Part I — Structured Data

Data Representation:

I.1 The entity-relationship (ER) data model

I.2 The relational model

Data Manipulation:

I.3 Relational algebra

I.4 Tuple-relational calculus

I.5 The SQL query language

Related reading: Chapter 4 of [DMS], §§ 4.3

Motivation

Tuple-relational calculus is another way of writing queries for relational data.

Its power lies in the fact that it is entirely *declarative*.

That is, we specify the properties of the data we are interested in retrieving, but we do not describe any particular method by which the data can be retrieved.

Basic format

Queries in the relational calculus are based on *tuple variables*.

Each tuple variable has an associated schema (i.e. a type). The variable ranges over all possible tuples of values matching the schema declaration.

A query in the calculus has the general form

$$\{T \mid p(T)\}$$

where T is a tuple variable and $p(T)$ is some formula of first-order predicate logic in which the tuple variable T occurs free.

The result of this query is the set of all possible tuples t (consistent with the schema of T) for which the formula $p(T)$ evaluates to true when $T = t$.

Simple example

Find all students at least 19 years old

$$\{S \mid S \in \texttt{Students} \wedge S.\texttt{age} > 18\}$$

In detail:

- S is a tuple variable
- S can take any value in the Students table
- Evaluate $S.\texttt{age} > 18$ on each such tuple
- That tuple should appear in the result if and only if the predicate evaluates to true

Formal syntax of atomic formulae

General formulae are built out of atomic formulae.

An *atomic formula* is one of the following:

- $R \in Rel$
- $R.a \;\; op \;\; S.b$
- $R.a \;\; op \;\; constant$
- $constant \;\; op \;\; S.b$

where: R, S are tuple variables, *Rel* is a relation name, a, b are attributes of R, S respectively, and *op* is any operator in the set $\{>, <, =, \neq, \geq, \leq\}$

Formal syntax of (composite) formulae

A *formula* is (recursively defined) to be one of the following:

- any atomic formula
- $\neg p, \quad p \wedge q, \quad p \vee q, \quad p \Rightarrow q$
- $\exists R.\, p(R), \quad \forall R.\, p(R)$

where $p(R)$ denotes a formula in which the variable R appears free.

N.B. Recall that Informatics 1: Computation & Logic introduced first-order logic in more detail. For notation, we follow Ramakrishnan & Gehrke "Database Management Systems" in using $\neg$ for *not*; $\wedge$ for *and*; $\vee$ for *or*; and $\Rightarrow$ for $\rightarrow$. The main difference from standard first-order logic is the use of variables ranging over tuples (rather than individuals), and the correspondingly specialized forms of atomic formulae.

A subtle point

In ordinary first-order logic we can, in principle, form quantifications $\exists \boldsymbol{R}.\ \boldsymbol{p}$ and $\forall \boldsymbol{R}.\ \boldsymbol{p}$ even when $\boldsymbol{R}$ does not occur in $\boldsymbol{p}$. (In practice, such quantifications are normally useless since they are trivial.)

In tuple-relational calculus we only allow $\exists \boldsymbol{R}.\ \boldsymbol{p}$ and $\forall \boldsymbol{R}.\ \boldsymbol{p}$ when $\boldsymbol{R}$ occurs free in $\boldsymbol{p}$. This is no great restriction, and it saves us explicitly declaring the schema of $\boldsymbol{R}$:

- Under this rule, every tuple variable $\boldsymbol{R}$ that appears in a formula is forced to appear in at least one atomic subformula. The atomic formulae in which $\boldsymbol{R}$ appears then determine the schema of $\boldsymbol{R}$. The schema is taken to be the smallest one containing all the fields that are declared as attributes of $\boldsymbol{R}$ within the formula itself.

Illustrative example

An example showing how to compute the minimal schema for a query:

$$\{P \mid \exists S \in \texttt{Students}\,(S.\texttt{age} > 20 \ \wedge\ P.\texttt{name} = S.\texttt{name} \ \wedge\ P.\texttt{age} = S.\texttt{age})\}$$

- The schema of S is that of the **Students** table. This is declared by the atomic formula $S \in \texttt{Students}$.
- The schema of P has just two fields **name** and **age**, with the same types as the corresponding fields in **Students**.
- The query returns a table with two fields **name** and **age** containing the names and ages of all students aged 21 or over.

Note the use of $\exists S \in \texttt{Students}\,(p)$ for $\exists S\,(S \in \texttt{Students} \wedge p)$.
We make free use of such (standard) abbreviations.

Further examples (1)

Query: Find the names of students who are taking Informatics 1

Relational algebra:

$$\pi_{\texttt{Students.name}}(\texttt{Students} \bowtie_{\texttt{Students.mn}=\texttt{Takes.mn}} (\texttt{Takes} \bowtie_{\texttt{Takes.code}=\texttt{Courses.code}} (\sigma_{\texttt{name}=\text{'Informatics 1'}}(\texttt{Courses}))))$$

Tuple-relational calculus:

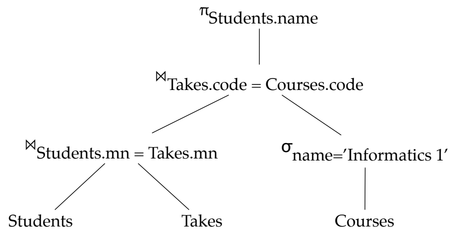
$$\{P \mid \exists S \in \texttt{Students}\ \exists T \in \texttt{Takes}\ \exists C \in \texttt{Courses}\ (C.\texttt{name} = \text{'Informatics 1'} \ \wedge\ C.\texttt{code} = T.\texttt{code} \ \wedge\ S.\texttt{mn} = T.\texttt{mn} \ \wedge\ P.\texttt{name} = S.\texttt{name})\}$$

Tree representation of algebraic expression (abstract syntax)

For the previous query, changing the bracketing does not change the query.

$$\pi_{\texttt{Students.name}}((\texttt{Students} \bowtie_{\texttt{Students.mn}=\texttt{Takes.mn}} \texttt{Takes}) \bowtie_{\texttt{Takes.code}=\texttt{Courses.code}} (\sigma_{\texttt{name}=\text{'Informatics 1'}}(\texttt{Courses})))$$

A tree representation can help one visualise a relational algebra query.



Further examples (2)

Query: Find the names of all courses taken by (everyone called) Joe

Relational algebra:

$$\pi_{\texttt{Courses.name}}((\sigma_{\texttt{name}=\text{'Joe'}}(\texttt{Students})) \bowtie_{\texttt{Students.mn}=\texttt{Takes.mn}} (\texttt{Takes} \bowtie_{\texttt{Takes.code}=\texttt{Courses.code}} \texttt{Courses}))$$

Tuple-relational calculus:

$$\{P \mid \exists S \in \texttt{Students}\ \exists T \in \texttt{Takes}\ \exists C \in \texttt{Courses}\ (S.\texttt{name} = \text{'Joe'} \ \wedge\ S.\texttt{mn} = T.\texttt{mn} \ \wedge\ C.\texttt{code} = T.\texttt{code} \ \wedge\ P.\texttt{name} = C.\texttt{name})\}$$

Further examples (3)

Query: Find the names of all students who are taking Informatics 1 or Geology 1

Relational algebra:

$$\pi_{\texttt{Students.name}}(\texttt{Students} \bowtie_{\texttt{Students.mn}=\texttt{Takes.mn}} (\texttt{Takes} \bowtie_{\texttt{Takes.code}=\texttt{Courses.code}} (\sigma_{\texttt{name}=\texttt{'Informatics 1'} \vee \texttt{name}=\text{'Geology 1'}}(\texttt{Courses}))))$$

Tuple-relational calculus:

$$\{P \mid \exists S \in \texttt{Students}\ \exists T \in \texttt{Takes}\ \exists C \in \texttt{Courses}\ ((C.\texttt{name} = \text{'Informatics 1'} \ \vee\ C.\texttt{name} = \text{'Geology 1'}) \ \wedge\ C.\texttt{code} = T.\texttt{code} \ \wedge S.\texttt{mn} = T.\texttt{mn} \ \wedge\ P.\texttt{name} = S.\texttt{name})\}$$

Further examples (4)

Query: Find the names of students who are taking both Informatics 1 and Geology 1

Relational algebra:

$$\pi_{\text{Students.name}}(\\ (\texttt{Students} \bowtie_{\texttt{Students.mn=Takes.mn}} \\ (\texttt{Takes} \bowtie_{\texttt{Takes.code=Courses.code}} \\ (\sigma_{\texttt{name}=\text{'Informatics 1'}}(\text{Courses})))) \\ \cap \\ (\texttt{Students} \bowtie_{\texttt{Students.mn=Takes.mn}} \\ (\texttt{Takes} \bowtie_{\texttt{Takes.code=Courses.code}} \\ (\sigma_{\texttt{name}=\text{'Geology 1'}}(\text{Courses})))) \ )$$

Further examples (4 continued)

Query: Find the names of students who are taking both Informatics 1 and Geology 1

Tuple-relational calculus:

$$\{P \mid \exists S \in \texttt{Students}\ (P.\texttt{name} = S.\texttt{name}\ \wedge \\ \forall C \in \texttt{Courses} \\ ((C.\texttt{name} = \text{'Informatics 1'}\ \vee\ C.\texttt{name} = \text{'Geology 1'}) \Rightarrow \\ (\exists T \in \texttt{Takes}\ (T.\texttt{mn} = S.\texttt{mn}\ \wedge\ T.\texttt{code} = C.\texttt{code})))) \ \}$$

Exercise. What does this query return in the case that there is no course in **Courses** called 'Geology 1'? Find a way of rewriting the query so that it only returns an answer if both 'Informatics 1' and 'Geology 1' courses exist.

Further examples (5)

Query: Find the names of all students who are taking all courses

Tuple-relational calculus:

$$\{P \mid \exists S \in \texttt{Students}\ (P.\texttt{name} = S.\texttt{name}\ \wedge \\ \forall C \in \texttt{Courses} \\ (\exists T \in \texttt{Takes}\ (T.\texttt{mn} = S.\texttt{mn}\ \wedge\ T.\texttt{code} = C.\texttt{code}))) \ \}$$

Exercise. Try to write this query in relational algebra.

Relational algebra and tuple-relational calculus compared

Relational algebra (RA) and tuple-relational calculus (TRC) have the *same* expressive power

That is, if a query can be expressed in RA, then it can be expressed in TRC, and vice-versa

Why is it useful to have both approaches?

Declarative versus procedural

Recall that TRC is *declarative* and RA is *procedural*.

This suggests the following methodology.

- *Specify* the data that needs to be retrieved using relational calculus.
- Translate this to an *equivalent query* in relational algebra.
- Rearrange that to obtain an *efficient* method to retrieve the data.

This approach underpins *query optimisation* in relational databases.

In practice, queries are written in SQL rather than TRC but these are then translated into algebraic operations.

The key observation is that succinctly and correctly *specifying* the queries is best done in one language, while efficiently *executing* those queries may require translating to a different one.