

Informatics 1

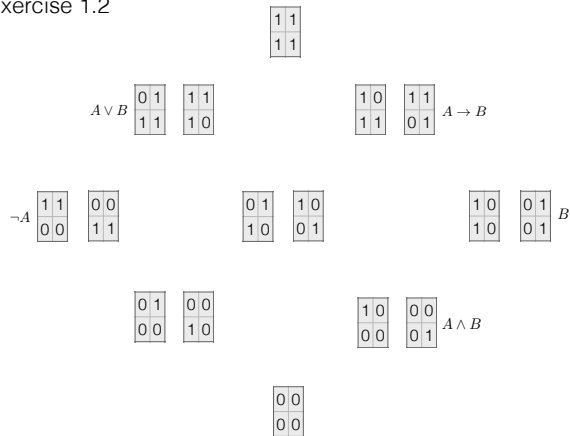
Computation and Logic

Entailment

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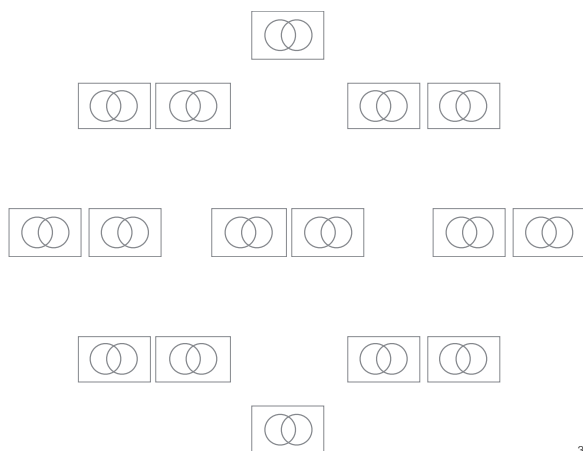
Exercise 1.2



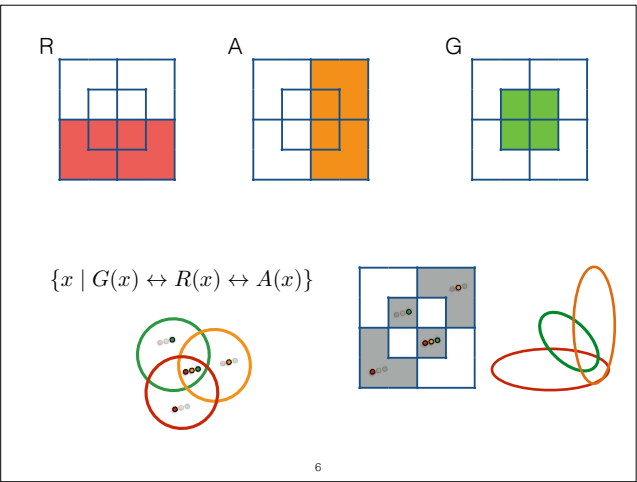
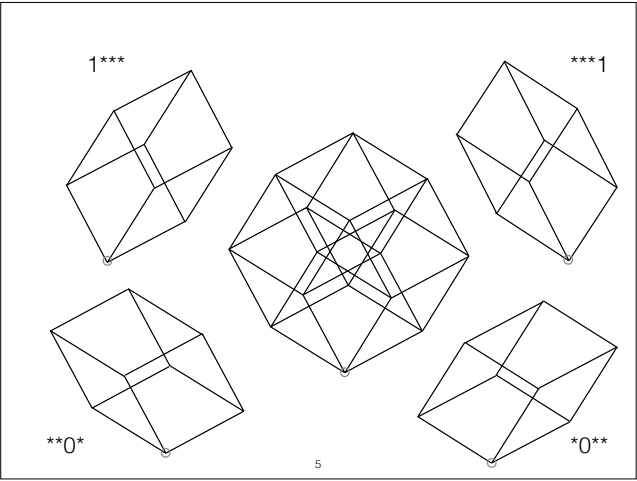
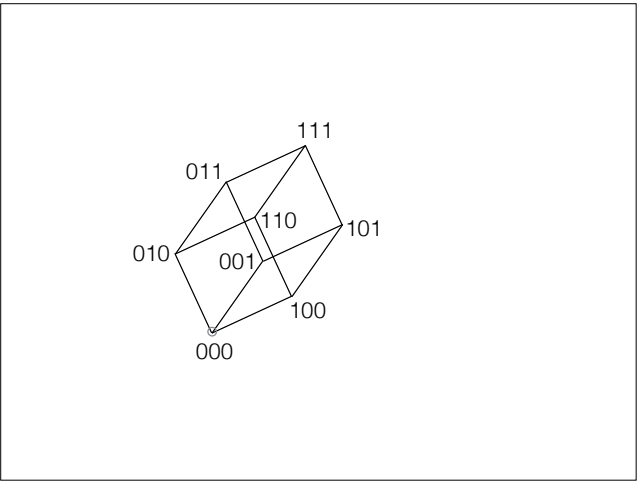
2

Each of the 16 2x2 tables above represents the truth table of a binary boolean operation.

Label each table with a boolean expression for which it is the truth table (five tables are already labelled – begin by checking whether these are consistent).



3

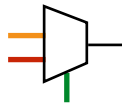
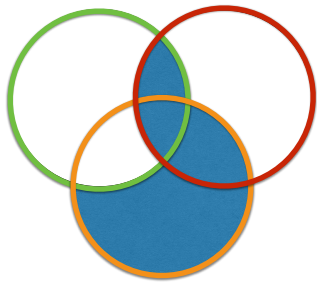




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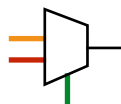
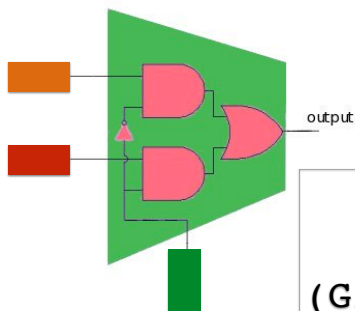
$(G?R:A)$

if G then R else A



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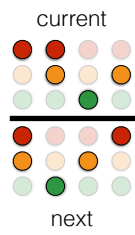
multiplexer – ITE



$$(G?R:A) = (G \wedge R) \vee (\neg G \wedge A)$$

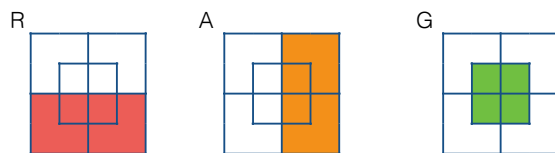
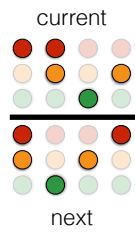
$(R? :)$

legal states

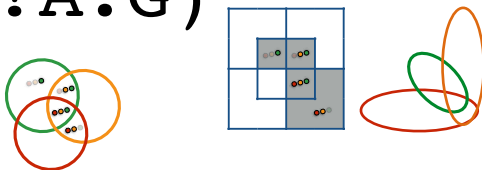


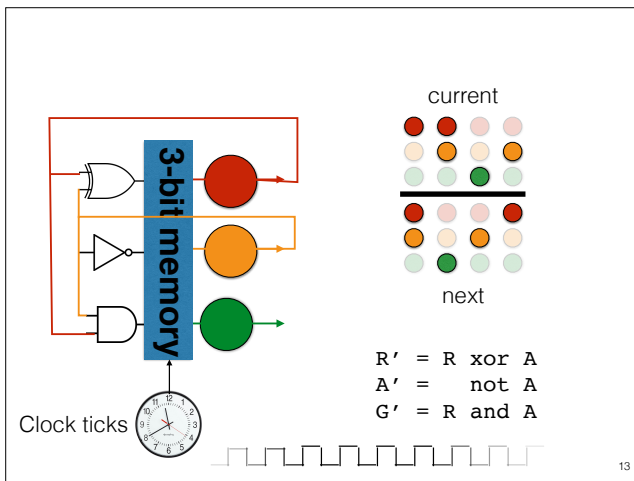
$(R? \neg G : A \oplus G)$

legal states



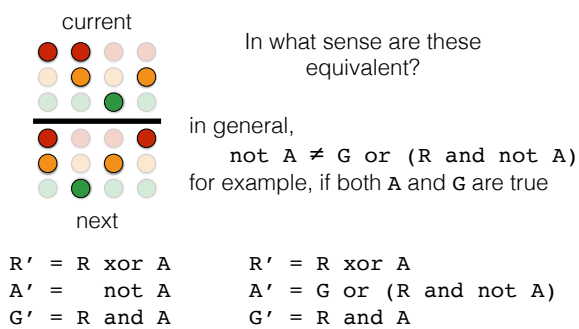
$(R?A : G)$





The next-state logic for sequencing our traffic lights can be implemented using three different gates. Many different technologies can be used to implement logic gates, some may use high and low voltages to represent binary values, others might use currents, but this logical

2 alternative implementations

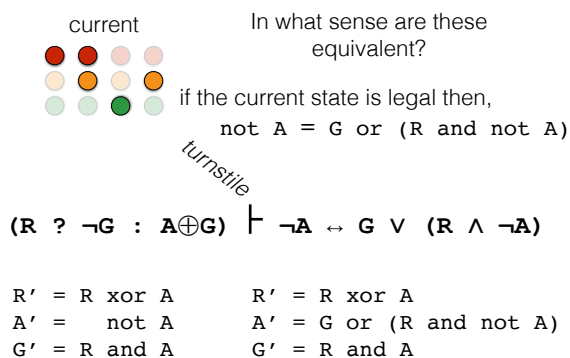


We have two implementations of the controller, with different next-state logic for the amber light, A.

$$A' = \text{not } A$$

$$A' = G \text{ or } (R \text{ and not } A).$$

2 alternative implementations



Slide 25 (lecture 1) shows an implementation of the traffic light controller.

We could have designed our logic differently.

For example, letting

$$A' = G \text{ or } (R \text{ and not } A).$$

Draw the circuit for this implementation.

Is this a correct

Entailment

In algebra, we consider expressions with variables, and write equations to express relationships between different expressions.

$$\text{LHS} = \text{RHS}$$

Boolean algebra, with equalities between expressions, gives us one way to express relationships between different logical expressions.

If we want to study logical arguments it is more natural to consider entailments.

$$\text{LHS} \vdash \text{RHS}$$

Entailment

If we want to study logical arguments it is more natural to consider entailments.

$$\text{LHS} \vdash \text{RHS}$$

The entailment is valid if any valuation that makes everything on the LHS true, makes the RHS true

$$\vdash \text{RHS}$$

an entailment with empty LHS is valid iff RHS is a **tautology**
i.e. every valuation makes it true

Is this a valid argument?

- Assumptions:
 - If I am clever then I will pass
 - If I will pass then I am clever,
 - Either I am clever or I will pass

- Conclusion:
 - I am clever and I will pass

$$\begin{array}{c} ? \\ C \rightarrow P, P \rightarrow C, C \vee P \vdash C \wedge P \end{array}$$

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Use the abbreviations C (I am clever) and P (I will pass). We argue as follows: The first two assumptions tell us that $C \leftrightarrow P$, so the truth value of C is equal to the truth value of P. The third assumption tells us that at least one of them is true, so both must be.

Is this a valid argument?

- Assumptions:
 - If the races are fixed or the gambling houses are crooked, then the tourist trade will decline.
 - If the tourist trade declines then the police force will be happy.
 - The police force is never happy.
- Conclusion:
 - The races are not fixed

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- Assumptions:
 - If the races are fixed or the gambling houses are crooked, then the tourist trade will decline.
 - If the tourist trade declines then the police force will be happy.
 - The police force is never happy.
 - Conclusion:
 - The races are not fixed
- a deduction
- $$\begin{array}{c}
 (RF \vee GC) \rightarrow TT \quad TT \rightarrow PH \quad \neg PH \\
 \hline
 \neg(TT) \\
 \hline
 \neg(RF \vee GC) \\
 \hline
 \neg RF \wedge \neg GC \\
 \hline
 \neg RF
 \end{array}$$
- the deduction is summarised in an entailment

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Here is a deduction, or proof. This sets out an argument, so that we can review how the steps fit together. Each line represents a simple step in the argument: we check that, if the propositions above the line are true then the conclusion below the line is true. The only propositions