \mathsf{S}\backslash\mathsf{NP}/\mathsf{VP}/\mathsf{NP} : \lambda x.\lambda f.\lambda y.helped'\,f\,x\,y \\
\text{Hans} &\vdash \mathsf{NP} : Hans' \\
\text{paint} &\vdash \mathsf{VP}/\mathsf{NP} : \lambda x.paint'\,x \\
\text{the house} &\vdash \mathsf{NP} : house'
\end{aligned}$$

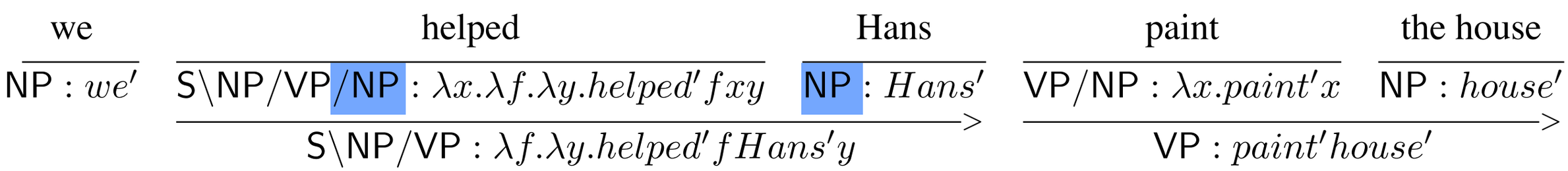
Categorial grammar

forward application

$$A/B : f \quad B : g \Rightarrow A : fg$$

backward application

$$B : g \quad A\backslash B : f \Rightarrow A : fg$$



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\text{we} &\vdash \mathsf{NP} : we' \\
\text{helped} &\vdash \mathsf{S\backslash NP/VP/NP} : \lambda x.\lambda f.\lambda y.helped' f x y \\
\text{Hans} &\vdash \mathsf{NP} : Hans' \\
\text{paint} &\vdash \mathsf{VP/NP} : \lambda x.paint' x \\
\text{the house} &\vdash \mathsf{NP} : house'
\end{aligned}$$

Categorial grammar

forward application

$$A/B : f \quad B : g \Rightarrow A : fg$$

backward application

$$B : g \quad A\backslash B : f \Rightarrow A : fg$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\text{we}}{\mathsf{NP} : we'} \quad \dfrac{\text{helped}}{\mathsf{S\backslash NP/VP/NP} : \lambda x.\lambda f.\lambda y.helped'\,f\,x\,y}}{\mathsf{S\backslash NP/VP} : \lambda f.\lambda y.helped'\,f\,Hans'\,y}>}{\mathsf{S\backslash NP} : \lambda y.helped'\,paint'\,Hans'\,y} \quad \dfrac{\dfrac{\text{Hans}}{\mathsf{NP} : Hans'}}{}}{}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\text{paint}}{\mathsf{VP/NP} : \lambda x.paint'\,x} \quad \dfrac{\text{the house}}{\mathsf{NP} : house'}}{\mathsf{VP} : paint'\,house'}>}{}}{}}{}$$

$$\begin{aligned}
\text{we} &\vdash \mathsf{NP} : we' \\
\text{helped} &\vdash \mathsf{S\backslash NP/VP/NP} : \lambda x.\lambda f.\lambda y.helped'\,f\,x\,y \\
\text{Hans} &\vdash \mathsf{NP} : Hans' \\
\text{paint} &\vdash \mathsf{VP/NP} : \lambda x.paint'\,x \\
\text{the house} &\vdash \mathsf{NP} : house'
\end{aligned}$$

Categorial grammar

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$$A/B : f \quad B : g \Rightarrow A : fg$$

backward application

$$B : g \quad A\backslash B : f \Rightarrow A : fg$$

$$\dfrac{\dfrac{\dfrac{\text{we}}{\mathsf{NP} : we'} \quad \dfrac{\text{helped}}{\mathsf{S\backslash NP/VP/NP} : \lambda x.\lambda f.\lambda y.helped'\,f\,x\,y}}{\mathsf{S\backslash NP/VP} : \lambda f.\lambda y.helped'\,f\,Hans'\,y}> \quad \dfrac{\dfrac{\text{Hans}}{\mathsf{NP} : Hans'} \quad \dfrac{\dfrac{\text{paint}}{\mathsf{VP/NP} : \lambda x.paint'\,x} \quad \dfrac{\text{the house}}{\mathsf{NP} : house'}}{\mathsf{VP} : paint'\,house'}>}{\mathsf{S\backslash NP} : \lambda y.helped'\,(paint'\,house')\,Hans'\,y}>$$

$$\begin{aligned}
\text{we} &\vdash \mathsf{NP} : we' \\
\text{helped} &\vdash \mathsf{S\backslash NP/VP/NP} : \lambda x.\lambda f.\lambda y.helped'\,f\,x\,y \\
\text{Hans} &\vdash \mathsf{NP} : Hans' \\
\text{paint} &\vdash \mathsf{VP/NP} : \lambda x.paint'\,x \\
\text{the house} &\vdash \mathsf{NP} : house'
\end{aligned}$$

Categorial grammar

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$$A/B : f \quad B : g \Rightarrow A : fg$$

backward application

$$B : g \quad A\backslash B : f \Rightarrow A : fg$$

we	helped	Hans	paint	the house
NP : we'	S\NP/VP/NP : $\lambda x.\lambda f.\lambda y.helped'fxy$	NP : $Hans'$	VP/NP : $\lambda x.paint'x$	NP : $house'$
	S\NP/VP : $\lambda f.\lambda y.helped'fHans'y$ (>)		VP : $paint'house'$ (>)	
	S\NP : $\lambda y.helped'(paint'house')Hans'y$ (>)			

$$\begin{aligned}
\text{we} &\vdash \text{NP} : we' \\
\text{helped} &\vdash \text{S\NP/VP/NP} : \lambda x.\lambda f.\lambda y.helped'fxy \\
\text{Hans} &\vdash \text{NP} : Hans' \\
\text{paint} &\vdash \text{VP/NP} : \lambda x.paint'x \\
\text{the house} &\vdash \text{NP} : house'
\end{aligned}$$

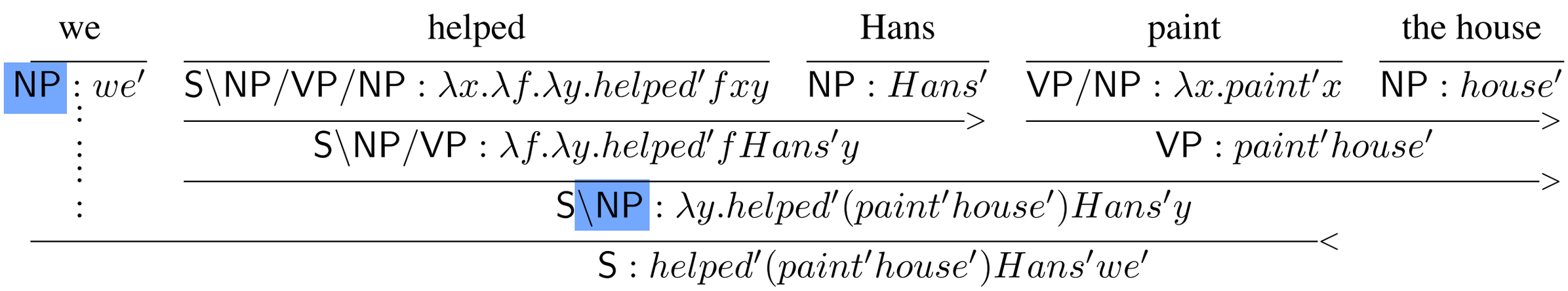
Categorial grammar

forward application

$$A/B : f \quad B : g \Rightarrow A : fg$$

backward application

$$B : g \quad A\backslash B : f \Rightarrow A : fg$$



$$\begin{aligned}
\text{we} &\vdash \mathsf{NP} : we' \\
\text{helped} &\vdash \mathsf{S\backslash NP/VP/NP} : \lambda x.\lambda f.\lambda y.helped'\,f\,x\,y \\
\text{Hans} &\vdash \mathsf{NP} : Hans' \\
\text{paint} &\vdash \mathsf{VP/NP} : \lambda x.paint'\,x \\
\text{the house} &\vdash \mathsf{NP} : house'
\end{aligned}$$

Categorial grammar

forward application

$$A/B : f \quad B : g \Rightarrow A : fg$$

backward application

$$B : g \quad A\backslash B : f \Rightarrow A : fg$$

$$\dfrac{\dfrac{\dfrac{\text{we}}{\mathsf{NP} : we'} \quad \dfrac{\dfrac{\text{helped}}{\mathsf{S\backslash NP/VP/NP} : \lambda x.\lambda f.\lambda y.helped'\,f\,x\,y} \quad \dfrac{\text{Hans}}{\mathsf{NP} : Hans'}}{\mathsf{S\backslash NP/VP} : \lambda f.\lambda y.helped'\,f\,Hans'\,y}\,{>} \quad \dfrac{\dfrac{\text{paint}}{\mathsf{VP/NP} : \lambda x.paint'\,x} \quad \dfrac{\text{the house}}{\mathsf{NP} : house'}}{\mathsf{VP} : paint'\,house'}\,{>}}{\mathsf{S\backslash NP} : \lambda y.helped'\,(paint'\,house')\,Hans'\,y}\,{>}}{\mathsf{S} : helped'\,(paint'\,house')\,Hans'\,we'}\,{<}$$

$$\begin{aligned}
\text{we} &\vdash \mathsf{NP} : we' \\
\text{helped} &\vdash \mathsf{S\backslash NP/VP/NP} : \lambda x.\lambda f.\lambda y.helped'\,f\,x\,y \\
\text{Hans} &\vdash \mathsf{NP} : Hans' \\
\text{paint} &\vdash \mathsf{VP/NP} : \lambda x.paint'\,x \\
\text{the house} &\vdash \mathsf{NP} : house'
\end{aligned}$$

Categorial grammar

CG is context-free (Bar-Hillel et al., 1964)

$$\dfrac{\dfrac{\text{we}}{\mathsf{NP} : we'} \quad \dfrac{\dfrac{\text{helped}}{\mathsf{S\backslash NP/VP/NP} : \lambda x.\lambda f.\lambda y.helped'\,f\,x\,y} \quad \dfrac{\text{Hans}}{\mathsf{NP} : Hans'}}{\mathsf{S\backslash NP/VP} : \lambda f.\lambda y.helped'\,f\,Hans'\,y}\,{>} \quad \dfrac{\dfrac{\text{paint}}{\mathsf{VP/NP} : \lambda x.paint'\,x} \quad \dfrac{\text{the house}}{\mathsf{NP} : house'}}{\mathsf{VP} : paint'\,house'}\,{>}}{\mathsf{S\backslash NP} : \lambda y.helped'\,(paint'\,house')\,Hans'\,y}\,{>}}{\mathsf{S} : helped'\,(paint'\,house')\,Hans'\,we'}\,{<}$$

$$\begin{aligned}
\text{we} &\vdash \mathsf{NP} : we' \\
\text{helped} &\vdash \mathsf{S\backslash NP/VP/NP} : \lambda x.\lambda f.\lambda y.helped'\,f\,x\,y \\
\text{Hans} &\vdash \mathsf{NP} : Hans' \\
\text{paint} &\vdash \mathsf{VP/NP} : \lambda x.paint'\,x \\
\text{the house} &\vdash \mathsf{NP} : house'
\end{aligned}$$

Categorial grammar

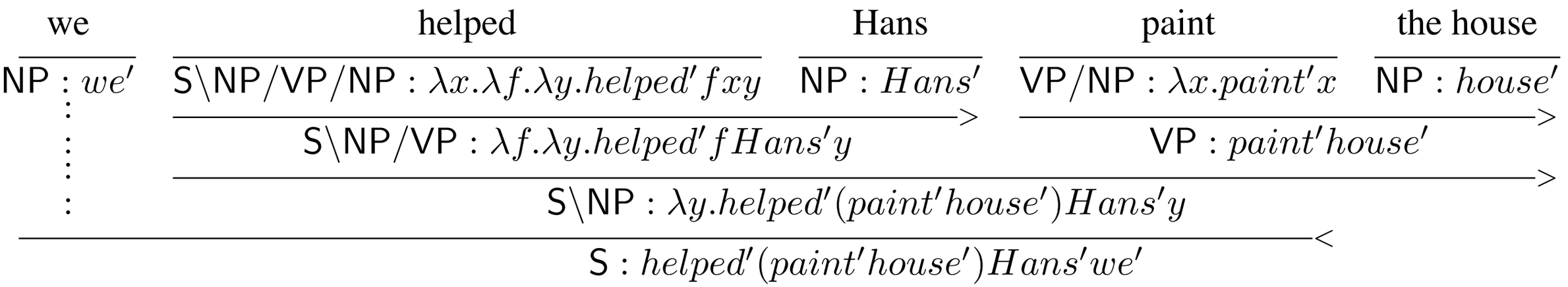
CG is context-free (Bar-Hillel et al., 1964)

$$\dfrac{\dfrac{\text{we}}{\mathsf{NP} : we'} \quad \dfrac{\dfrac{\text{helped}}{\mathsf{S\backslash NP/VP/NP} : \lambda x.\lambda f.\lambda y.helped'\,f\,x\,y} \quad \dfrac{\text{Hans}}{\mathsf{NP} : Hans'}}{\mathsf{S\backslash NP/VP} : \lambda f.\lambda y.helped'\,f\,Hans'\,y}\,{>} \quad \dfrac{\dfrac{\text{paint}}{\mathsf{VP/NP} : \lambda x.paint'\,x} \quad \dfrac{\text{the house}}{\mathsf{NP} : house'}}{\mathsf{VP} : paint'\,house'}\,{>}}{\mathsf{S\backslash NP} : \lambda y.helped'\,(paint'\,house')\,Hans'\,y}\,{>}}{\mathsf{S} : helped'\,(paint'\,house')\,Hans'\,we'}\,{<}$$

$$\begin{aligned}
\text{we} &\vdash \mathsf{NP} : we' & \mathsf{NP} &\rightarrow \text{we} \\
\text{helped} &\vdash \mathsf{S\backslash NP/VP/NP} : \lambda x.\lambda f.\lambda y.helped'\,f\,x\,y & \mathsf{S} &\rightarrow \mathsf{NP}\ \text{helped}\ \mathsf{NP}\ \mathsf{VP} \\
\text{Hans} &\vdash \mathsf{NP} : Hans' & \mathsf{NP} &\rightarrow \text{Hans} \\
\text{paint} &\vdash \mathsf{VP/NP} : \lambda x.paint'\,x & \mathsf{VP} &\rightarrow \text{paint}\ \mathsf{NP} \\
\text{the house} &\vdash \mathsf{NP} : house' & \mathsf{NP} &\rightarrow \text{house}
\end{aligned}$$

Categorial grammar

CG is context-free (Bar-Hillel et al., 1964)



we $\vdash$ NP : we'

helped $\vdash$ S\NP/VP/NP : $\lambda x.\lambda f.\lambda y.helped'\,f\,x\,y$

Hans $\vdash$ NP : $Hans'$

paint $\vdash$ VP/NP : $\lambda x.paint'\,x$

the house $\vdash$ NP : $house'$

NP → we

S → NP helped NP VP

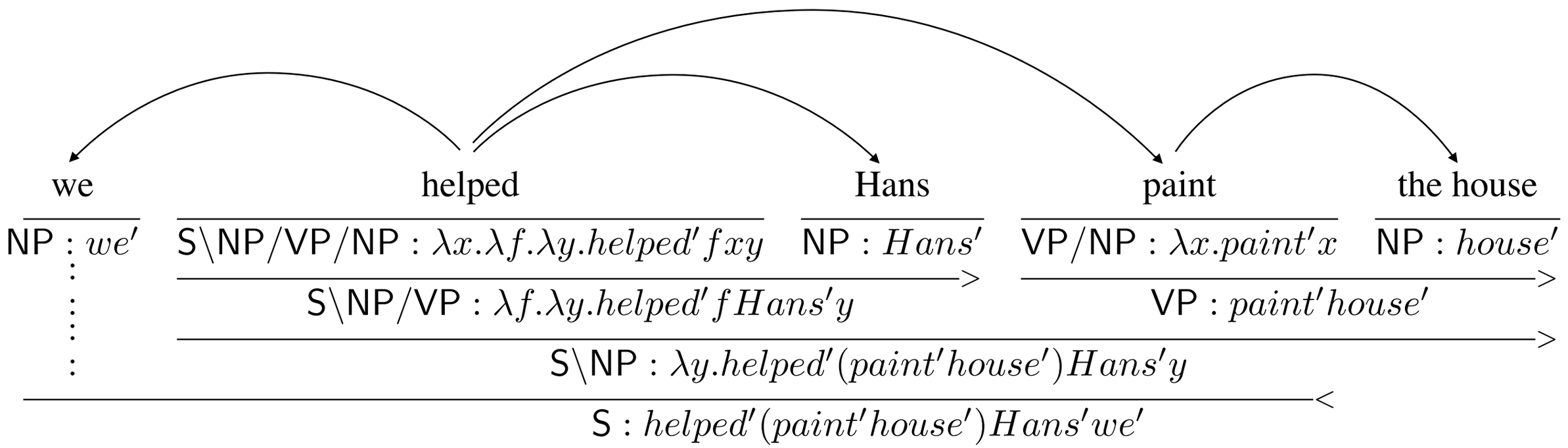
NP → Hans

VP → paint NP

NP → house

Categorial grammar

CG is context-free (Bar-Hillel et al., 1964)

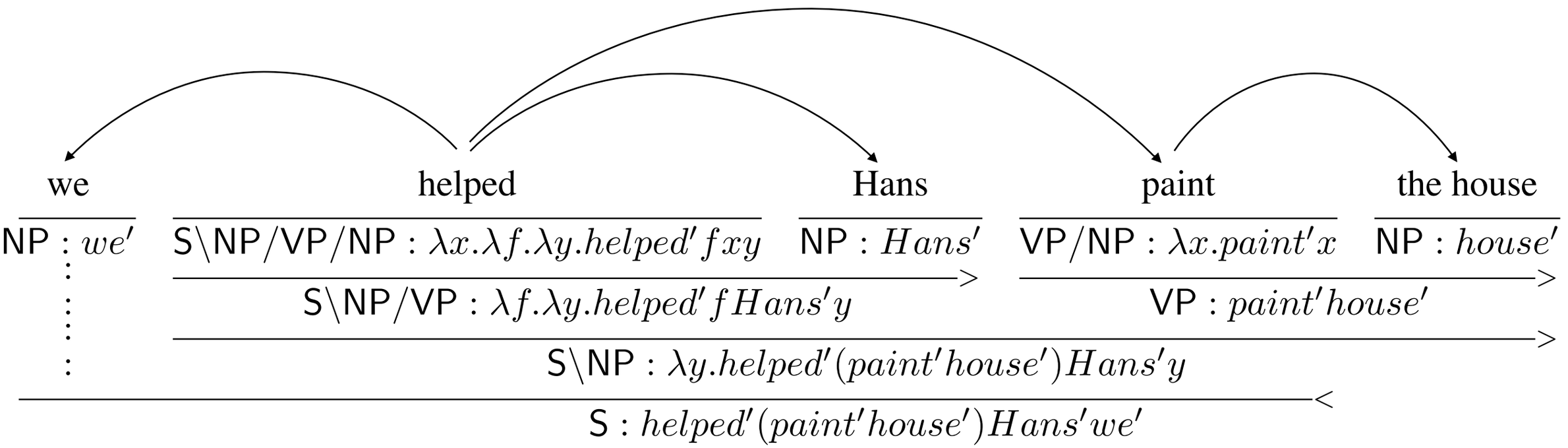


we	$\vdash$ NP : we'	NP $\rightarrow$	we
helped	$\vdash$ S\NP/VP/NP : $\lambda x.\lambda f.\lambda y.helped'\,f\,x\,y$	S $\rightarrow$	NP helped NP VP
Hans	$\vdash$ NP : $Hans'$	NP $\rightarrow$	Hans
paint	$\vdash$ VP/NP : $\lambda x.paint'\,x$	VP $\rightarrow$	paint NP
the house	$\vdash$ NP : $house'$	NP $\rightarrow$	house

Categorial grammar

CG is context-free (Bar-Hillel et al., 1964)

it is also a projective *dependency grammar* (Hays, 1964; Gaifman, 1965)



we	$\vdash$ NP : we'	NP $\rightarrow$	we
helped	$\vdash$ S\NP/VP/NP : $\lambda x.\lambda f.\lambda y.helped'\,f\,x\,y$	S $\rightarrow$	NP helped NP VP
Hans	$\vdash$ NP : $Hans'$	NP $\rightarrow$	Hans
paint	$\vdash$ VP/NP : $\lambda x.paint'\,x$	VP $\rightarrow$	paint NP
the house	$\vdash$ NP : $house'$	NP $\rightarrow$	house

Combinatory categorial grammar

mer em Hans es huus hälfed aastriiche

$$\begin{aligned}
\text{mer} &\vdash \mathsf{NP} : we' \\
\text{em Hans} &\vdash \mathsf{NP} : Hans' \\
\text{es huus} &\vdash \mathsf{NP} : house' \\
\text{hälfed} &\vdash \mathsf{S\backslash NP\backslash NP/VP} : \lambda f.\lambda x.\lambda y.helped'\, f\, x\, y \\
\text{aastriiche} &\vdash \mathsf{VP\backslash NP} : \lambda x.paint'\, x
\end{aligned}$$

Combinatory categorial grammar

mer	em Hans	es huus	hälfed	aastriiche
NP : we'	NP : $Hans'$	NP : $house'$	S\NP\NP/VP : $\lambda f.\lambda x.\lambda y.helped'\,f\,x\,y$	VP\NP : $\lambda x.paint'\,x$

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Combinatory categorial grammar

forward composition (degree n)

$$A/B : f \quad B|_1C_1...|_nC_n : \lambda x_n...\lambda x_1.g(x_1,...,x_n)$$
$$\Rightarrow A|_1C_1...|_nC_n : \lambda x_n...\lambda x_1.fg(x_1,...,x_n)$$

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				$>\mathbf{B}_\times$
				S\NP\NP\NP : $\lambda z.\lambda x.\lambda y.helped'(paint'\,z)\,x\,y$

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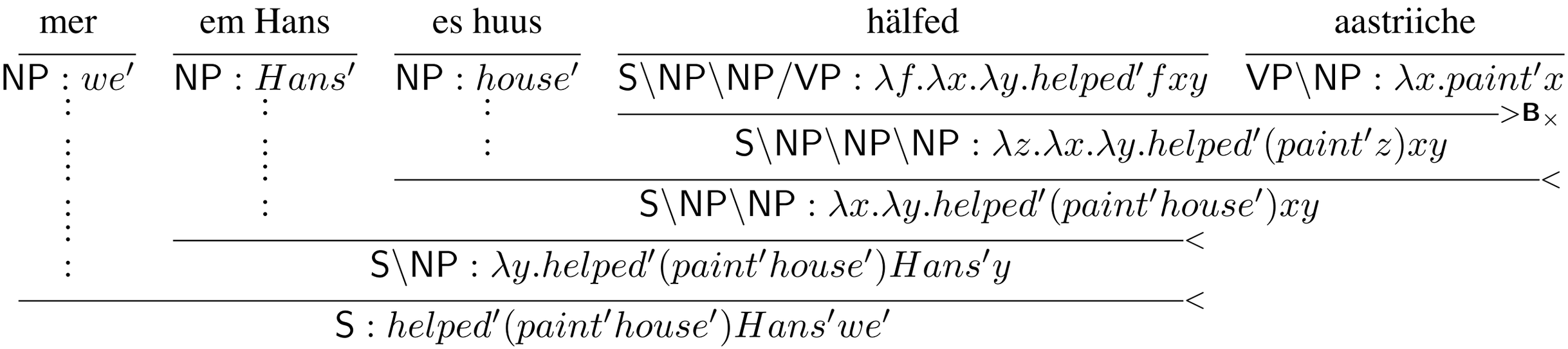

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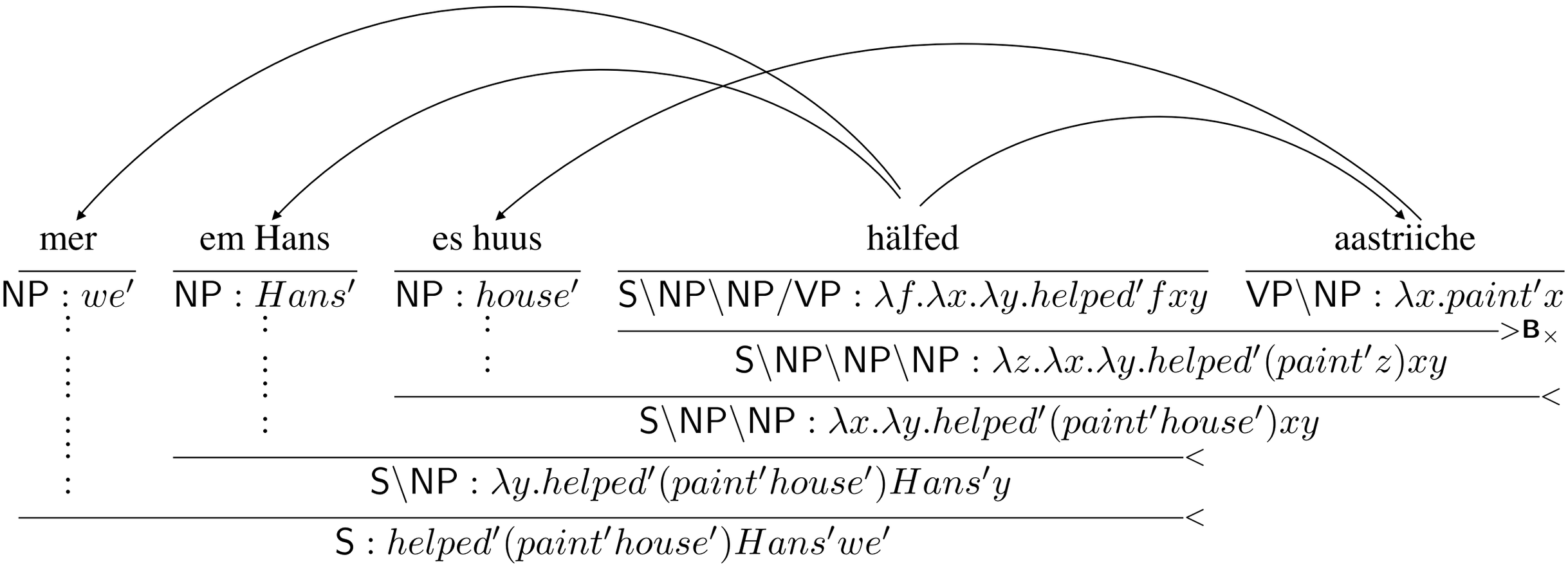
Combinatory categorial grammar

is mildly non-projective dependency grammar (Kuhlmann, 2013)



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es huus $\vdash$ NP : $house'$

hälfed $\vdash$ S\NP\NP/VP : $\lambda f.\lambda x.\lambda y.helped'\,f\,x\,y$

aastriiche $\vdash$ VP\NP : $\lambda x.paint'\,x$

NP → we

NP → em Hans

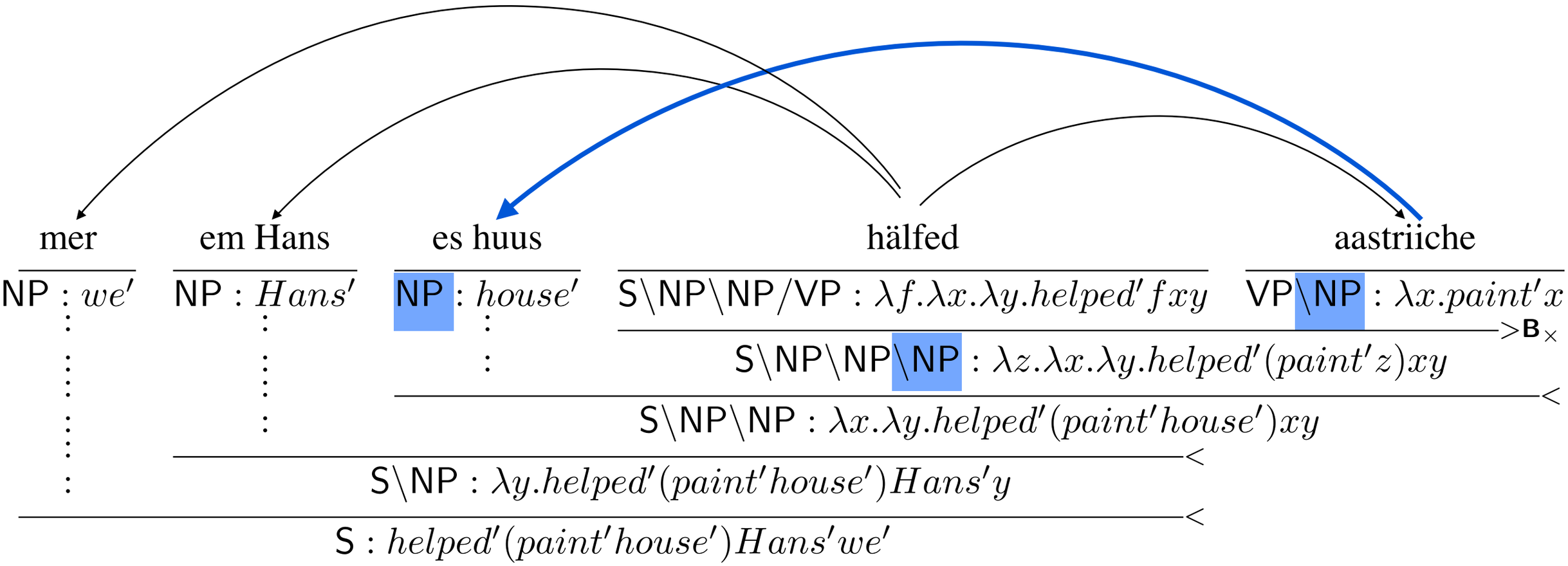
NP → es huus

S → NP NP $VP_1^{(2)}$ hälfed $VP_2^{(2)}$

$VP^{(2)}$ → NP, aastriiche

Combinatory categorial grammar

is mildly non-projective dependency grammar (Kuhlmann, 2013)



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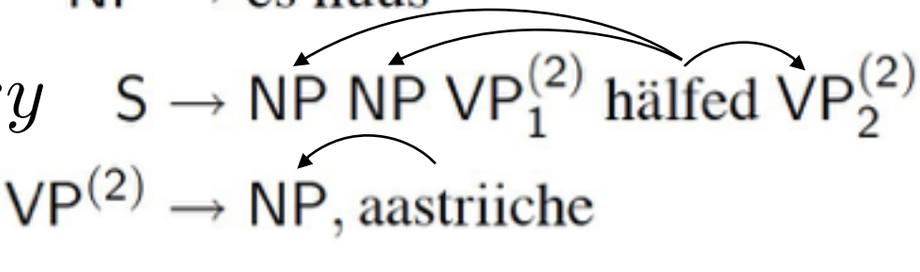

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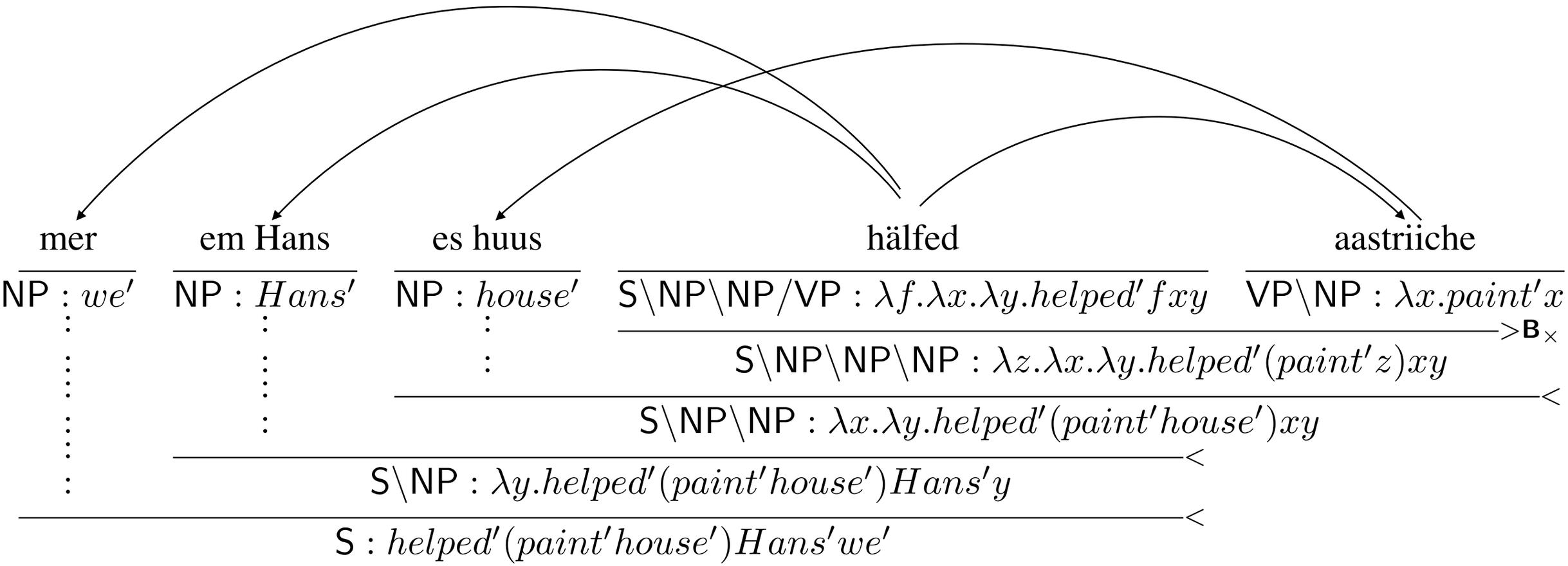

CCG is not context-free

$a \vdash A$
$b \vdash S\backslash A/C$
$b \vdash S\backslash A/C/S$
$c \vdash C$

intersected with $a^*b^*c^*$ $=$ $a^nb^nc^n$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{a \quad a \quad a \quad b \quad b \quad b \quad c \quad c \quad c}{A \quad A \quad A \quad S\backslash A/C/S \quad S\backslash A/C/S \quad S\backslash A/C \quad C \quad C \quad C}\,{>}\mathbf{B}}{S\backslash A/C\backslash A/C/S}\,{>}\mathbf{B}}{S\backslash A/C\backslash A/C\backslash A/C}\,{>}}{S\backslash A/C\backslash A/C\backslash A}\,{<}}{S\backslash A/C\backslash A/C}\,{>}}{S\backslash A/C\backslash A}\,{<}}{S\backslash A/C}\,{>}}{S\backslash A}\,{<}}{S}$$