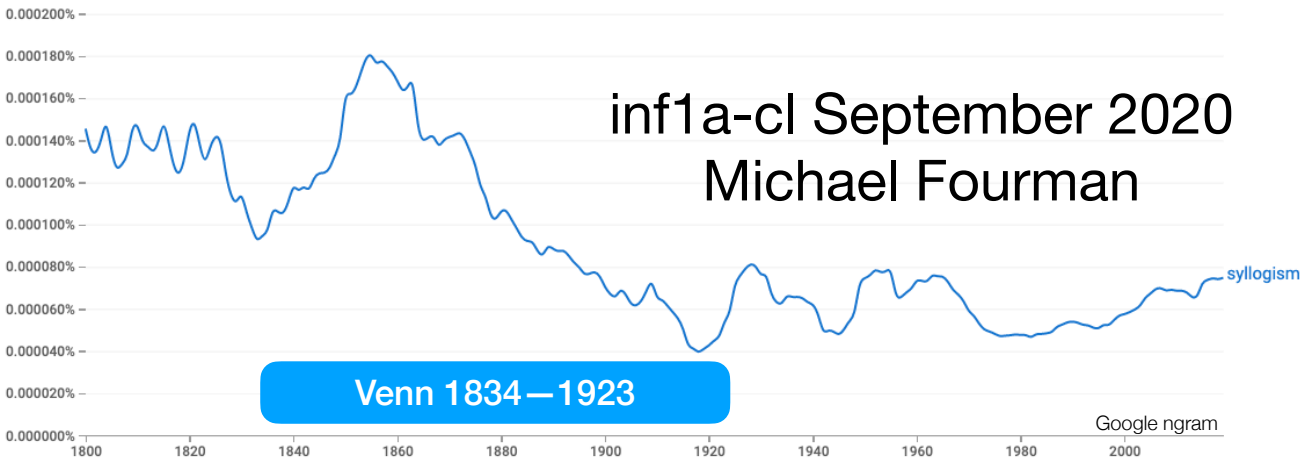
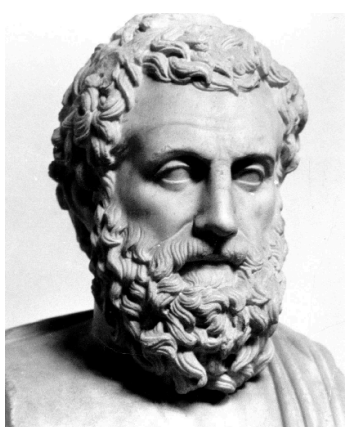


inf1a-cl September 2020
Michael Fourman

INF1A

Aristotle
via Venn

Negation and contraposition of predicates

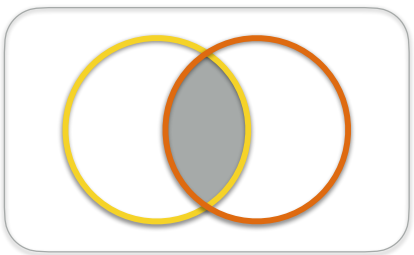


Aristotle
384-322 BC

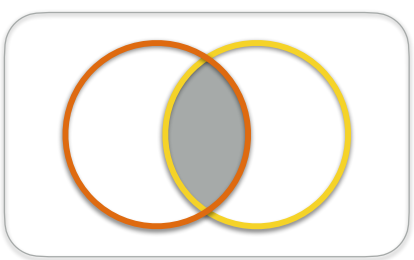
no **a** is **b** **iff** no **b** is **a**

contraposition of predicates

no **a** is **b**

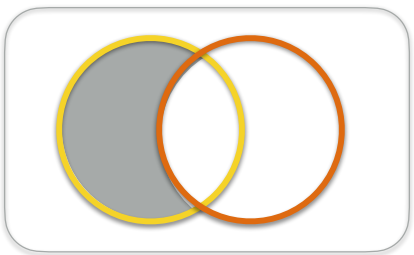


$$\frac{a \vDash \neg b}{b \vDash \neg a}$$

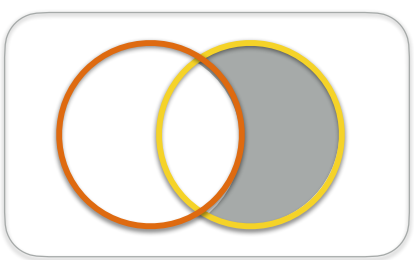


no **b** is **a**

all **a** is **b**
no **a** is not **b**

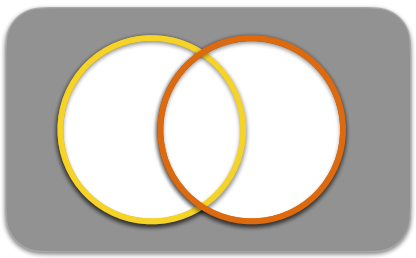


$$\frac{a \vDash b}{\neg b \vDash \neg a}$$

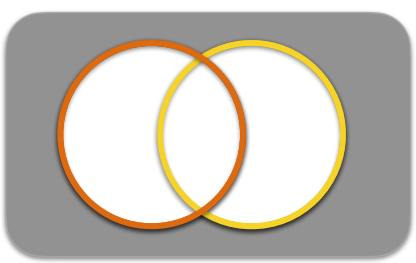


all **not b** is **not a**
no **not b** is **a**

all **not a** is **b**



$$\frac{\neg a \vDash b}{\neg b \vDash a}$$



all **not b** is **a**

INF1A

syllogism

The first rule of boolean algebra

$$\neg\neg\neg a = a$$

The second rule of boolean logic

the first is *barbara*

$$\dfrac{\dfrac{\dfrac{a \models b}{\neg b \models \neg a}}{\neg\neg a \models \neg\neg b}}{a \models b}$$

$$\dfrac{a \models b}{\neg b \models \neg a}$$

$$\dfrac{a \models b}{\neg b \models \neg a}$$

contraposition

INF1A

syllogism

The first rule of boolean algebra

$$\neg\neg\neg a = a$$

The second rule of boolean logic
(the first rule of logic is *barbara*)

$$\dfrac{\dfrac{\dfrac{a \models b}{\neg b \models \neg a}}{\neg\neg a \models \neg\neg b}}{a \models b}$$

$$\dfrac{a \models b}{\neg b \models \neg a}$$

$$\dfrac{a \models b}{\neg b \models \neg a}$$

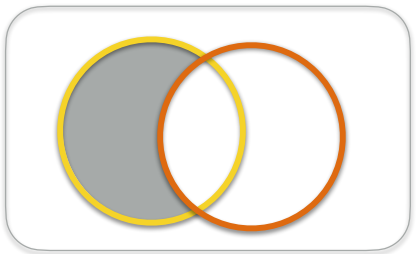
contraposition

… to every affirmation there corresponds exactly one denial such that that denial denies exactly what that affirmation affirms.

Aristotle

predicates a, b …

Venn diagram



every a *satisfies* b

a relation
between predicates

$a \models b$

INF1A

syllogism

negation

$$(\neg a)x = \texttt{not}(a\ x)$$

$$\neg\neg a = a$$

valid rules

$$\frac{a \models b \quad b \models c}{a \models c}$$

$$\frac{a \models b \quad b \models \neg c}{a \models \neg c}$$

$$\frac{a \models b}{\neg b \models \neg a} \qquad \frac{a \models \neg b}{b \models \neg a} \qquad \frac{\neg a \models b}{\neg b \models a}$$

contraposition