

Tautology

Satisfied for all valuations of the atoms

Satisfiable

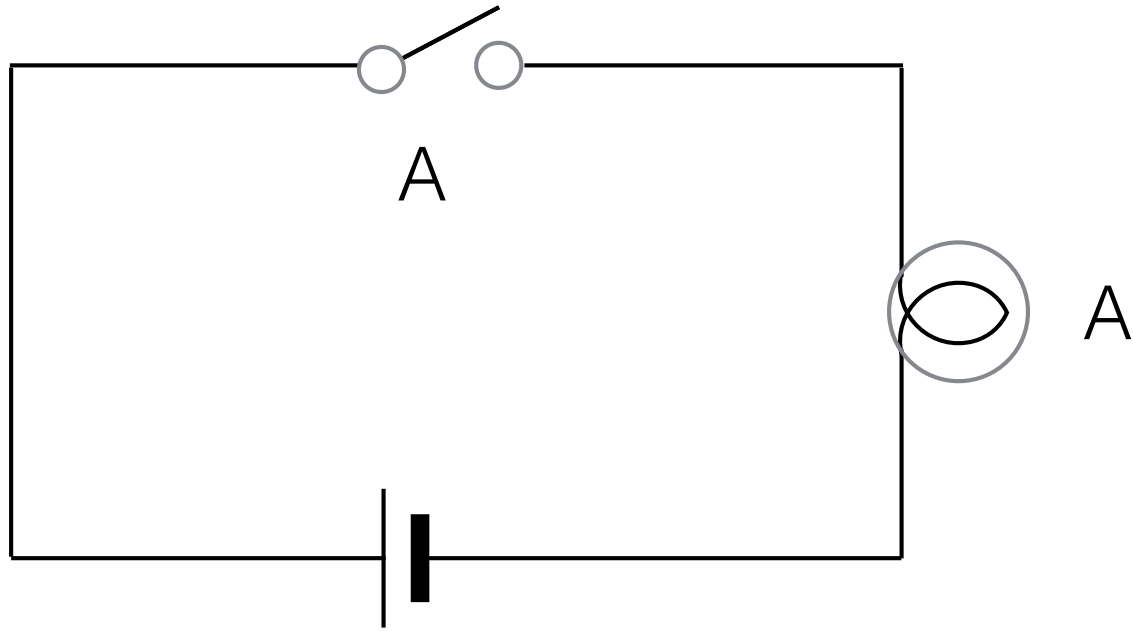
Satisfied for some valuations of the atoms

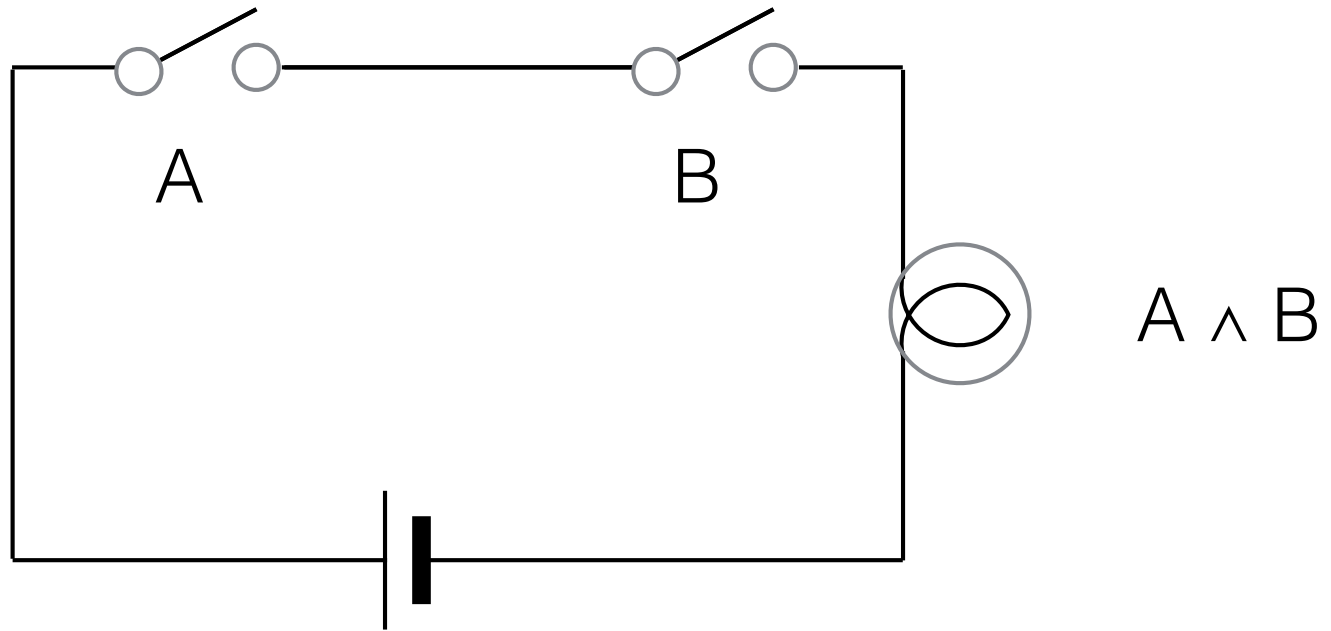
Contingent

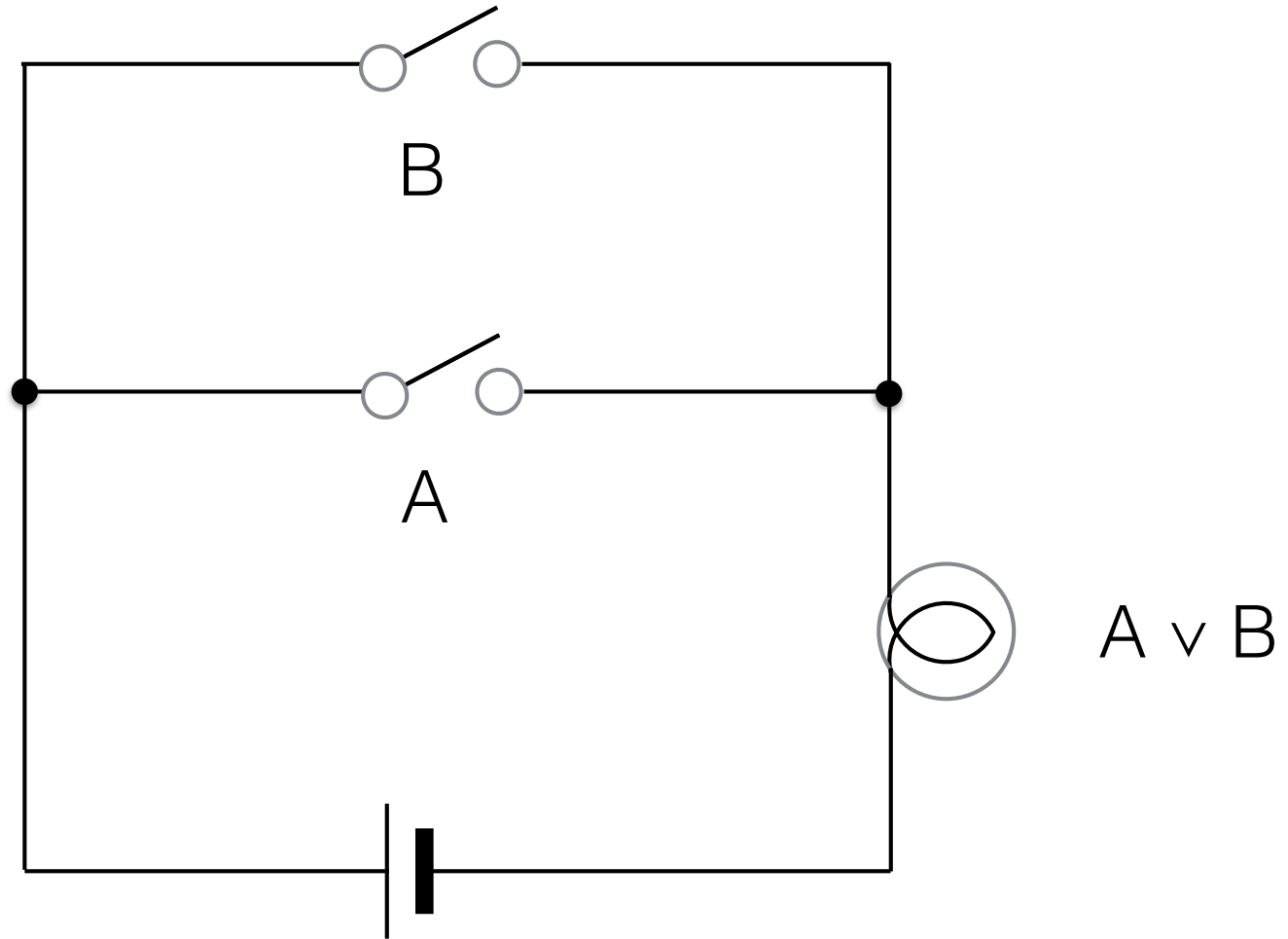
Satisfied for some but not all valuations of the atoms

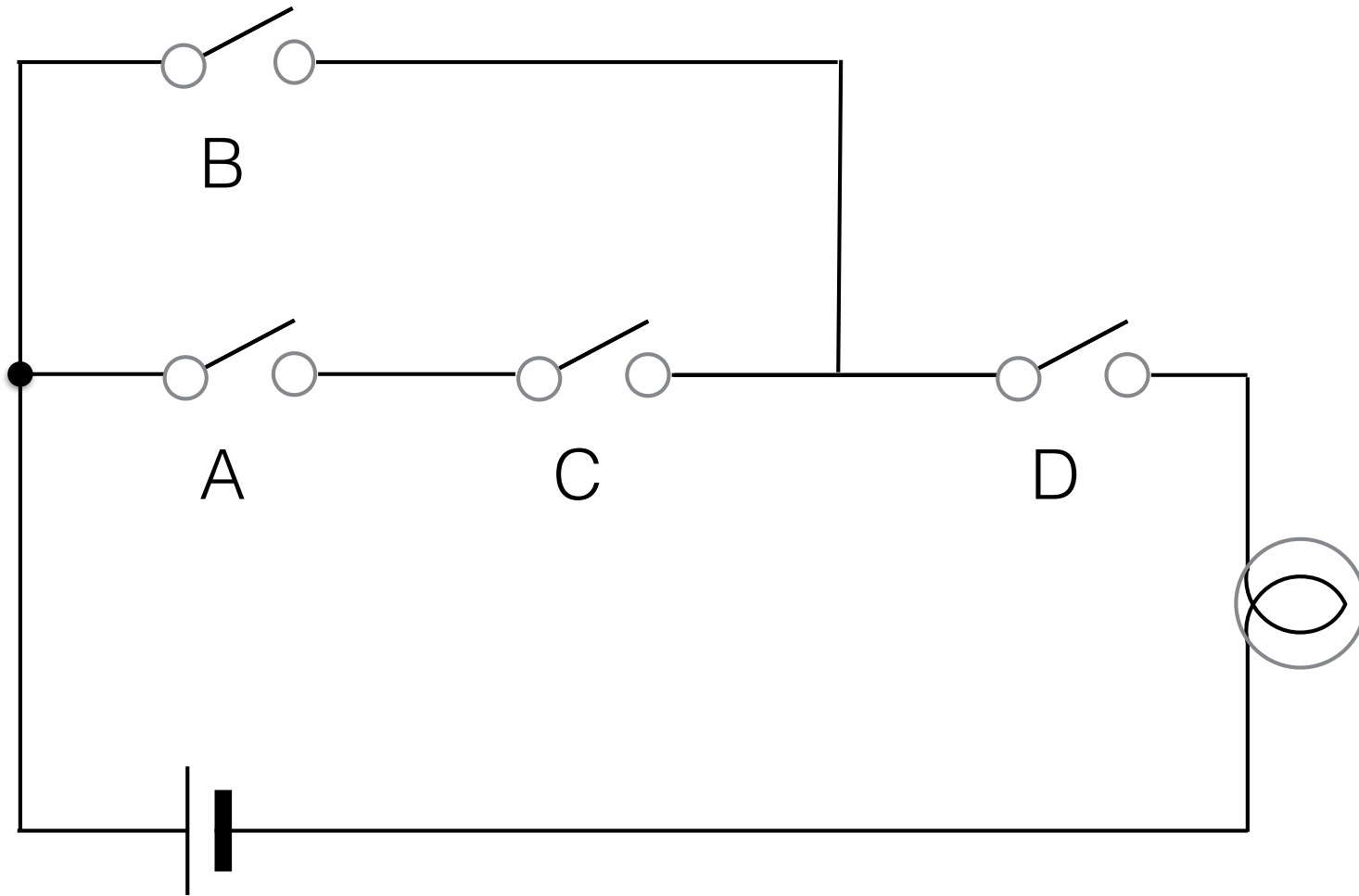
Contradiction

Satisfied for no valuations of the atoms

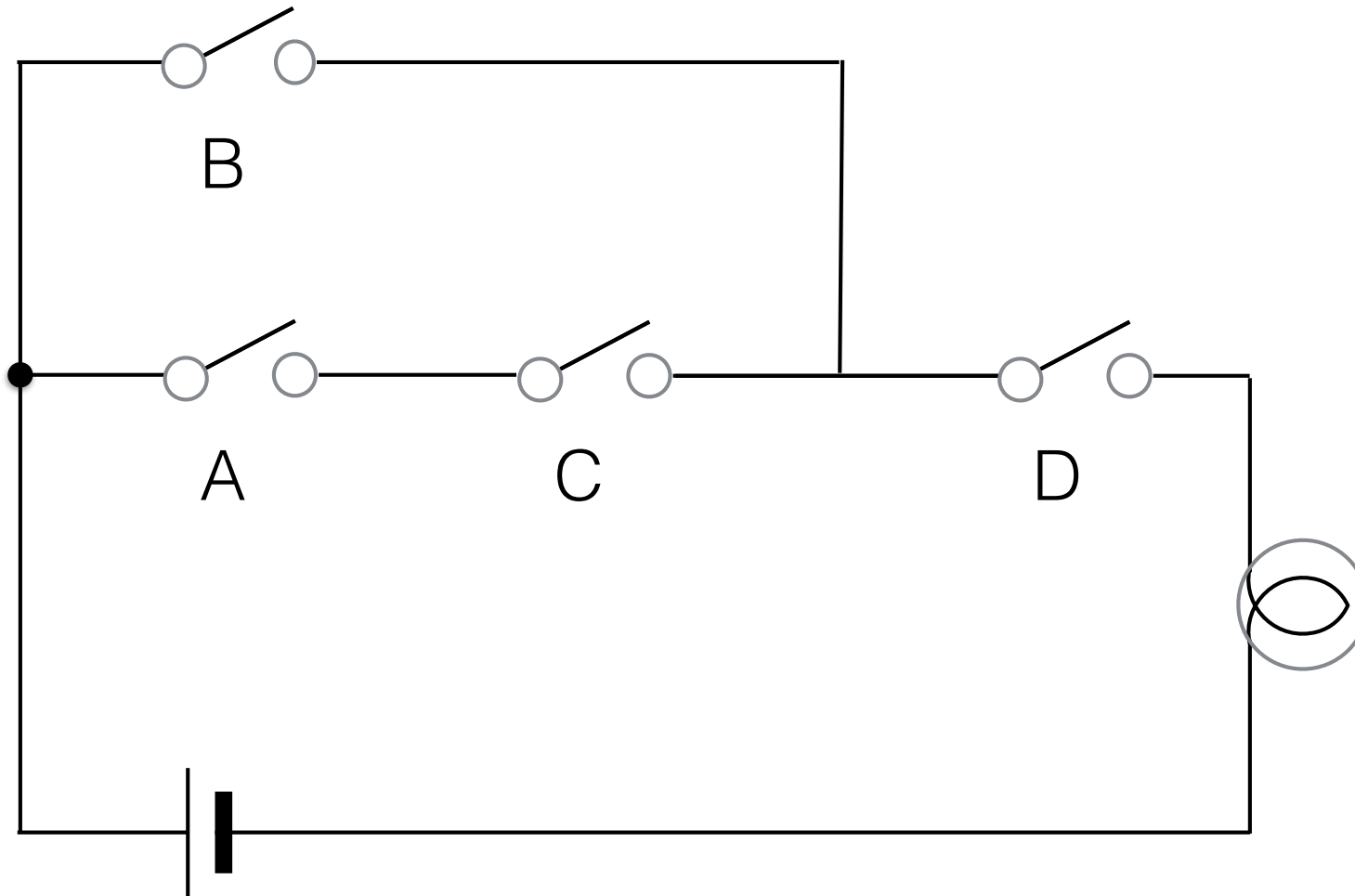


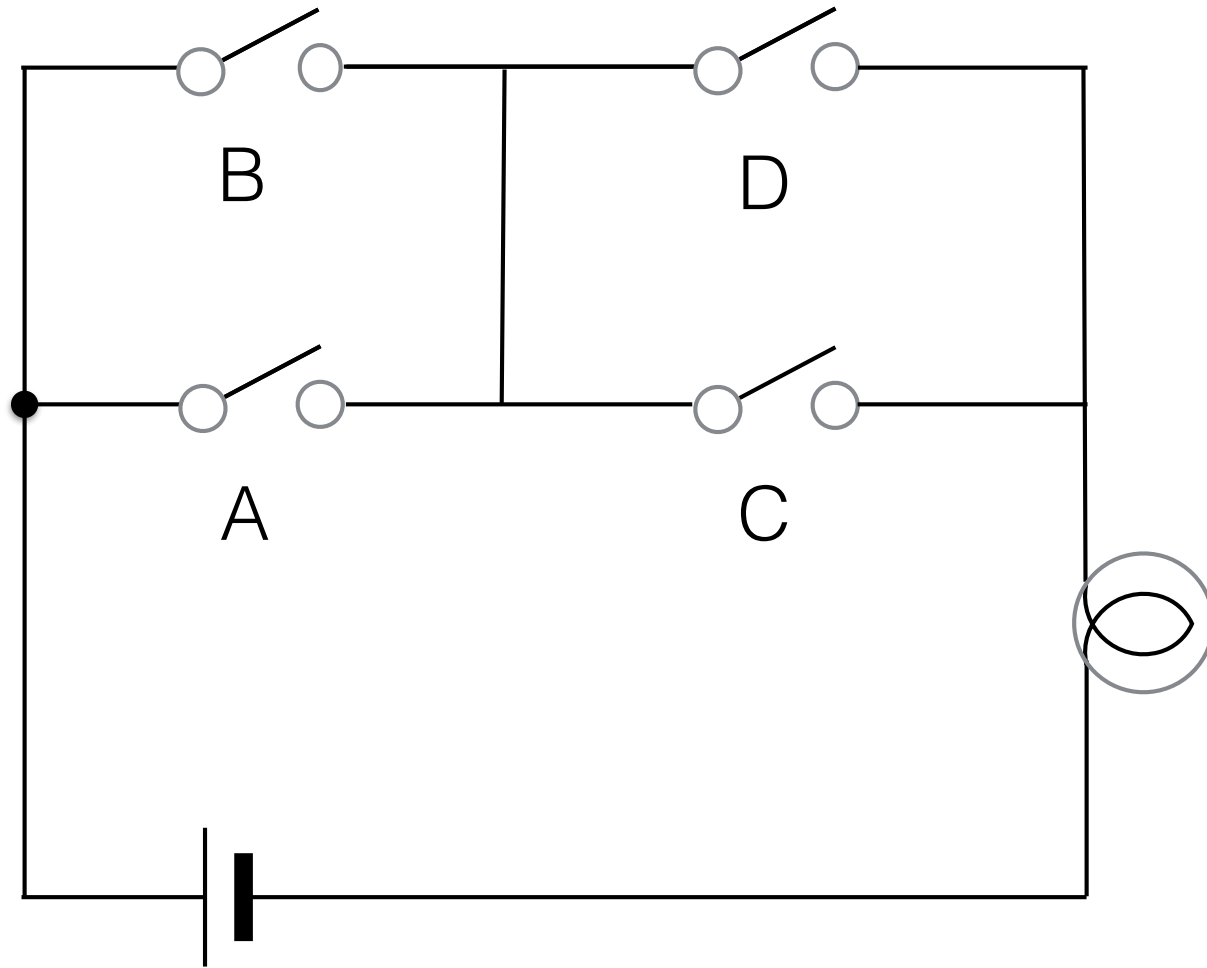






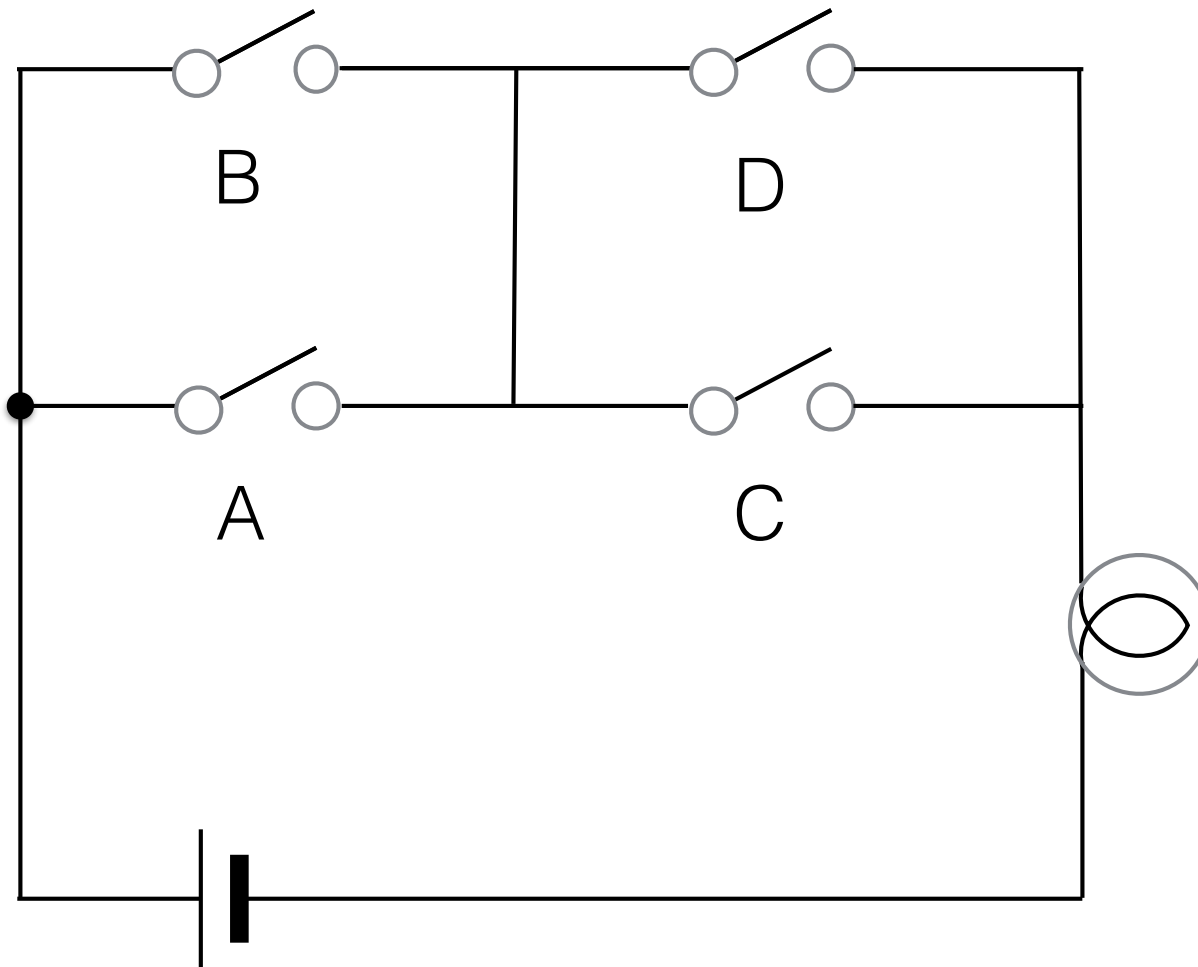
$$(B \vee (A \wedge C)) \wedge D$$





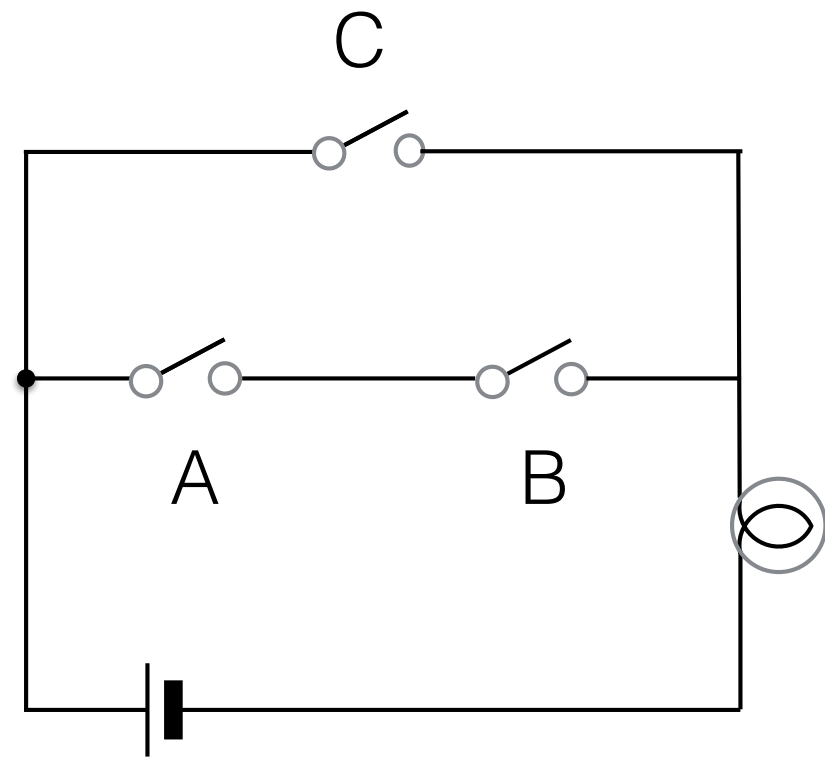
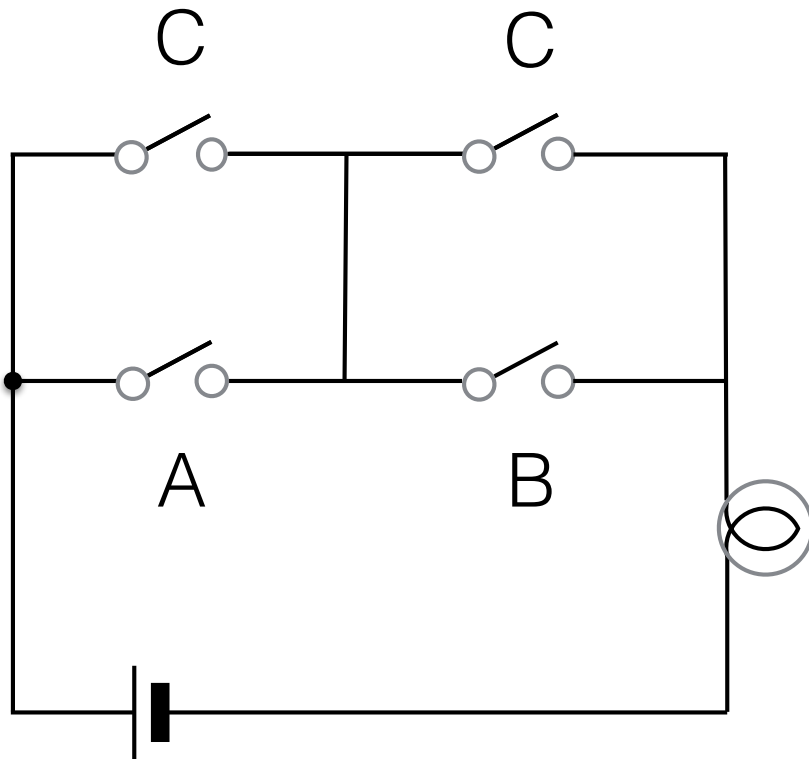
$$(A \wedge C) \vee (B \wedge D) \vee (B \wedge C) \vee (A \wedge D)$$

$$= (A \vee B) \wedge (C \vee D)$$



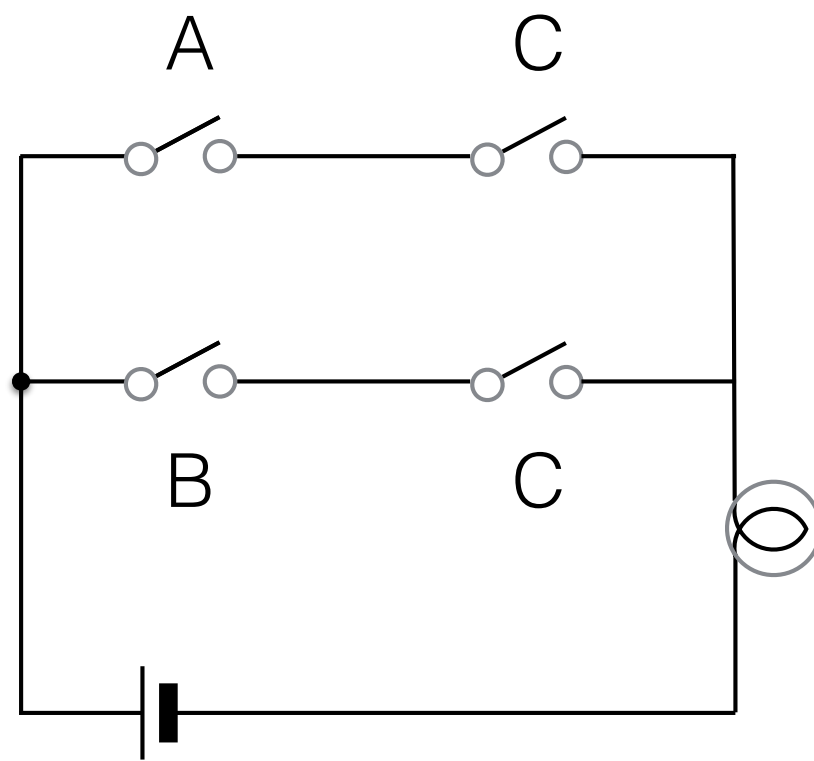
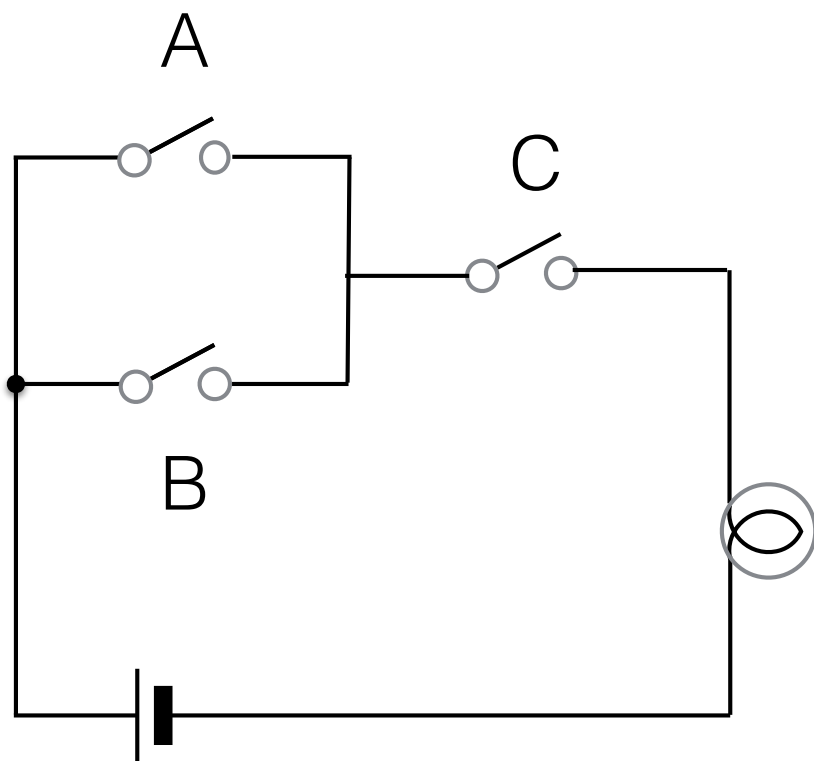
$$(A \vee C) \wedge (B \vee C)$$

$$= (A \wedge B) \vee C$$



$$(A \vee B) \wedge C$$

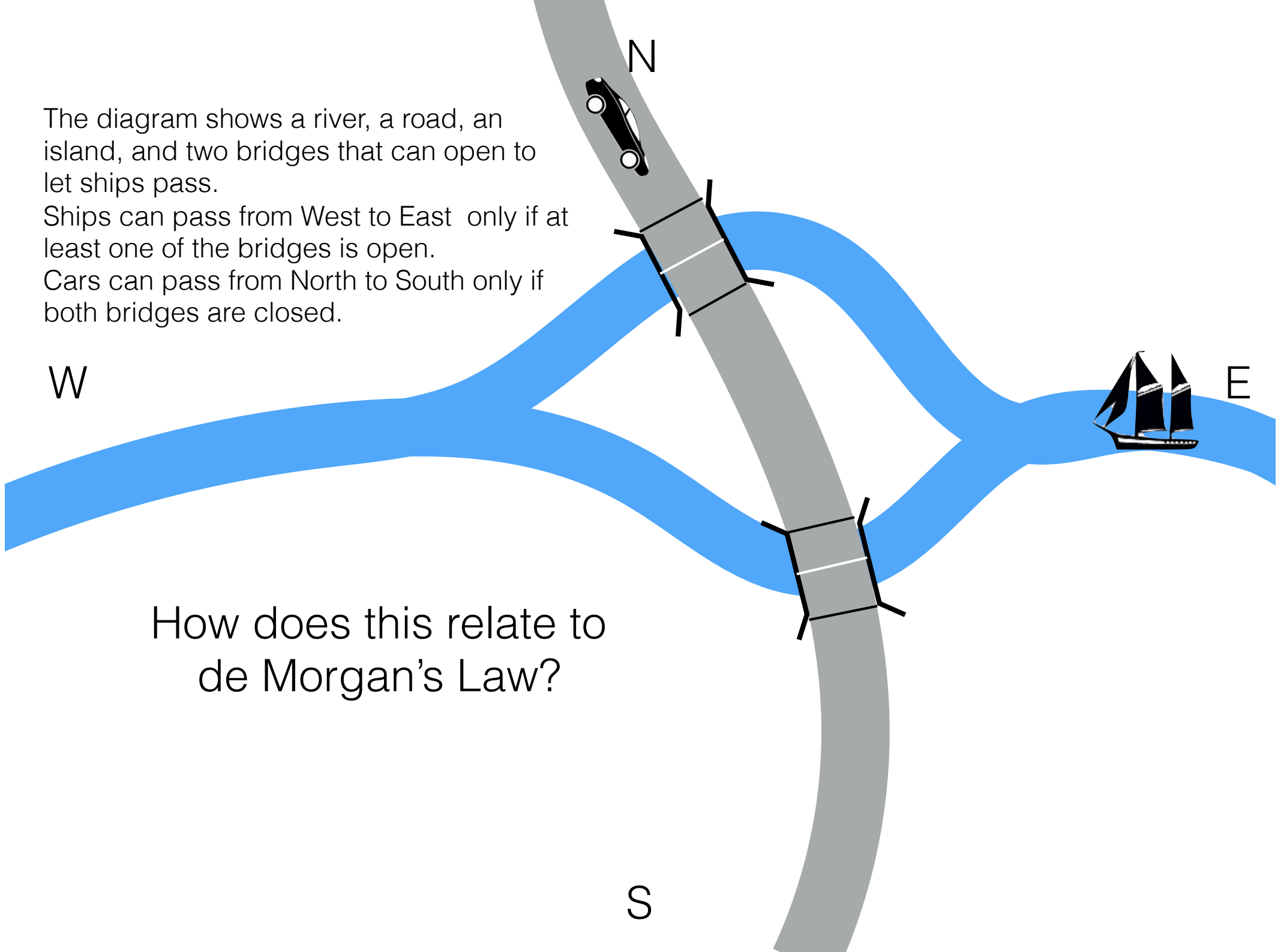
$$= (A \wedge B) \vee C$$



The diagram shows a river, a road, an island, and two bridges that can open to let ships pass.

Ships can pass from West to East only if at least one of the bridges is open.

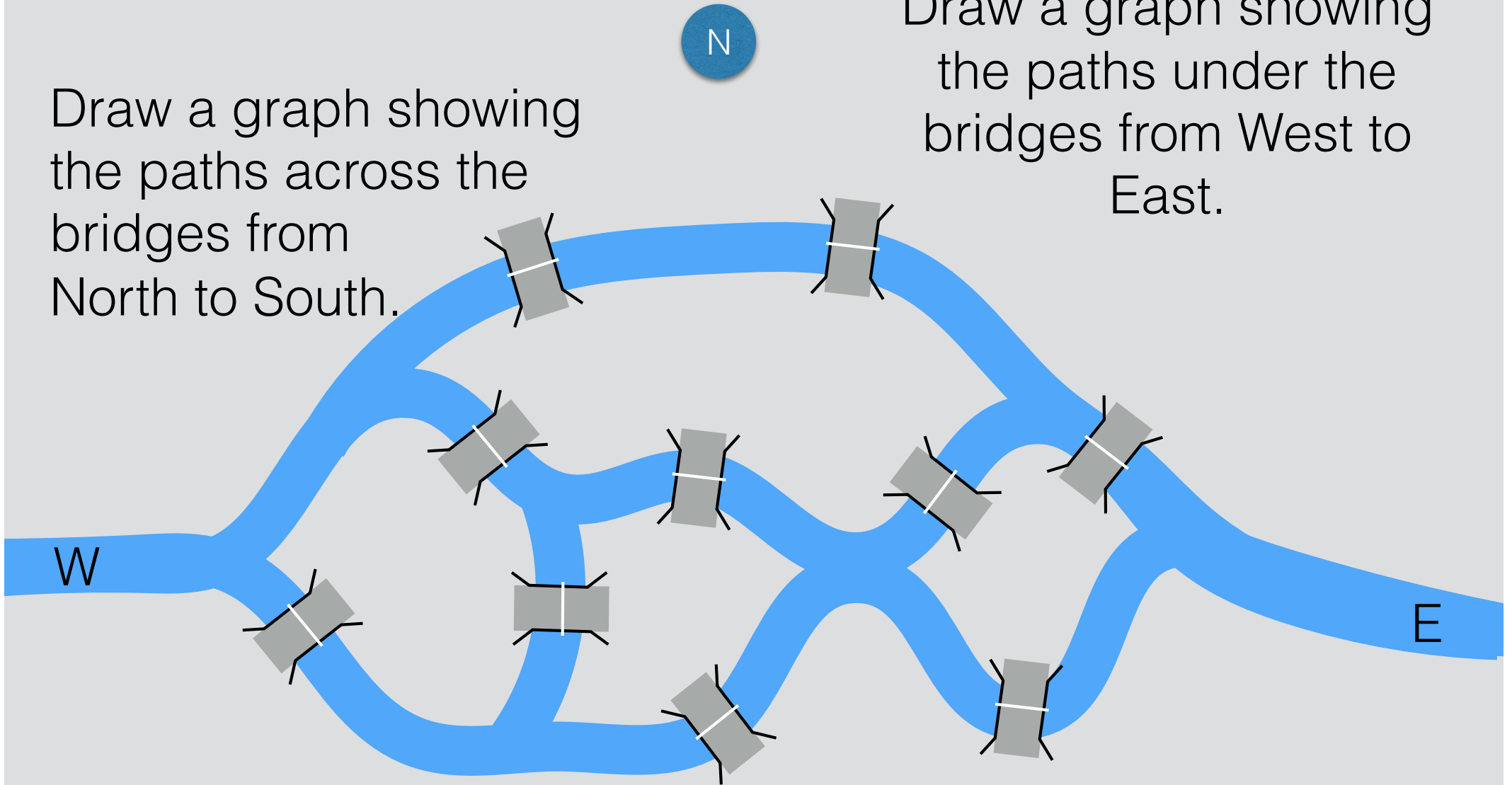
Cars can pass from North to South only if both bridges are closed.



How does this relate to de Morgan's Law?

Draw a graph showing the paths across the bridges from North to South.

Draw a graph showing the paths under the bridges from West to East.

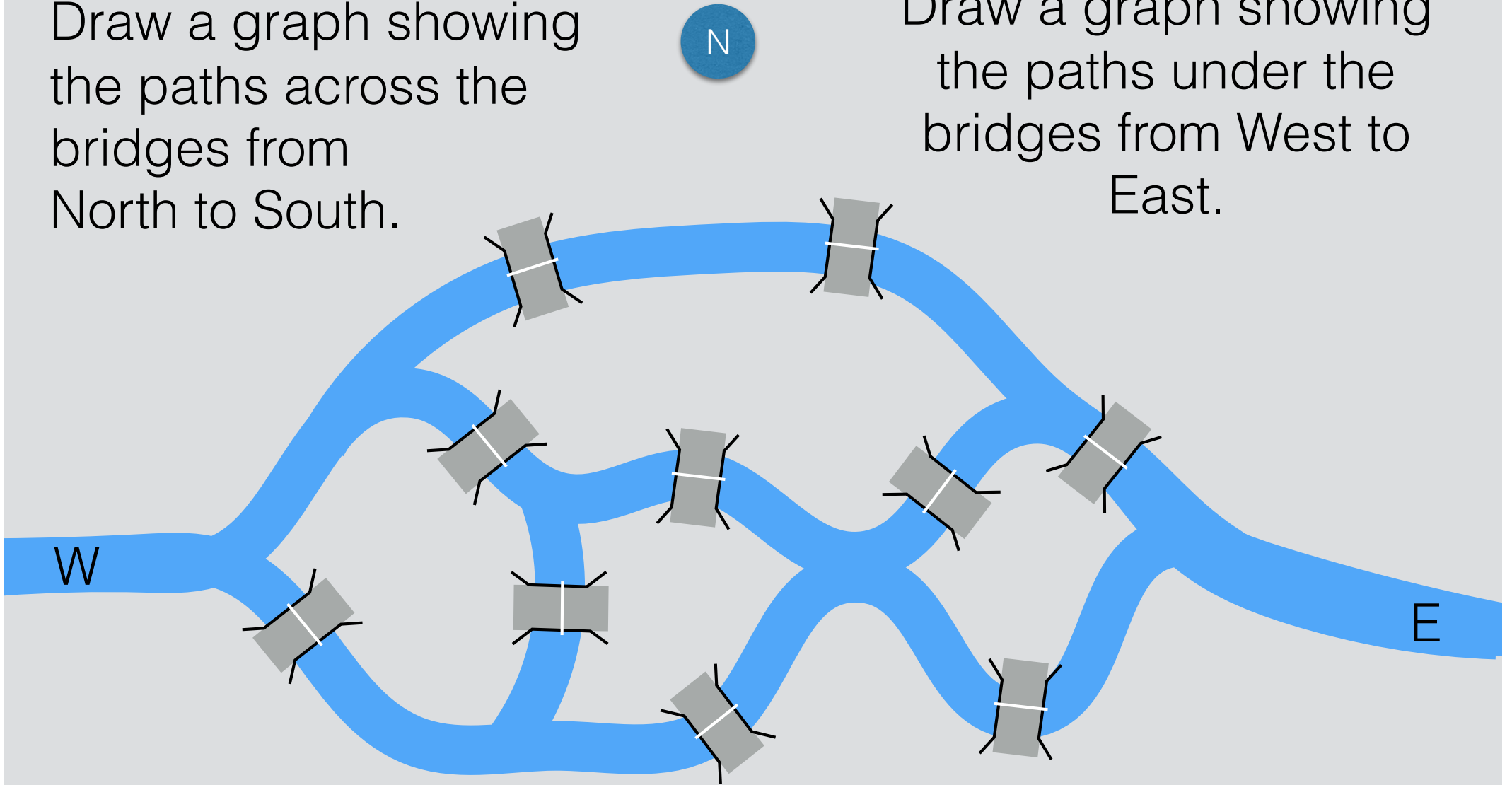


In each case, the bridges correspond to edges of the graph.

What is the logical relationship between the two graphs?

Draw a graph showing the paths across the bridges from North to South.

Draw a graph showing the paths under the bridges from West to East.



In each case, the bridges correspond to edges of the graph.

What is the logical relationship between the two graphs?

To produce conjunctive normal form
(CNF)

eliminate \leftrightarrow \rightarrow
push negations in
push \vee inside \wedge

$$\neg(a \rightarrow b) = a \wedge \neg b$$

$$a \leftrightarrow b = (a \rightarrow b) \wedge (b \rightarrow a)$$

$$a \rightarrow b = \neg a \vee b$$

$$\neg(a \vee b) = \neg a \wedge \neg b$$

$$\neg(a \wedge b) = \neg a \vee \neg b$$

$$\neg 0 = 1$$

$$\neg\neg a = a$$

$$\neg 1 = 0$$

$$a \vee 1 = 1$$

$$a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$$

$$a \wedge 0 = 0$$

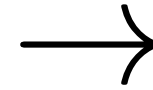
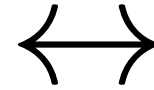
$$a \vee 0 = a$$

$$a \vee \neg a = 1$$

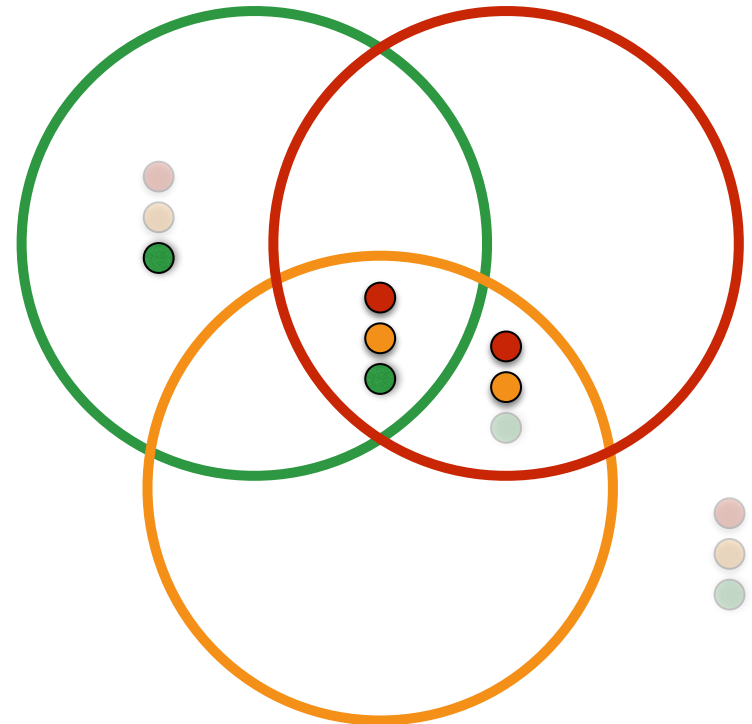
$$a \wedge \neg a = 0$$

$$a \wedge 1 = a$$

eliminate



$$\boxed{R \leftrightarrow A} = (R \rightarrow A) \wedge (A \rightarrow R)$$
$$= \boxed{(\neg R \vee A) \wedge (\neg A \vee R)}$$



eliminate \leftrightarrow \rightarrow

$$\begin{aligned} R \leftrightarrow A &= (R \rightarrow A) \wedge (A \rightarrow R) \\ &= (\neg R \vee A) \wedge (\neg A \vee R) \end{aligned}$$

$$\begin{aligned} G \leftrightarrow (R \leftrightarrow A) &= (\neg G \vee ((\neg R \vee A) \wedge (\neg A \vee R))) \\ &\wedge \\ &(\neg((\neg R \vee A) \wedge (\neg A \vee R)) \vee G) \end{aligned}$$

push negations in

$$\begin{aligned} G &\leftrightarrow (R \leftrightarrow A) \\ &= (\neg G \vee ((\neg R \vee A) \wedge (\neg A \vee R))) \\ &\quad \wedge \\ &\quad (\neg((\neg R \vee A) \wedge (\neg A \vee R)) \vee G) \\ &= (\neg G \vee ((\neg R \vee A) \wedge (\neg A \vee R))) \\ &\quad \wedge \\ &\quad ((\neg(\neg R \vee A) \vee \neg(\neg A \vee R)) \vee G) \end{aligned}$$

push negations in

$$\begin{aligned} G &\leftrightarrow (R \leftrightarrow A) \\ &= (\neg G \vee ((\neg R \vee A) \wedge (\neg A \vee R))) \\ &\quad \wedge \\ &\quad (\neg((\neg R \vee A) \wedge (\neg A \vee R)) \vee G) \\ &= (\neg G \vee ((\neg R \vee A) \wedge (\neg A \vee R))) \\ &\quad \wedge \\ &\quad ((\neg(\neg R \vee A) \vee \neg(\neg A \vee R)) \vee G) \\ &= (\neg G \vee ((\neg R \vee A) \wedge (\neg A \vee R))) \\ &\quad \wedge \\ &\quad (((R \wedge \neg A) \vee (A \wedge \neg R)) \vee G) \end{aligned}$$

push \vee inside \wedge

$$\begin{aligned} G &\leftrightarrow (R \leftrightarrow A) \\ &= (\neg G \vee ((\neg R \vee A) \wedge (\neg A \vee R))) \\ &\quad \wedge \\ &\quad (((R \wedge \neg A) \vee (A \wedge \neg R)) \vee G) \\ &= (((\neg G \vee \neg R \vee A) \wedge (\neg G \vee \neg A \vee R))) \\ &\quad \wedge \\ &\quad (((R \wedge \neg A) \vee (A \wedge \neg R)) \vee G) \end{aligned}$$

push \vee inside \wedge

$$\begin{aligned} G &\leftrightarrow (R \leftrightarrow A) \\ &= (\neg G \vee \neg R \vee A) \wedge (\neg G \vee \neg A \vee R) \\ &\quad \wedge \\ &\quad \left(((R \wedge \neg A) \vee (A \wedge \neg R)) \vee G \right) \\ &= (\neg G \vee \neg R \vee A) \wedge (\neg G \vee \neg A \vee R) \\ &\quad \wedge \\ &\quad \left(((R \vee A) \wedge (\neg A \vee A) \wedge (R \vee \neg R) \wedge (\neg A \vee \neg R)) \vee G \right) \end{aligned}$$

simplify

$$\neg A \vee A = \top$$

$$R \vee \neg R = \top$$

$$x \wedge \top = x$$

$$G \leftrightarrow (R \leftrightarrow A)$$

$$= (\neg G \vee \neg R \vee A) \wedge (\neg G \vee \neg A \vee R)$$

\wedge

$$\left(((R \vee A) \wedge (\neg A \vee A) \wedge (R \vee \neg R) \wedge (\neg A \vee \neg R)) \vee G \right)$$

$$= (\neg G \vee \neg R \vee A) \wedge (\neg G \vee \neg A \vee R)$$

\wedge

$$\left(((R \vee A) \wedge (\neg A \vee \neg R)) \vee G \right)$$

push \vee inside \wedge

$$\begin{aligned} G &\leftrightarrow (R \leftrightarrow A) \\ &= (\neg G \vee \neg R \vee A) \wedge (\neg G \vee \neg A \vee R) \\ &\quad \wedge \\ &\quad \boxed{(((R \vee A) \wedge (\neg A \vee \neg R)) \vee G)} \\ &= (\neg G \vee \neg R \vee A) \wedge (\neg G \vee \neg A \vee R) \\ &\quad \wedge \\ &\quad \boxed{(R \vee A \vee G) \wedge (\neg A \vee \neg R \vee G)} \end{aligned}$$

check!

$$G \leftrightarrow (R \leftrightarrow A) =$$



$$(\neg G \vee \neg R \vee A)$$



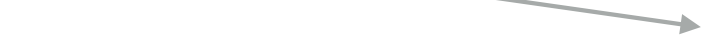
\wedge



$$(\neg G \vee \neg A \vee R)$$



\wedge



$$(R \vee A \vee G)$$



\wedge



$$(\neg A \vee \neg R \vee G)$$



$A \vee B \vee C \vee D$

$(A \vee B \vee L) \wedge (L = C \vee D)$

$L = C \vee D$

$(L \rightarrow C \vee D) \wedge (C \vee D \rightarrow L)$

$(\neg L \vee C \vee D) \wedge (\neg(C \vee D) \vee L)$

$(\neg L \vee C \vee D) \wedge ((\neg C \wedge \neg D) \vee L)$

$(\neg L \vee C \vee D) \wedge (\neg C \vee L) \wedge (\neg D \vee L)$

$A \vee B \vee C \vee D \equiv$

$(A \vee B \vee L) \wedge (\neg L \vee C \vee D) \wedge (\neg C \vee L) \wedge (\neg D \vee L)$