Informatics 1B, 2008

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Data and Analysis

Note 3

Relational Algebra

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Part I — Structured Data

Data Representation:

Note 1 The entity-relationship (ER) data model

Note 2 The relational model

Data Manipulation:

Note 3 Relational algebra

Note 4 Tuple relational calculus

Note 5 The SQL query language

Querying

Once data is organised in a relational schema, the natural next step is to *manipulate* data. For our purposes, this means querying.

Querying is the process of identifying the parts of stored data that have properties of interest

We consider three approaches.

- Relational algebra (this note): a *procedural* way of expressing queries over relationally represented data
- Tuple-relational calculus (note 4): a *declarative* way of expressing queries, tightly coupled to first order predicate logic
- SQL (note 5): a widely implemented query language influenced by relational algebra and relational calculus

Operators

The key concept in relational algebra is an *operator*

Operators accept a single relation or a pair of relations as input

Operators produce a single relation as output

Operators can be *composed* by using one operator's output as input to another operator (composition of functions)

There are five basic operators: *selection*, *projection*, *union*, *cross-product*, and *difference*

Other operators can be defined as composites of these five, but are so frequently used that they are often treated as fundamental

Selection and projection: σ and π

Recall that relational data is stored in *tables*

Selection and *projection* allow one to isolate any "rectangular subset" of a single table

- Selection identifies *rows* of interest
- Projection identifies *columns* of interest

If both are used on a single table, we extract a *rectangular subset* of the table

Selection: example

mn	name	age	email
s0456782	John	18	john@inf
s0412375	Mary	18	mary@inf
s0378435	Helen	20	helen@phys
s0189034	Peter	22	peter@math

Students

name	age
John	18
Mary	18
Helen	20
Peter	22

$\pi_{\text{name, age}}(\text{Students})$

mn	name	age	email
s0378435	Helen	20	helen@phys
s0189034	Peter	22	peter@math

$\sigma_{\text{age}>18}(\text{Students})$

name	age
Helen	20
Peter	22

Combination

Selection: general form

General form: $\sigma_{\text{predicate}}$(Relation instance)

A *predicate* is a condition that is applied on each row of the table

- It should evaluate to either true or false
- If it evaluates to true, the row is propagated to the output, if it evaluates to false the row is dropped
- The output table may thus have lower cardinality than the input

Predicates are written in the Boolean form

$$\text{term}_1 \ \texttt{bop} \ \text{term}_2 \ \texttt{bop} \ \ldots \ \texttt{bop} \ \text{term}_m$$

- Where $\texttt{bop} \in \{\vee, \wedge\}$
- term_i's are of the form attribute $\texttt{rop}$ constant or attribute_1 $\texttt{rop}$ attribute_2 (where $\texttt{rop} \in \{>, <, =, \neq, \geq, \leq\}$)

Projection: example

mn	name	age	email
s0456782	John	18	john@inf
s0412375	Mary	18	mary@inf
s0378435	Helen	20	helen@phys
s0189034	Peter	22	peter@math

Students

name	age
John	18
Mary	18
Helen	20
Peter	22

$\pi_{\text{name, age}}(\text{Students})$

mn	name	age	email
s0378435	Helen	20	helen@phys
s0189034	Peter	22	peter@math

$\sigma_{\text{age}>18}(\text{Students})$

name	age
Helen	20
Peter	22

Combination

Projection: general form

General form: $\pi_{\text{column list}}(\text{Relation instance})$

All rows of the input are propagated in the output

Only columns appearing in the *column list* appear in the output

Thus the *arity* of the output table may be lower than that of the input table

The resulting relation has a different schema!

Selection and projection: example

mn	name	age	email
s0456782	John	18	john@inf
s0412375	Mary	18	mary@inf
s0378435	Helen	20	helen@phys
s0189034	Peter	22	peter@math

Students

name	age
John	18
Mary	18
Helen	20
Peter	22

$\pi_{\text{name, age}}(\text{Students})$

mn	name	age	email
s0378435	Helen	20	helen@phys
s0189034	Peter	22	peter@math

$\sigma_{\text{age}>18}(\text{Students})$

name	age
Helen	20
Peter	22

Combination

Note the *algebraic equivalence* between:

- $\sigma_{\text{age}>18}(\pi_{\text{name,age}}(\text{Students}))$
- $\pi_{\text{name,age}}(\sigma_{\text{age}>18}(\text{Students}))$

Set operations

There are three basic set operations in relational algebra:

- *union*
- *difference*
- *cross-product*

A fourth, *intersection*, can be expressed in terms of the others

All these set operations are binary.

Essentially, they are the well-known set operations from set theory, but extended to deal with tuples

Union

Let $\boldsymbol{R}$ and $\boldsymbol{S}$ be two relations. For union, set difference and intersection $\boldsymbol{R}$ and $\boldsymbol{S}$ are required to have compatible schemata:

- Two schemata are said to be *compatible* if they have the same number of fields and corresponding fields in a left-to-right order have the same domains. N.B., the names of the fields are not used

The *union* $\boldsymbol{R} \cup \boldsymbol{S}$ of $\boldsymbol{R}$ and $\boldsymbol{S}$ is a new relation with the same schema as $\boldsymbol{R}$. It contains exactly the tuples that appear in at least one of the relations $\boldsymbol{R}$ and $\boldsymbol{S}$

N.B. For naming purposes it is assumed that the output relation inherits the field names from the relation appearing first in the specification ($\boldsymbol{R}$ in the previous case)

Union example

mn	*name*	*age*	*email*
s0456782	John	18	john@inf
s0412375	Mary	18	mary@inf
s0378435	Helen	20	helen@phys
s0189034	Peter	22	peter@math

S_1

mn	*name*	*age*	*email*
s0489967	Basil	19	basil@inf
s0412375	Mary	18	mary@inf
s9989232	Ophelia	24	oph@bio
s0189034	Peter	22	peter@math
s0289125	Michael	21	mike@geo

S_2

mn	*name*	*age*	*email*
s0456782	John	18	john@inf
s0412375	Mary	18	mary@inf
s0378435	Helen	20	helen@phys
s0189034	Peter	22	peter@math
s0489967	Basil	19	basil@inf
s9989232	Ophelia	24	oph@bio
s0289125	Michael	21	mike@geo

$S_1 \cup S_2$

Set difference and intersection

The *set difference* $\boldsymbol{R} - \boldsymbol{S}$ and *intersection* $\boldsymbol{R} \cap \boldsymbol{S}$ are also new relations with the same schema as $\boldsymbol{R}$ and $\boldsymbol{S}$.

$\boldsymbol{R} - \boldsymbol{S}$ contains exactly those tuples that appear in $\boldsymbol{R}$ but which do not appear in $\boldsymbol{S}$

$\boldsymbol{R} \cap \boldsymbol{S}$ contains exactly those tuples that appear in both $\boldsymbol{R}$ and $\boldsymbol{S}$

For both operations, the same naming conventions apply as for union

Note that intersection can be defined from set difference by
$\boldsymbol{R} \cap \boldsymbol{S} = \boldsymbol{R} - (\boldsymbol{R} - \boldsymbol{S})$

Set difference example

mn	*name*	*age*	*email*
s0456782	John	18	john@inf
s0412375	Mary	18	mary@inf
s0378435	Helen	20	helen@phys
s0189034	Peter	22	peter@math

S_1

mn	*name*	*age*	*email*
s0489967	Basil	19	basil@inf
s0412375	Mary	18	mary@inf
s9989232	Ophelia	24	oph@bio
s0189034	Peter	22	peter@math
s0289125	Michael	21	mike@geo

S_2

mn	*name*	*age*	*email*
s0456782	John	18	john@inf
s0378435	Helen	20	helen@phys

S_1-S_2

Intersection example

mn	*name*	*age*	*email*
s0456782	John	18	john@inf
s0412375	Mary	18	mary@inf
s0378435	Helen	20	helen@phys
s0189034	Peter	22	peter@math

S_1

mn	*name*	*age*	*email*
s0489967	Basil	19	basil@inf
s0412375	Mary	18	mary@inf
s9989232	Ophelia	24	oph@bio
s0189034	Peter	22	peter@math
s0289125	Michael	21	mike@geo

S_2

mn	*name*	*age*	*email*
s0412375	Mary	18	mary@inf
s0189034	Peter	22	peter@math

$S_1 \cap S_2$

Cross product

The *cross-product* (also known as the *Cartesian product*) $\boldsymbol{R} \times \boldsymbol{S}$ of two relations $\boldsymbol{R}$ and $\boldsymbol{S}$ is a new relation where

- The schema of the relation is obtained by first listing all the fields of $\boldsymbol{R}$ (in order) followed by all the fields of $\boldsymbol{S}$ (in order).
- The resulting relation contains one tuple $\langle \boldsymbol{r}, \boldsymbol{s} \rangle$ for each pair of tuples $\boldsymbol{r} \in \boldsymbol{R}$ and $\boldsymbol{s} \in \boldsymbol{S}$. (Here $\langle \boldsymbol{r}, \boldsymbol{s} \rangle$ denotes the tuple obtained by appending $\boldsymbol{r}$ and $\boldsymbol{s}$ together, with $\boldsymbol{r}$ first and $\boldsymbol{s}$ second.)

Note that if there is a field name common to $\boldsymbol{R}$ and $\boldsymbol{S}$ then two separate columns with this name appear in the cross-product schema.

N.B. The two relations need not have the same schema to begin with.

Cross-product example

mn	*name*	*age*	*email*
s0456782	John	18	john@inf
s0412375	Mary	18	mary@inf
s0378435	Helen	20	helen@phys
s0189034	Peter	22	peter@math

S_1

code	*name*	*year*
inf1	Informatics 1	1
math1	Mathematics 1	1

R

mn	*name*	*age*	*email*	*code*	*name*	*year*
s0456782	John	18	john@inf	inf1	Informatics 1	1
s0456782	John	18	john@inf	math1	Mathematics 1	1
s0412375	Mary	18	mary@inf	inf1	Informatics 1	1
s0412375	Mary	18	mary@inf	math1	Mathematics 1	1
s0378435	Helen	20	helen@phys	inf1	Informatics 1	1
s0378435	Helen	20	helen@phys	math1	Mathematics 1	1
s0189034	Peter	22	peter@math	inf1	Informatics 1	1
s0189034	Peter	22	peter@math	math1	Mathematics 1	1

$S_1 \times R$

Renaming

The renaming operator changes the names of tables and columns.

This can be used to avoid *naming conflicts* when the application of an operator results in a schema with duplicate column names

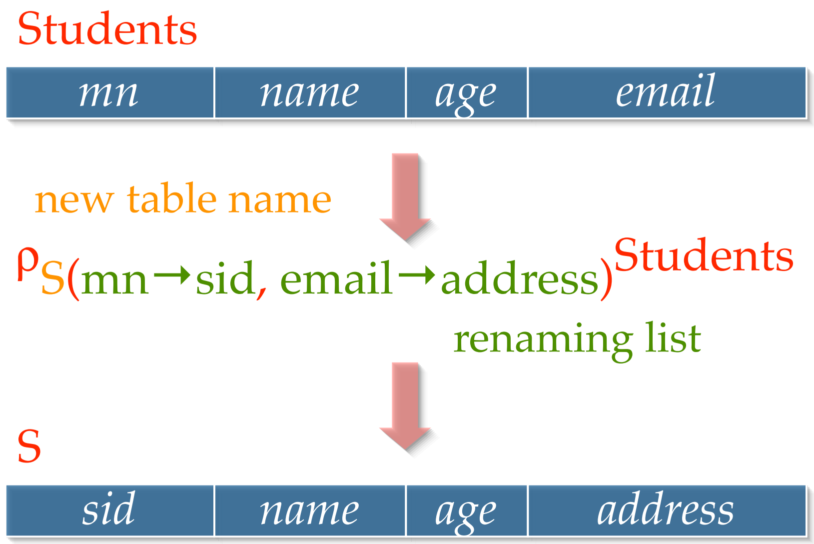
General form

$$\rho_{\text{New-relation-name}(\text{renaming-list})}(\text{Original-relation-name})$$

Semantics:

- The relation is assigned the new relation name
- The renaming list consists of terms of the form oldname $\longrightarrow$ newname which rename a field named oldname to newname
- For ρ to be well-defined there should be no naming conflicts in the output

Renaming example



N.B.

- The types of the columns do not change
- Either the renaming list, or the new table name may be empty

Join

The *relational join* $\boldsymbol{R} \bowtie_{\boldsymbol{p}} \boldsymbol{S}$ is the most frequently used relational operator.

It is a *derived operator*, it can be defined in terms of cross-product and selection.

The format for a join is $\boldsymbol{R} \bowtie_{\boldsymbol{p}} \boldsymbol{S}$ where $\boldsymbol{R}$ and $\boldsymbol{S}$ are relations and the *join predicate* $\boldsymbol{p}$ is a predicate (as defined on slide 3.7) that applies to the schema of $\boldsymbol{R} \times \boldsymbol{S}$.

For example, $\boldsymbol{p}$ may have the form $\mathrm{col}_{\mathbf{1}}\, \texttt{rop}\, \mathrm{col}_{\mathbf{2}}$ where $\mathrm{col}_{\mathbf{1}}$, $\mathrm{col}_{\mathbf{2}}$ are columns of $\boldsymbol{R}$, $\boldsymbol{S}$ and $\texttt{rop} \in \{>, <, =, \neq, \geq, \leq\}$

Formally, the relational join is *defined* by:

$$\boldsymbol{R} \bowtie_{\boldsymbol{p}} \boldsymbol{S} = \boldsymbol{\sigma}_{\boldsymbol{p}}(\boldsymbol{R} \times \boldsymbol{S})$$

Join example

mn	*name*	*age*	*email*
s0456782	John	18	john@inf
s0412375	Mary	18	mary@inf
s0378435	Helen	20	helen@phys
s0189034	Peter	22	peter@math

Students

mn	*code*	*mark*
s0412375	inf1	80
s0378435	math1	70

Takes

mn	*name*	*age*	*email*	*mn*	*code*	*mark*
s0456782	John	18	john@inf	s0412375	inf1	80
s0456782	John	18	john@inf	s0378435	math1	70
s0412375	Mary	18	mary@inf	s0412375	inf1	80
s0412375	Mary	18	mary@inf	s0378435	math1	70
s0378435	Helen	20	helen@phys	s0412375	inf1	80
s0378435	Helen	20	helen@phys	s0378435	math1	70
s0189034	Peter	22	peter@math	s0412375	inf1	80
s0189034	Peter	22	peter@math	s0378435	math1	70

$$\mathit{Students} \bowtie_{\mathit{Students.mn} = \mathit{Takes.mn}} \mathit{Takes}$$

Equijoin

An *equijoin* is a commonly occurring join operation in which the predicate is a conjunction of equalities of the form $\boldsymbol{R}.\texttt{name}_1 = \boldsymbol{S}.\texttt{name}_2$.
(A *conjunction* is a list of conditions connected by $\wedge$.)

The schema of the equijoin consists of the fields of $\boldsymbol{R}$, followed by just those fields of $\boldsymbol{S}$ that are not mentioned in the join equalities. The equijoin is computed by *projecting* the join onto the fields that remain (all those of $\boldsymbol{R}$, and those from $\boldsymbol{S}$ that have not been removed). Put more simply: remove from the join those columns labelled with $\boldsymbol{S}$-fields that appear in the equalities.

Note that the example on the previous slide,
$\texttt{Students} \bowtie_{\texttt{Students.mn = Takes.mn}} \texttt{Takes}$, is naturally treated as an equijoin . The resulting relation is then as before, but with the second column labelled `mn` removed.

Natural join

The *natural join* is a special equijoin in which the equalities are between *all* fields that have the same name in $\boldsymbol{R}$ and $\boldsymbol{S}$.

We simply write $\boldsymbol{R} \bowtie \boldsymbol{S}$ for such an equijoin.

Note that the equijoin version of the example on slide 3.22 is in fact the natural join `Students` $\bowtie$ `Takes`. (The common field name is `mn`.)

This is a very natural way of joining two relations, hence the name. It frequently occurs when joining two tables in which one has a foreign key constraint referencing the other.