UNIVERSITY OF EDINBURGH

COLLEGE OF SCIENCE AND ENGINEERING

SCHOOL OF INFORMATICS

INFORMATICS 1 — COMPUTATION & LOGIC

Saturday 1 st April 2017

00:00 to 00:00

INSTRUCTIONS TO CANDIDATES

This is a take-home exercise. It will not be marked anonymously. The examination, which will be marked anonymously, will have a similar format.

Bring your completed script with you to class on Thursday 24th November. You will mark your paper in class and then submit it for feedback on any outstanding queries.

In addition to answering these questions you may find it useful to create your own variations on these examples.

Please give your student number and tutorial group below.

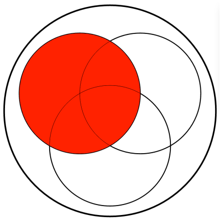
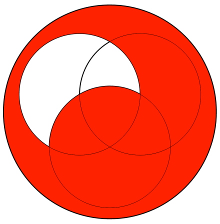
Student ID:

Tutorial group:

THIS EXAMINATION WILL BE MARKED ANONYMOUSLY

This is a take-home exercise. The examination will have a similar format.
Bring your completed script with you to class on Thursday 24th November.

1. (a) The entailment $B \vdash A \to C$, is invalid.

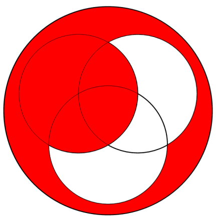
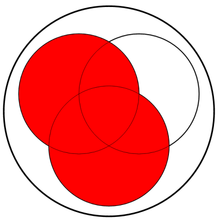
B	$A \to C$	$A \vdash B \to C$
		Valid ☐ Invalid ☒

How is this invalidity shown by comparing the two Venn diagrams above? [*4 marks*]

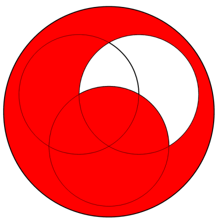
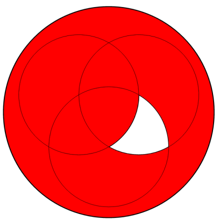
(b) For each of the following entailments, complete the two Venn diagrams to represent the assumption and conclusion, and place a mark in one of the check boxes provided to indicate whether the entaiment is valid.

(You should use the same encoding as in the example above, where each circle represents one of the propositions A, B, C.)

i. $(B \lor C) \to A \vdash \neg C \to A$ [*8 marks*]

$(B \lor C) \to A$	$\neg C \to A$	$(B \lor C) \to A \vdash \neg C \to A$
		Valid ☐ Invalid ☐

ii. $(B \to C) \vdash (B \land C) \to A$ [*8 marks*]

$B \to C$	$(B \land C) \to A$	$(B \to C) \vdash (B \land C) \to A$
		Valid ☐ Invalid ☐

2. This question concerns the 256 possible truth valuations of the following eight propositional letters A, B, C, D, E, F, G, H. For each of the following expressions, say how many of the 256 valuations satisfy the expression, and briefly explain your reasoning. For example, the expression D is satisfied by half of the valuations, that is 128 of the 256, since for each valuation that makes D true there is a matching valuation that make D false.

(a) $A \wedge B$ — Answer [1 mark]

(b) $(A \vee B) \wedge C$ — Answer [2 marks]

(c) $(A \to B) \to C$ — Answer [2 marks]

(d) $(A \to B) \wedge (B \to A) \wedge (C \to D) \wedge (D \to F) \wedge (E \to F) \wedge (F \to G) \wedge (G \to H)$ [5 marks]

Answer

(e) $(A \to B) \wedge (B \to C) \wedge (C \to D) \wedge (D \to C) \wedge (E \to F) \wedge (F \to G) \wedge (F \to H)$ [5 marks]

Answer

(f) $(H \to D) \wedge (A \to B) \wedge (B \to D) \wedge (C \to E) \wedge (D \to F) \wedge (F \to (H \wedge G)$ [5 marks]

Answer

3. You are given the following inference rules: (Γ, Δ vary over finite sets of expressions; A, B vary over expressions):

$$\frac{}{\Gamma, A \vdash \Delta, A}\ (I)$$

$$\frac{\Gamma, A, B \vdash \Delta}{\Gamma, A \wedge B \vdash \Delta}\ (\wedge L) \qquad \frac{\Gamma \vdash A, B, \Delta}{\Gamma \vdash A \vee B, \Delta}\ (\vee R)$$

$$\frac{\Gamma, A \vdash \Delta \quad \Gamma, B \vdash \Delta}{\Gamma, A \vee B \vdash \Delta}\ (\vee L) \qquad \frac{\Gamma \vdash A, \Delta \quad \Gamma \vdash B, \Delta}{\Gamma \vdash A \wedge B, \Delta}\ (\wedge R)$$

$$\frac{\Gamma \vdash A, \Delta \quad \Gamma, B \vdash \Delta}{\Gamma, A \to B \vdash \Delta}\ (\to L) \qquad \frac{\Gamma, A \vdash B, \Delta}{\Gamma \vdash A \to B, \Delta}\ (\to R)$$

$$\frac{\Gamma \vdash A, \Delta}{\Gamma, \neg A \vdash \Delta}\ (\neg L) \qquad \frac{\Gamma, A \vdash \Delta}{\Gamma \vdash \neg A, \Delta}\ (\neg R)$$

(Where A and B are propositional expressions, Γ, Δ are sets of expressions, and Γ, A refers to $\Gamma \cup \{A\}$.) This question concerns the use of these rules to prove the following entailment. This is your **goal**.

$$S \to \neg(P \wedge Q),\ \neg R \vee (Q \to P) \vdash Q \to \neg(R \wedge S) \tag{1}$$

(a) Which of these rules have a conclusion matching the goal (1)?
For each such rule complete a line in the table below showing the name of the rule and the expressions matched with Γ, Δ, A, B [*10 marks*]

Rule	Γ	Δ	A	B

(b) Use the rules given to construct a formal proof with the goal as conclusion, making any remaining assumptions as simple as possible.
Label each step in your proof with the name of the rule being applied. [*10 marks*]

$$\overline{}\quad S \to \neg(P \land Q),\ \neg R \lor (Q \to P) \vdash Q \to \neg(R \land S)$$

4. (a) What is a counterexample to an entailment? *[2 marks]*

Answer

(b) How can resolution be used to determine whether a clausal form is satisfiable? *[2 marks]*

Answer

(c) Use resolution either to show this entailment is valid or to produce a counterexample: $P \to (Q \to R) \vdash (P \to Q) \to (P \to R)$

P Q R

Answer

Counterexample?

[2 marks]

(d) Use resolution either to show this entailment is valid or to produce a counterexample: $P \to (R \to S), Q \to \neg(R \wedge S) \to (P \vee \neg B) \vdash (R \vee S) \to \neg(P \vee \neg Q)$

P Q R S

Answer

Counterexample?

[2 marks]

(e) The resolution procedure is sound and complete. What would it mean to say the resolution procedure was

i. **not sound** *[2 marks]*

Answer

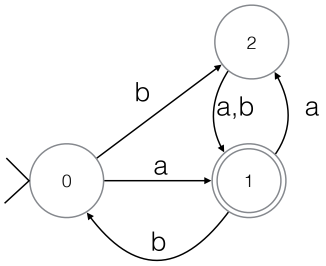
ii. **not complete**. *[2 marks]*

Answer

5. Each diagram shows an FSM. In each case give a regular expression for the language accepted by the FSM, make a mark in the check box against each string that it accepts (and no mark against those strings it does not accept), make a mark in the DFA check box if it is deterministic, and draw an equivalent DFA if it is not.

[4 marks]

(a)



`aab` ☐

`aba` ☐

`bab` ☐

`aaa` ☐

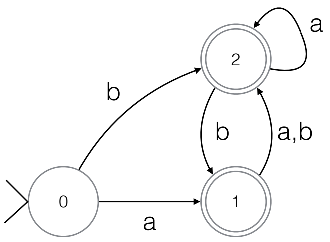
`bbb` ☐

DFA ☐

`regex:`

[4 marks]

(b)



`aab` ☐

`aba` ☐

`bab` ☐

`aaa` ☐

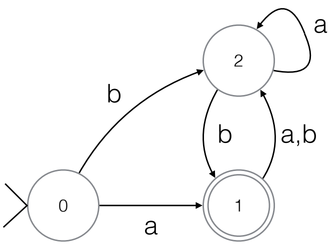
`bbb` ☐

DFA ☐

`regex:`

[4 marks]

(c)



`aab` ☐

`aba` ☐

`bab` ☐

`aaa` ☐

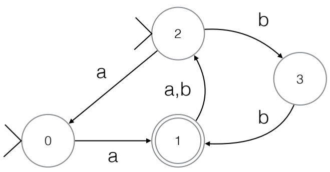
`bbb` ☐

DFA ☐

`regex:`

[4 marks]

(d)



`aab` ☐

`aba` ☐

`bab` ☐

`aaa` ☐

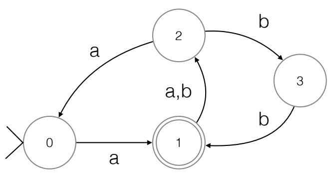
`bbb` ☐

DFA ☐

`regex:`

[4 marks]

(e)



`aab` ☐

`aba` ☐

`bab` ☐

`aaa` ☐

`bbb` ☐

DFA ☐

`regex:`