Informatics 1 - Computation & Logic: Tutorial 2

Propositional Logic: Truth Tables

Week 4: 12-18 October 2014

Please attempt the entire worksheet in advance of the tutorial, and bring with you all work, including (if a computer is involved) printouts of code and test results. Tutorials cannot function properly unless you do the work in advance.

You may work with others, but you must understand the work; you can't phone a friend during the exam.

Assessment is formative, meaning that marks from coursework do not contribute to the final mark. But coursework is not optional. If you do not do the coursework you are unlikely to pass the exams.

Attendance at tutorials is **obligatory**; please let your tutor know if you cannot attend.

1. Look up the following terms, on the internet or elsewhere, and describe, in words, when an expression in propositional logic is:

   (a) Satisfiable:

   (b) Tautologous:

(c) Contingent:

(d) Inconsistent:

2. Which combinations of these four properties, SATISFIABLE/TAUTOLOGOUS/CONTINGENT/INCONSISTENT, are possible?

   Your answer should be a boolean formula, using the propositional letters S, T, C, I (with the obvious interpretations), that characterises the possible combinations of these properties.

3. Construct truth tables for the following expressions of propositional logic, and use these to decide whether the expressions are satisfiable, tautologous, contingent, or inconsistent:

   (a) $(A \to B) \lor (\neg A \lor \neg B)$

   Draw the truth table here:

   This expression is SATISFIABLE/TAUTOLOGOUS/CONTINGENT/INCONSISTENT

(b) $\neg(A \wedge \neg B) \leftrightarrow \neg(\neg A \vee B)$

Draw the truth table here:

This expression is
SATISFIABLE/TAUTOLOGOUS/CONTINGENT/INCONSISTENT

(c) $A \rightarrow (B \wedge (A \vee B))$

Draw the truth table here:

This expression is
SATISFIABLE/TAUTOLOGOUS/CONTINGENT/INCONSISTENT

(d) $(\neg A \wedge B) \vee C \leftrightarrow ((A \vee \neg B) \rightarrow C)$

Draw the truth table here:

This expression is
SATISFIABLE/TAUTOLOGOUS/CONTINGENT/INCONSISTENT

4. This question concens the same expressions of propositional logic as the previous exercise. In each case, use the laws of Boolean Algebra to derive an equivalent CNF for the given expression. Eliminate arrows; push negations down; push disjunctions down; and simplify, using commutativity and associativity of $\vee, \wedge$, together with the following rules:

$$\neg(a \to b) = a \wedge \neg b \qquad a \leftrightarrow b = (a \to b) \wedge (b \to a) \qquad a \to b = \neg a \vee b$$

$$\neg(a \vee b) = \neg a \wedge \neg b \qquad \neg(a \wedge b) = \neg a \vee \neg b$$

$$\neg 0 = 1 \qquad \neg\neg a = a \qquad \neg 1 = 0$$

$$a \vee 1 = 1 \qquad a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c) \qquad a \wedge 0 = 0$$

$$a \vee 0 = a \qquad a \vee \neg a = 1 \qquad a \wedge \neg a = 0 \qquad a \wedge 1 = a$$

(a) $(A \to B) \vee (\neg A \vee \neg B)$

Show your working here:

This expression is
SATISFIABLE/TAUTOLOGOUS/CONTINGENT/INCONSISTENT

(b) $\neg(A \wedge \neg B) \leftrightarrow \neg(\neg A \vee B)$

Show your working here:

This expression is
SATISFIABLE/TAUTOLOGOUS/CONTINGENT/INCONSISTENT

(c) $A \to (B \land (A \lor B))$

Show your working here:

This expression is
SATISFIABLE/TAUTOLOGOUS/CONTINGENT/INCONSISTENT

(d) $(\neg A \land B) \lor C \leftrightarrow ((A \lor \neg B) \to C)$

Show your working here:

This expression is
SATISFIABLE/TAUTOLOGOUS/CONTINGENT/INCONSISTENT

5. (a) How many rows will a truth table for an expression in propositional logic with n atomic propositions have? Why?

(b) In general, is this a limitation? Why?

6. An *entailment* of propositional logic is of the form

$$\phi_1, \ldots, \phi_n \models \psi$$

where ϕ_i, ψ are all expressions of propositional logic. The ϕ_i expressions are the *premises* of the entailment and ψ is the *conclusion*. An entailment is *valid* if and only if there is no possible assignment of truth values to atomic propositional symbols such that the premises are all true and the conclusion false.

Using a truth table, determine whether the following entailments are valid or invalid:

(a) $(A \wedge B) \rightarrow A,\ B \vee \neg A \ \models\ A \vee B$

This entailment is VALID/INVALID

(b) $\neg A \vee (B \to C),\ B \wedge C,\ C \to A \ \models\ A$

This entailment is VALID/INVALID

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Summary of useful symbols

Capital	Lowercase	Name
A	α	alpha
B	β	beta
Γ	γ	gamma
Δ	δ	delta
E	ϵ	epsilon
Z	ζ	zeta
H	η	eta
Θ	θ	theta
I	ι	iota
K	κ	kappa
Λ	λ	lambda
M	μ	mu
N	ν	nu
Ξ	ξ	xi
Π	π	pi
P	ρ	rho
Σ	σ	sigma
T	τ	tau
Υ	υ	upsilon
Φ	ϕ	phi
X	χ	chi
Ψ	ψ	psi
Ω	ω	omega

Symbol	Meaning	Example
$\neg$	not	$\neg A$
$\wedge$	and	$A \wedge B$
$\vee$	or	$A \vee B$
$\rightarrow$	implies	$A \rightarrow B$
$\models$	entails	$[\beta_1, ..., \beta_n] \models \alpha$
$\leftrightarrow$	equivalent	$A \leftrightarrow B$
$\vdash$	can be proved	$[\beta_1, ..., \beta_n] \vdash \alpha$