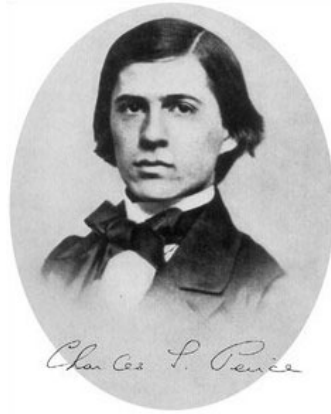


Karnaugh Maps



John Venn
1834—1923



Charles Peirce 1839—1914

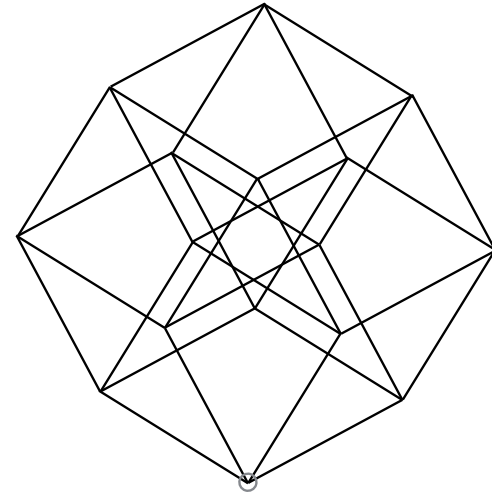


Allan Marquand (1853-1924)



Maurice Karnaugh
4 October 1924 —

inf1a-cl-6
Michael Fourman





Gottfried Wilhelm Leibniz

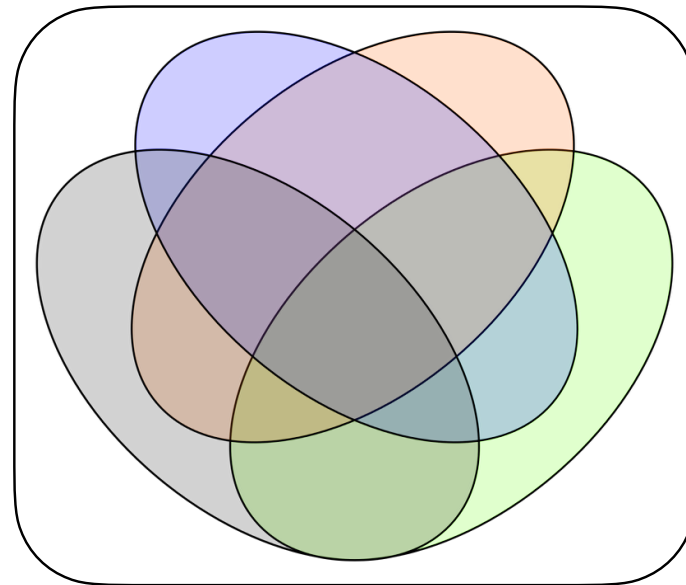
1646–1716

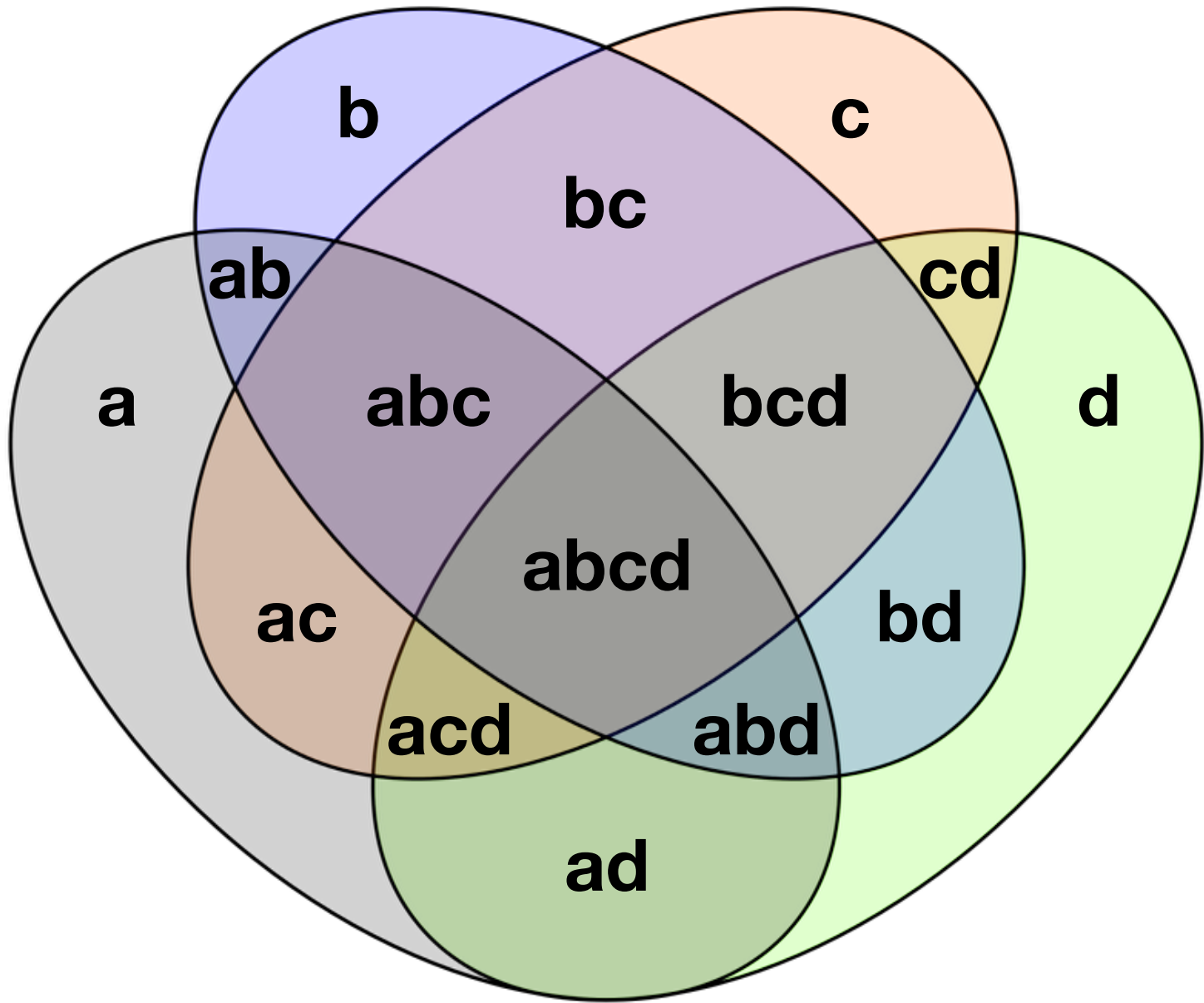
**How many kinds of thing
can we discern with four
predicates?**

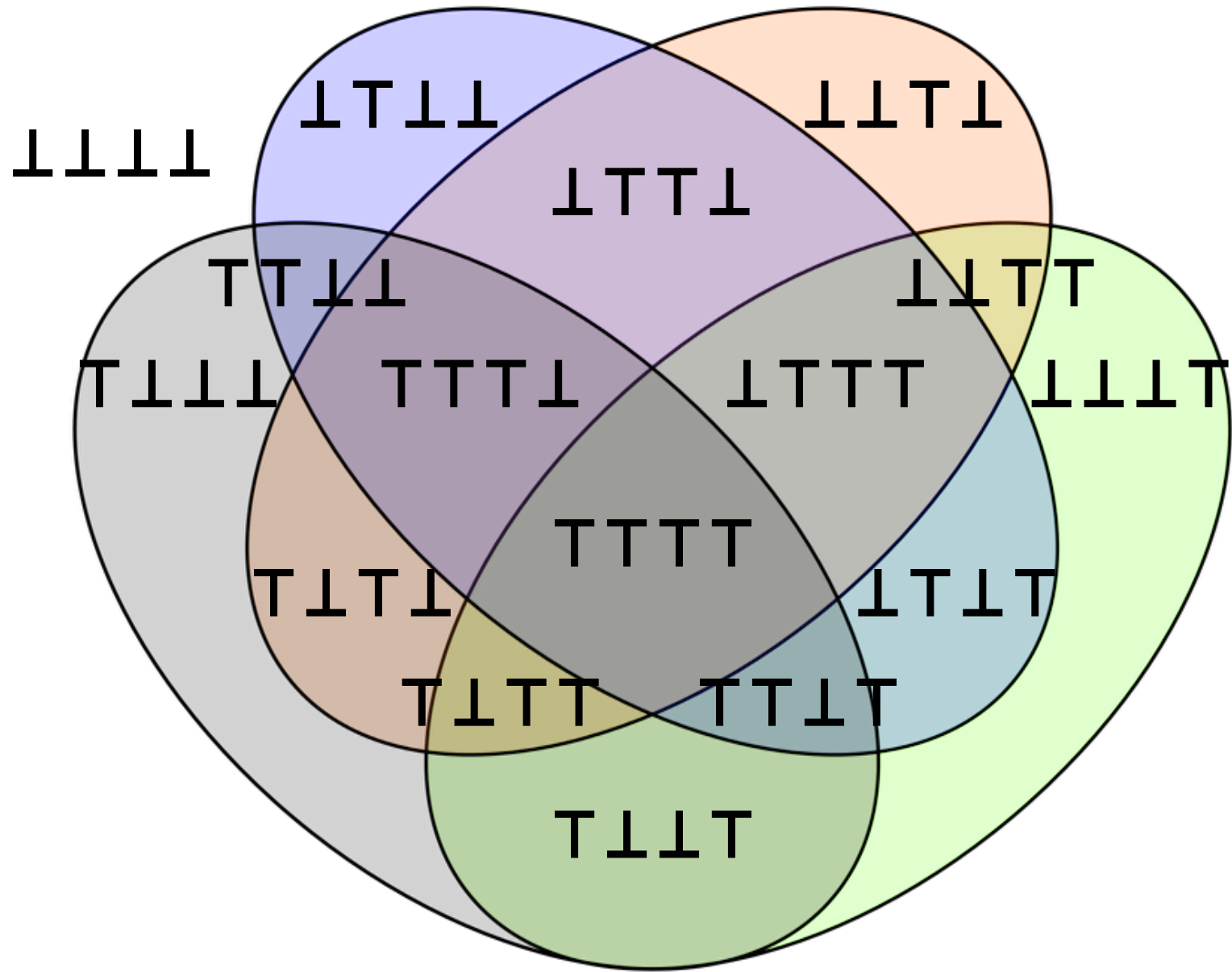
- Identity of Indiscernables:
“No two distinct things exactly resemble one another.”
— Leibniz

That is, two objects are identical if and only if they satisfy the same properties.

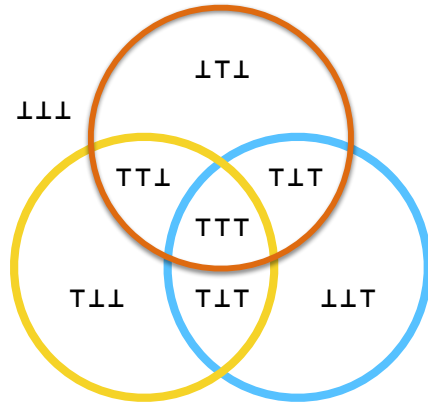
- “A difference that makes no difference is no difference.” — Spock
- “Equals may be substituted for equals.”







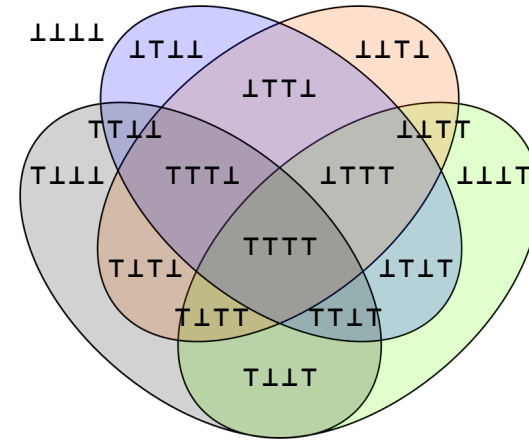
pms



How many universes can we distinguish with three predicates?

There are 256 universes can we distinguish them with Aristotelian categorical propositions?

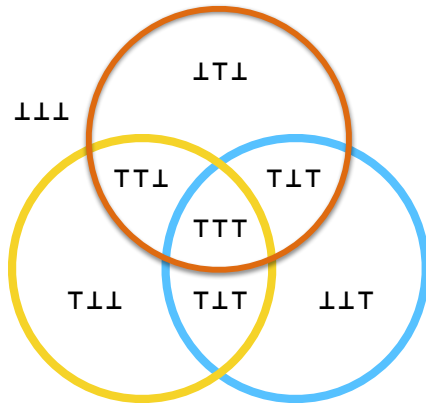
abcd



How many universes can we distinguish with four predicates?

There are 64 Ki universes can we distinguish them with Aristotelian categorical propositions?

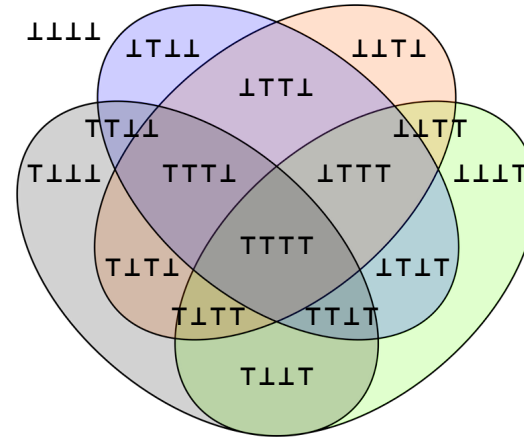
pms



(Bool, Bool, Bool)
 How many predicates are
 there for this universe?

$$2^8 = 256$$

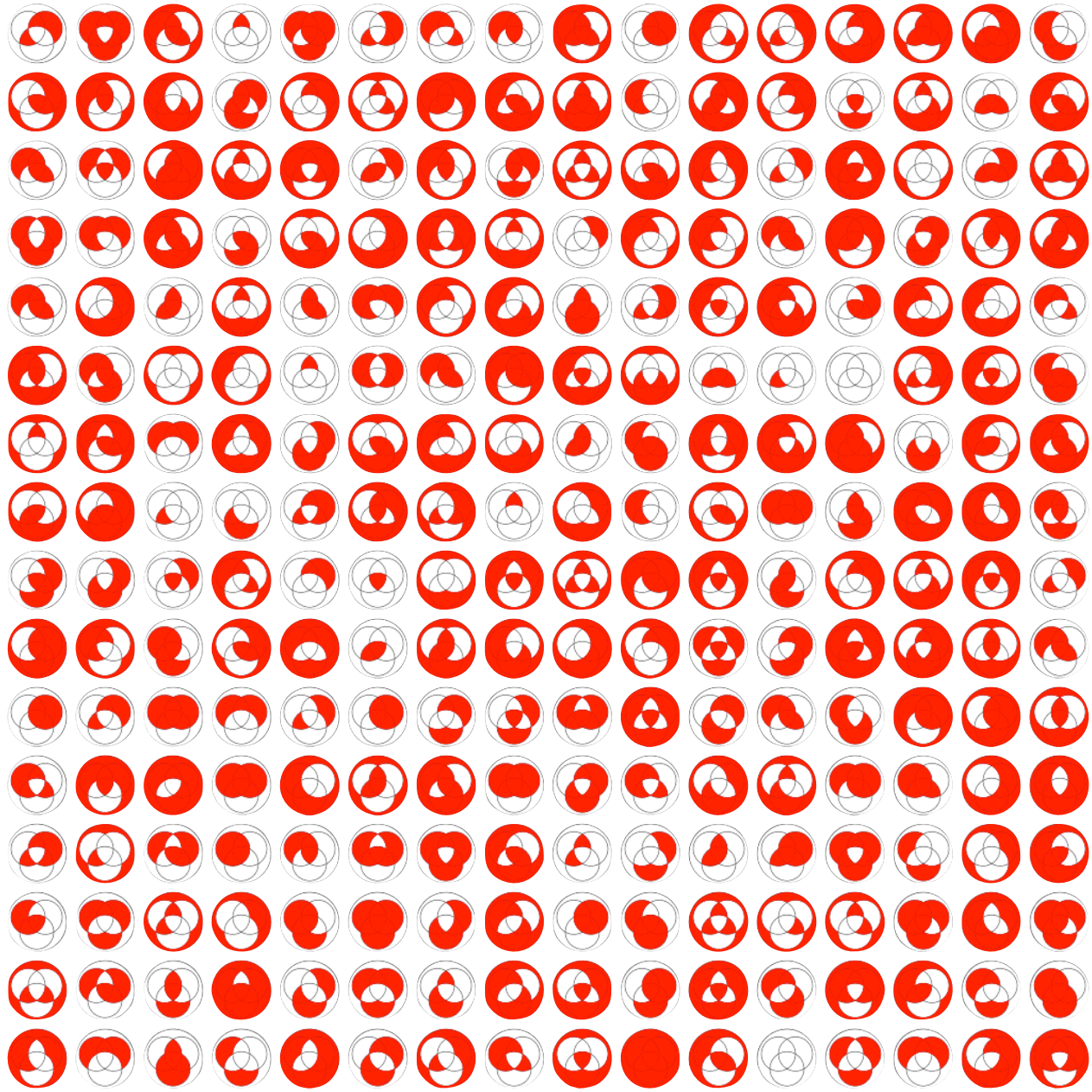
abcd



(Bool, Bool, Bool, Bool)
 How many predicates are
 there for this universe?

$$64 \text{ Ki} \\ 2^{16} = 64 \times 1024 = 65,536$$

They can all be written in terms of $\neg \wedge \vee$

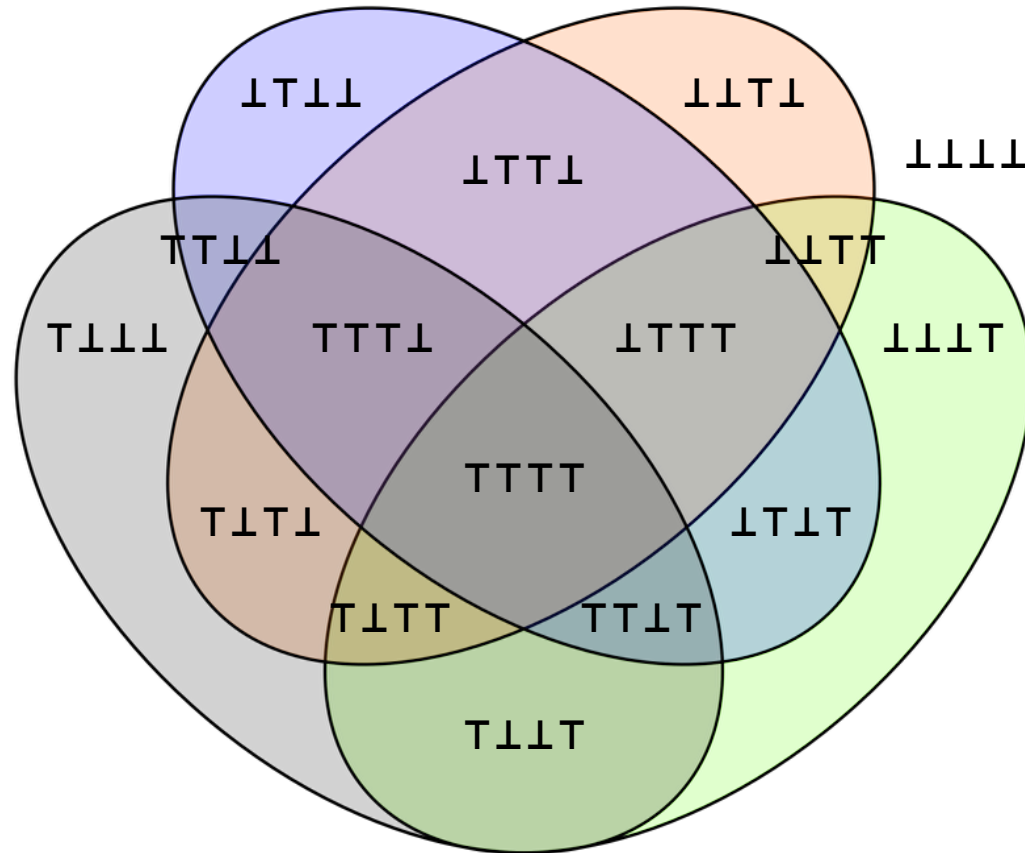


Each region is represented by a *state*, or *valuation*

e.g. $\top \perp \perp \top$
valuation
a function

```
data Atom =
  A | B | C | D
v :: Atom -> Bool
v A = True
v B = False
v C = False
v D = True
```

e.g. $\top \perp \perp \top$
state $A \wedge \neg B \wedge \neg C \wedge D$
conjunction of literals

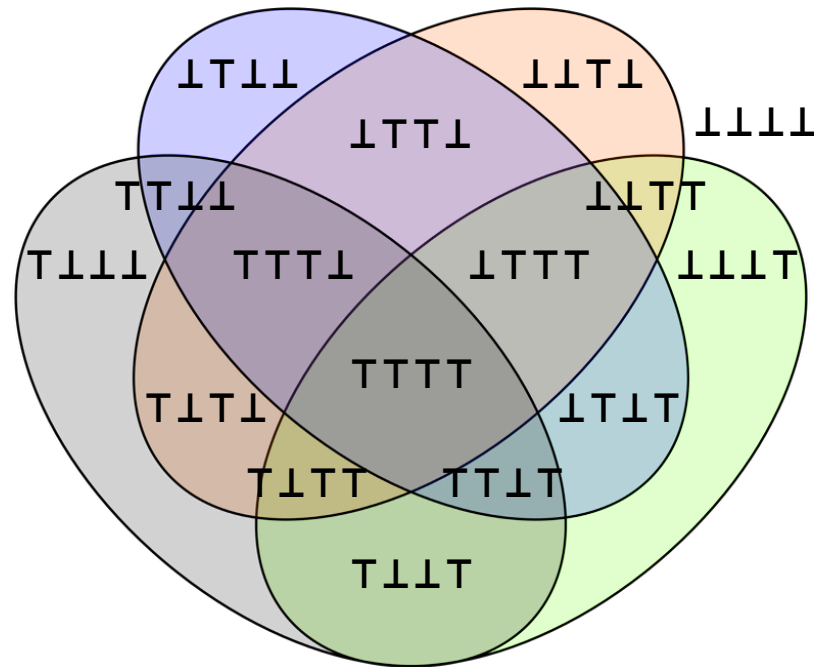


Each region is represented by a *state*, or *valuation*

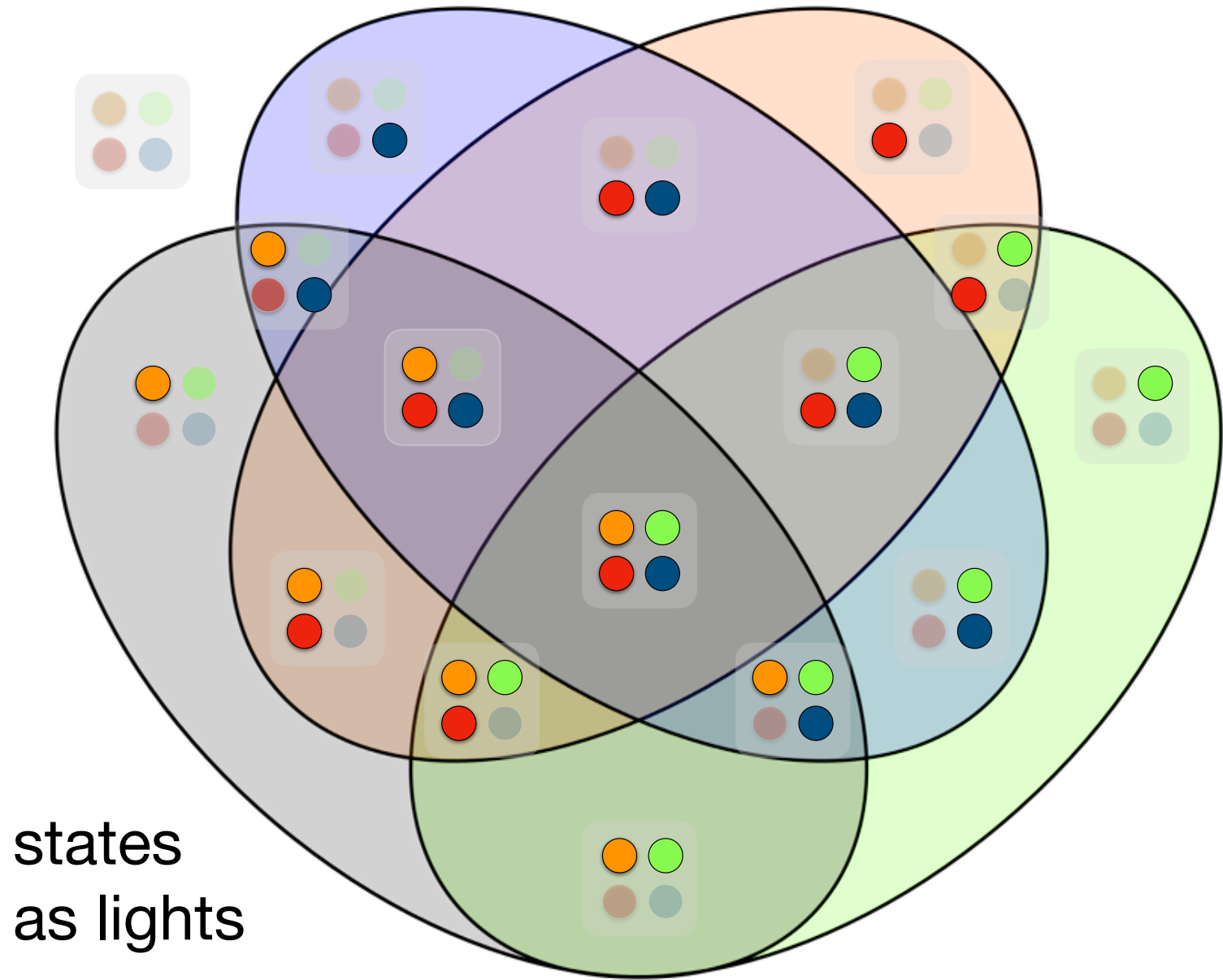
e.g. $\top \perp \perp \top$
 valuation
 a function

```
data Atom =
  A | B | C | D
v :: Atom -> Bool
v A = True
v B = False
v C = False
v D = True
```

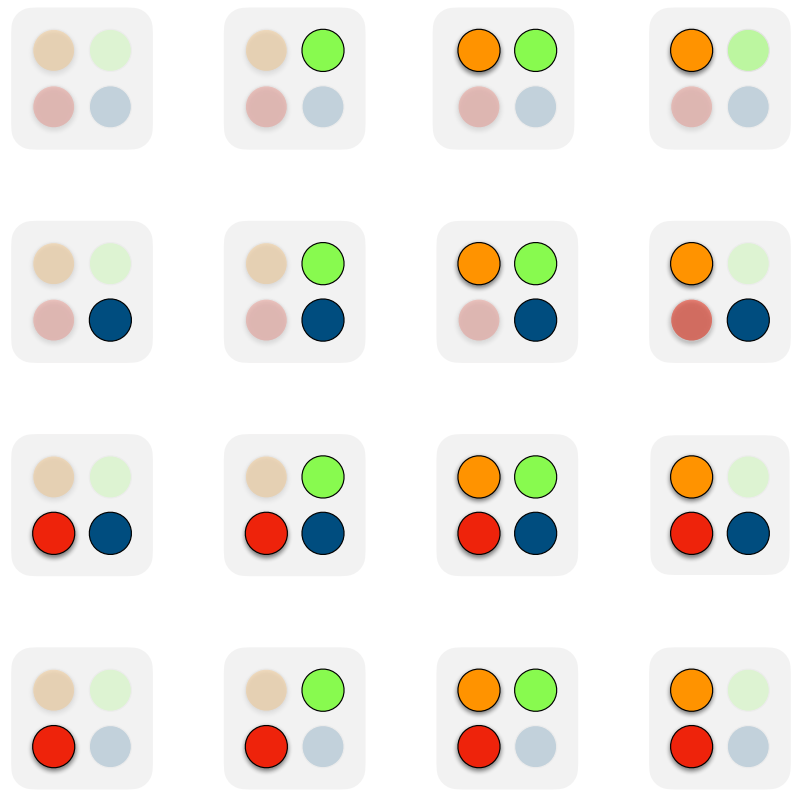
e.g. $\top \perp \perp \top$
 state $A \wedge \neg B \wedge \neg C \wedge D$
 conjunction of literals



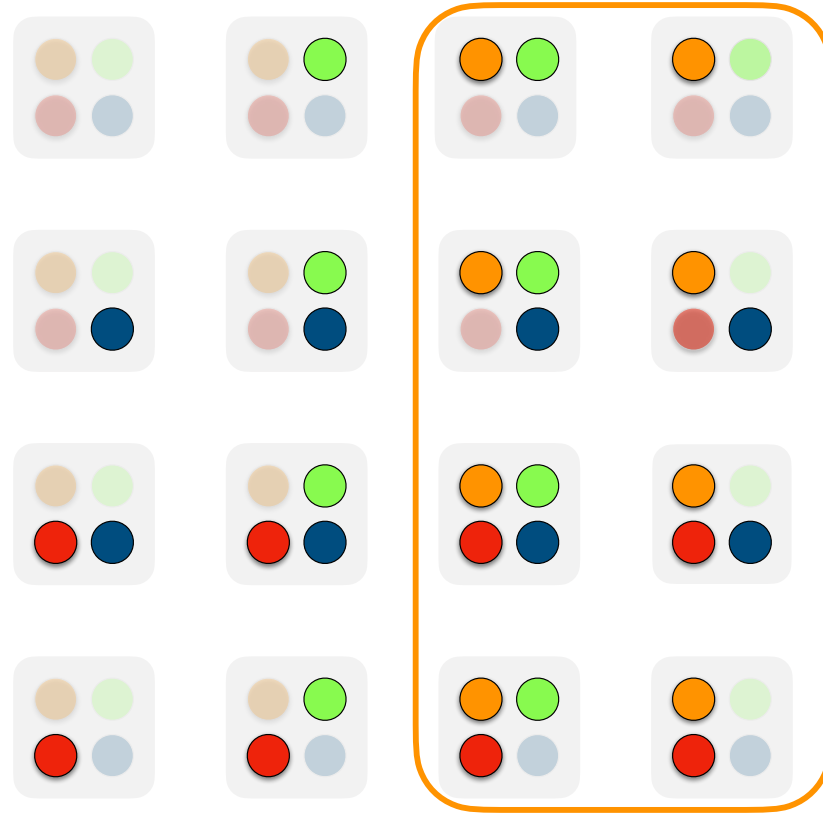
$A, \neg B, \neg C, D \models p$ iff $p(\top \perp \perp \top)$



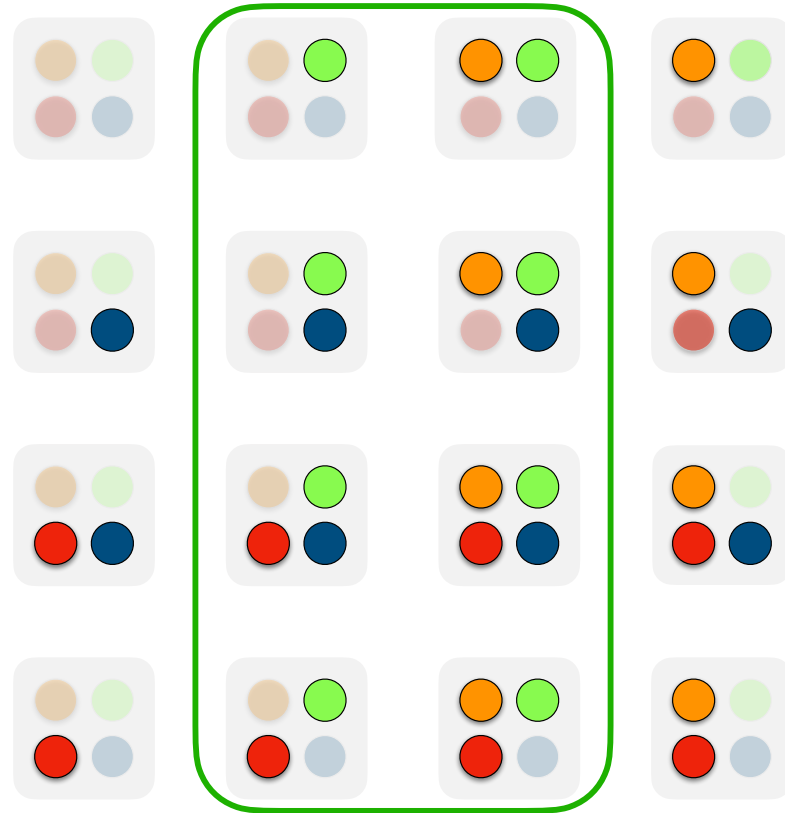
states
as lights



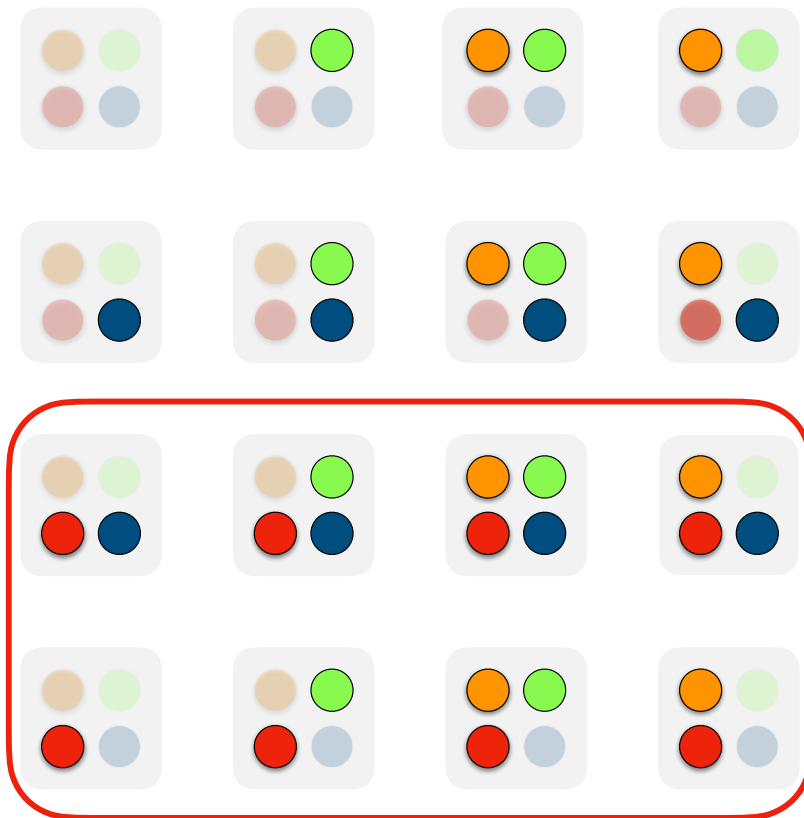
A



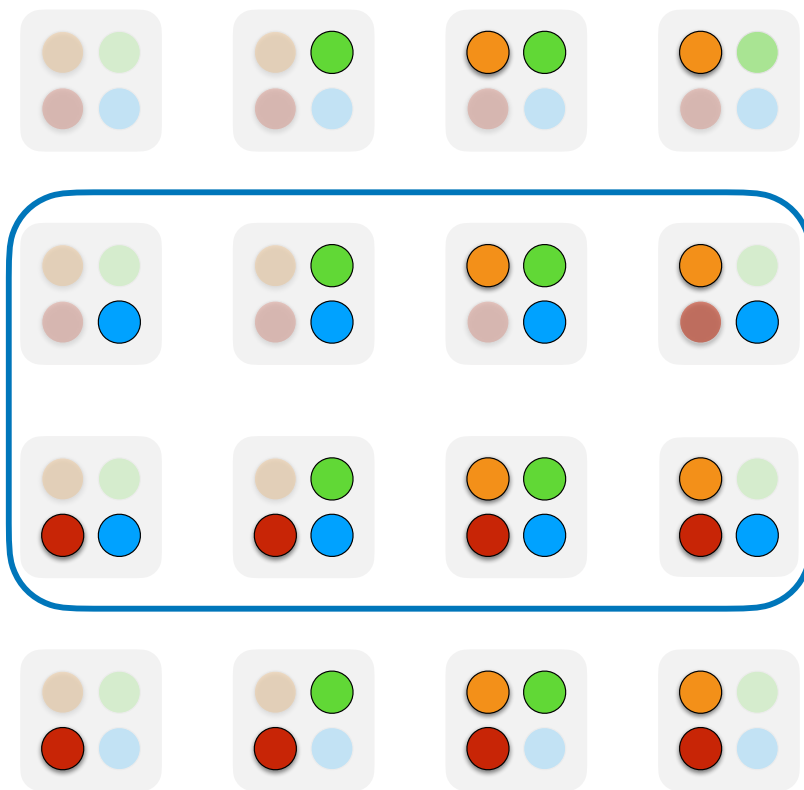
G



R



B

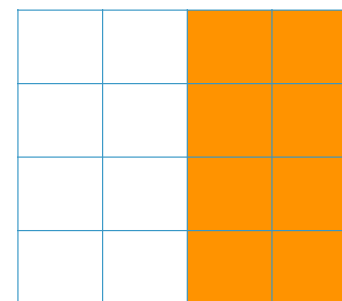


AG

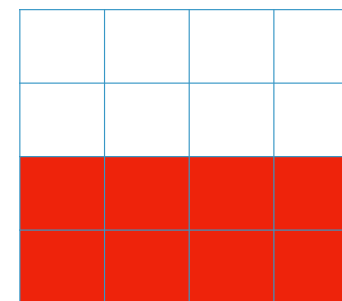
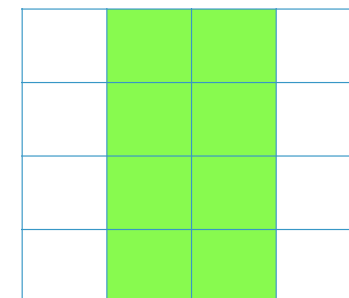
	00	01	11	10
00				
01				
11				
10				

RB

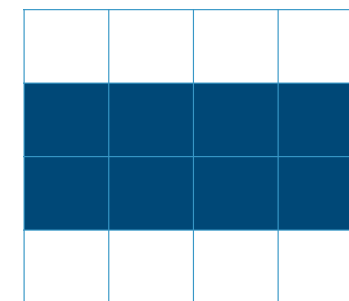
Amber



Green



Red



Blue

Karnaugh Maps

	AG			
	00	01	11	10
RB	00			
	01			
	11			
	10			

4 atoms: 16 states: 64Ki subsets

Amber

Green

Red

Blue

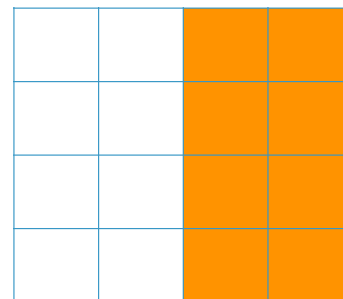
Karnaugh Maps

		AG			
		00	01	11	10
RB	00	0	0	0	0
	01	0	1	1	0
	11	0	1	1	0
	10	0	0	0	0

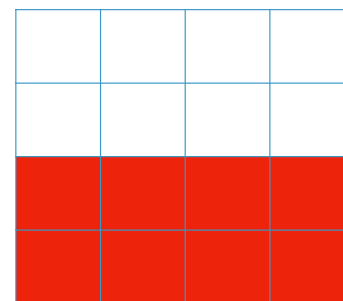
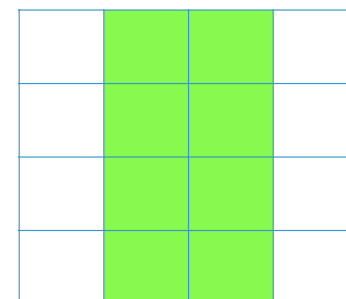
$$G \wedge B$$

4 atoms: 16 states: 64Ki subsets

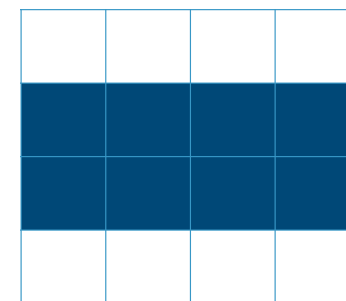
Amber



Green



Red



Blue

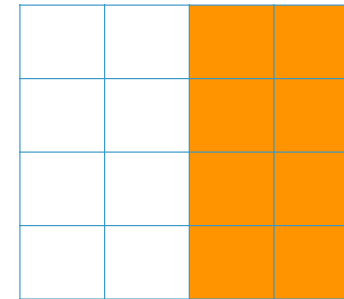
Karnaugh Maps

		AG			
		00	01	11	10
RB	00	0	0	0	0
	01	1	1	0	0
	11	1	1	0	0
	10	0	0	0	0

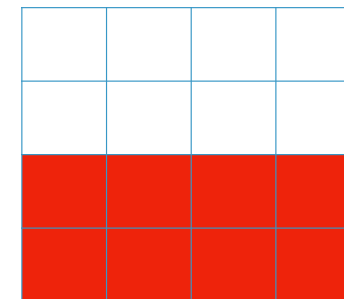
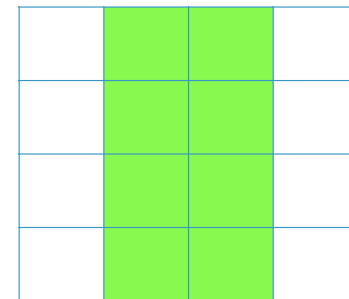
$$\neg A \wedge B$$

4 atoms: 16 states: 64Ki subsets

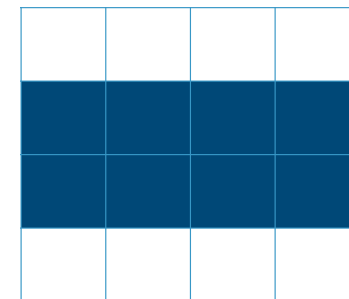
Amber



Green



Red



Blue

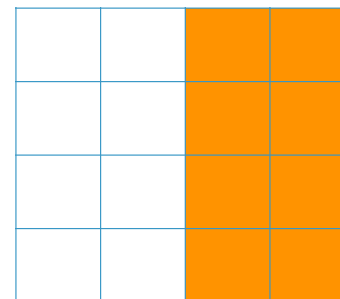
Karnaugh Maps

		AG			
		00	01	11	10
RB	00	1	1	0	0
	01	1	1	0	0
	11	0	0	0	0
	10	0	0	0	0

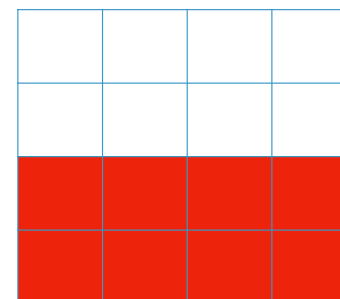
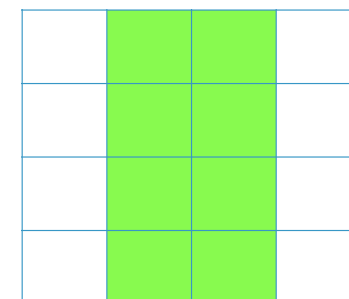
$$\neg A \wedge \neg R$$

4 atoms: 16 states: 64Ki subsets

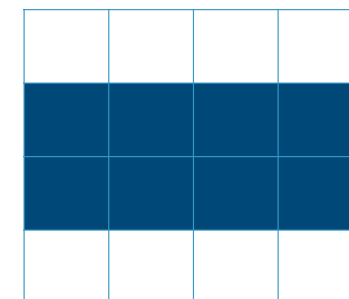
Amber



Green



Red



Blue

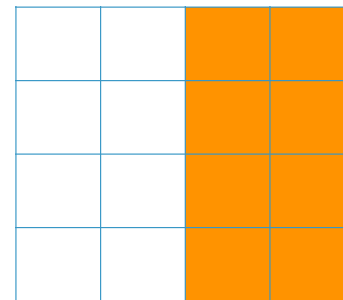
Karnaugh Maps

		AG			
		00	01	11	10
RB	00	1	1	1	1
	01	0	0	0	0
	11	0	0	0	0
	10	0	0	0	0

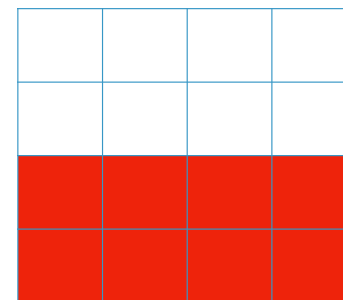
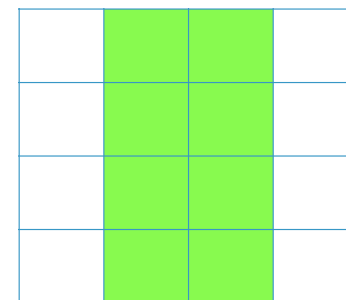
$$\neg B \wedge \neg R$$

4 atoms: 16 states: 64Ki subsets

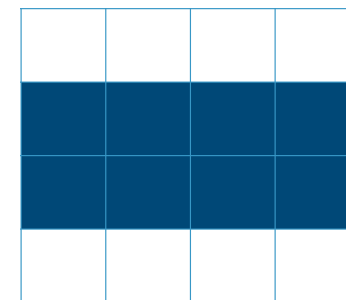
Amber



Green



Red



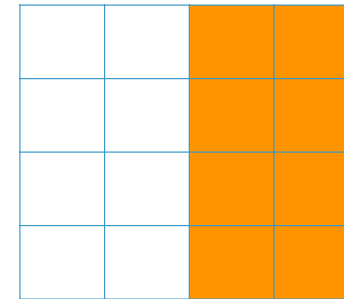
Blue

Karnaugh Maps

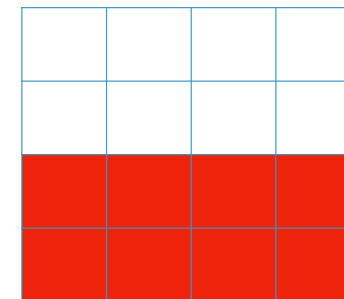
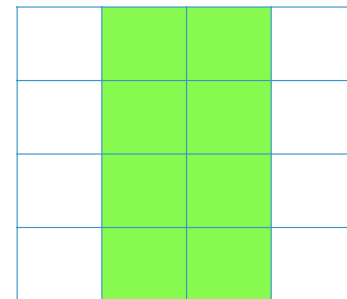
		AG			
		00	01	11	10
RB	00	1	0	0	1
	01	1	0	0	1
	11	0	0	0	0
	10	0	0	0	0

$$\neg G \wedge \neg R$$

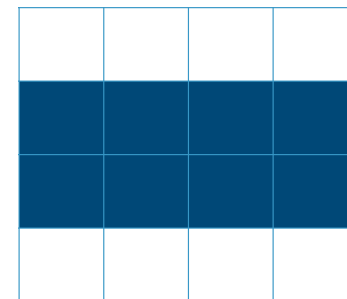
Amber



Green

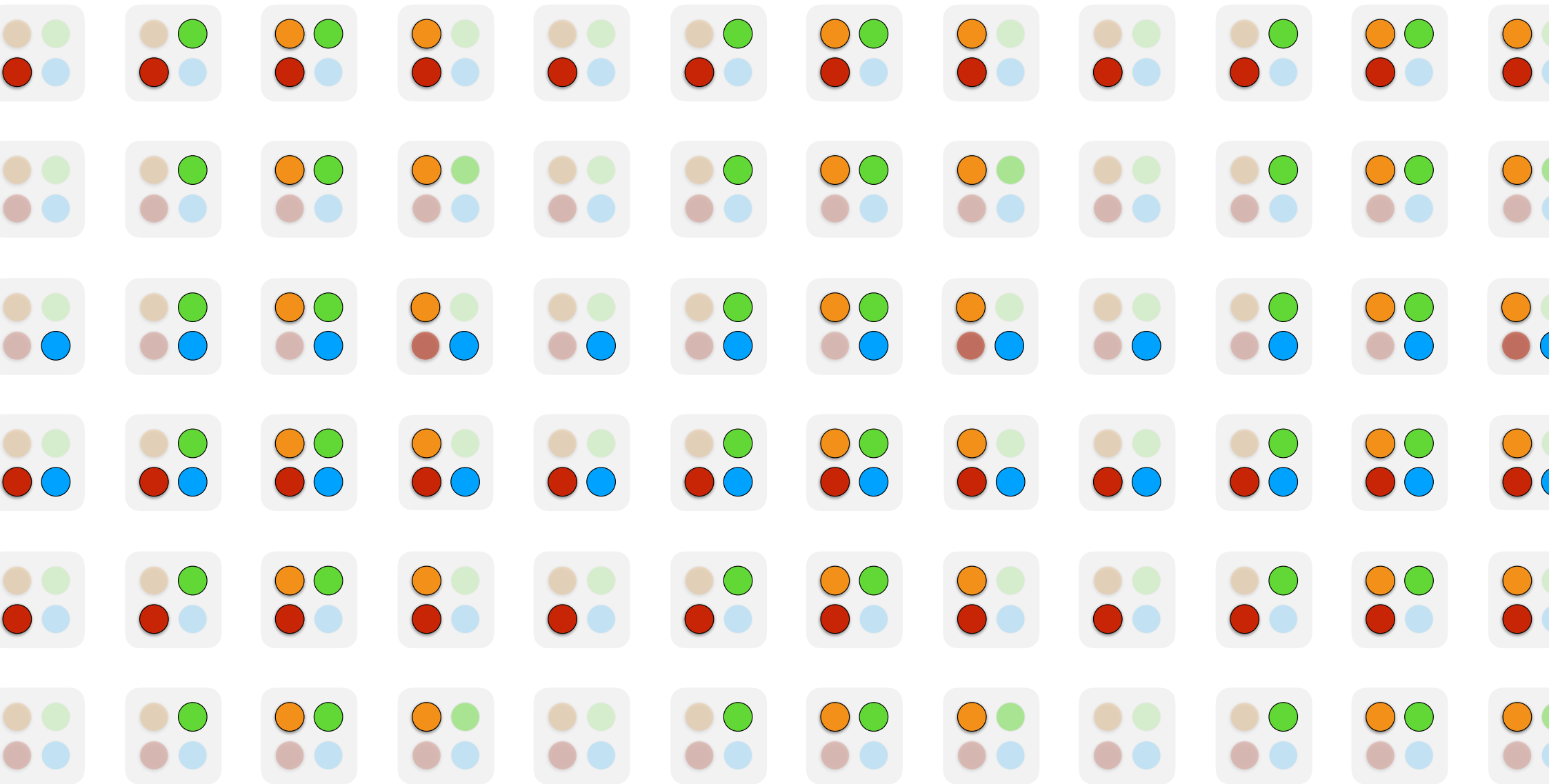


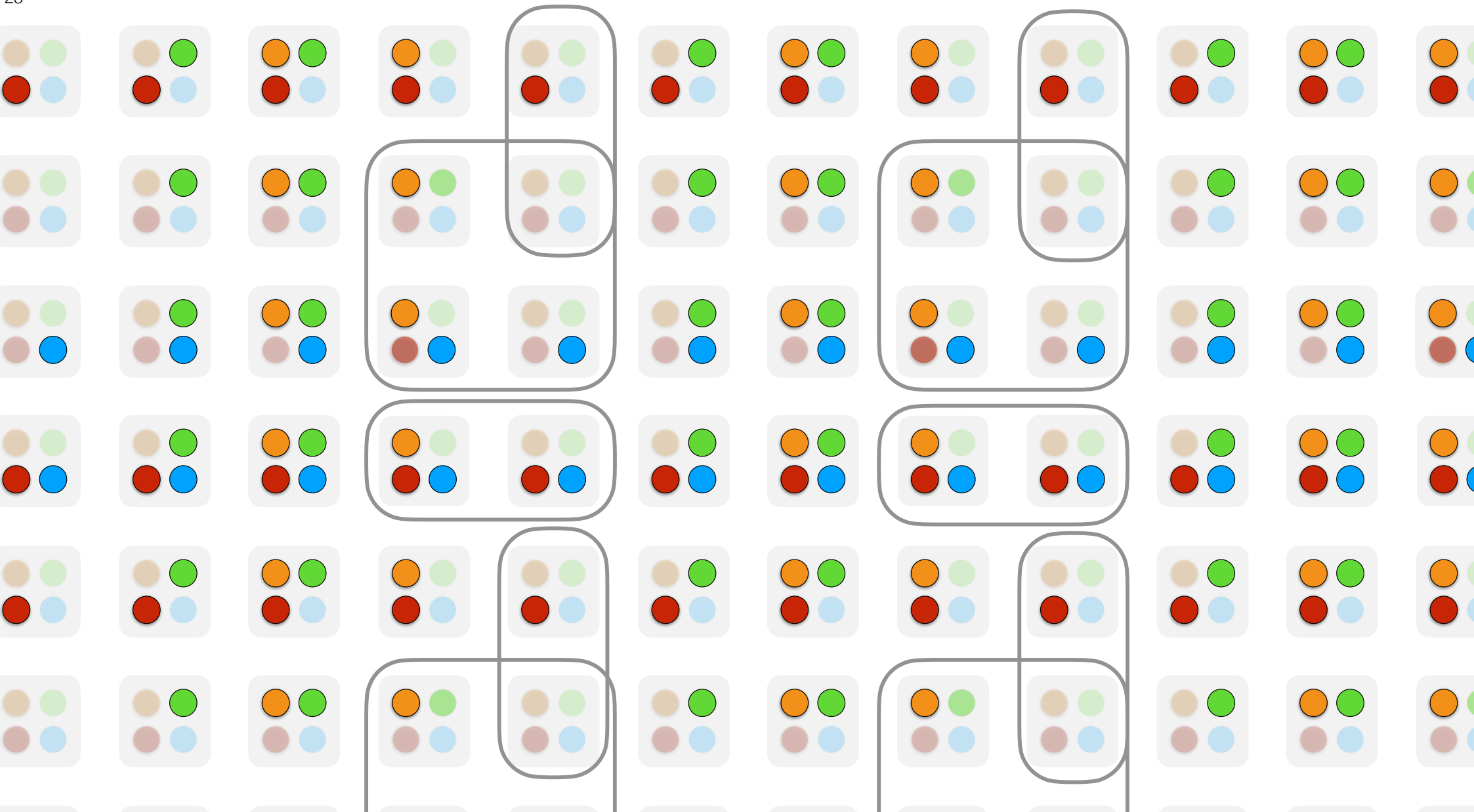
Red



Blue



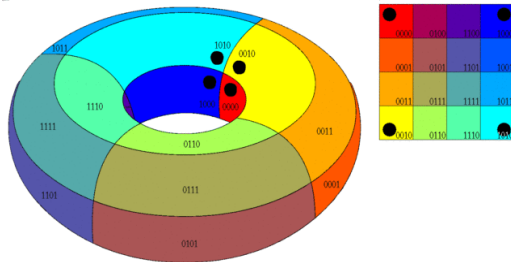




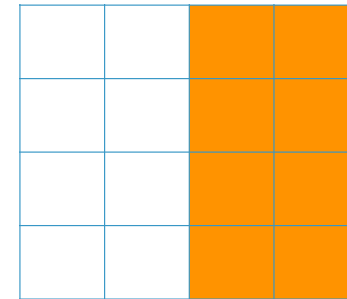
Karnaugh Maps

		AG			
		00	01	11	10
RB	00	1	0	0	1
	01	1	0	0	1
	11	0	0	0	0
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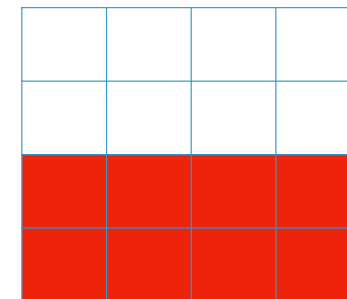
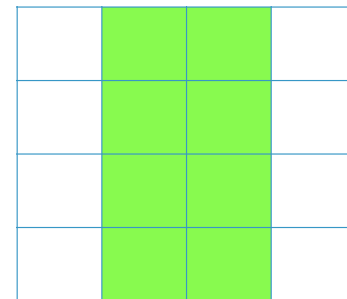
$$\neg G \wedge \neg R$$



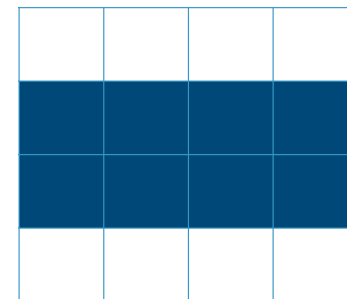
Amber



Green



Red

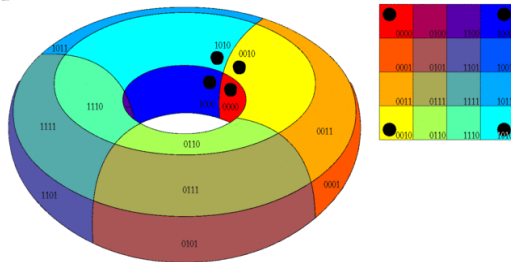


Blue

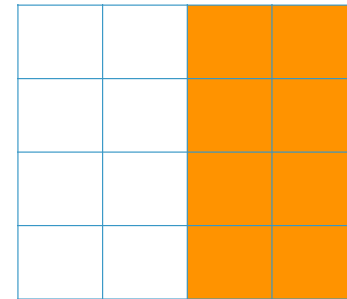
Karnaugh Maps

		AG			
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RB	00	1	0	0	1
	01	0	0	0	0
	11	0	0	0	0
	10	1	0	0	1

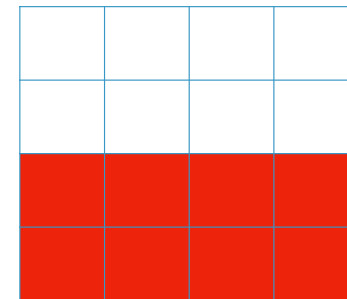
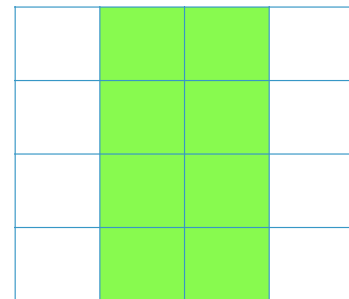
$$\neg G \wedge \neg B$$



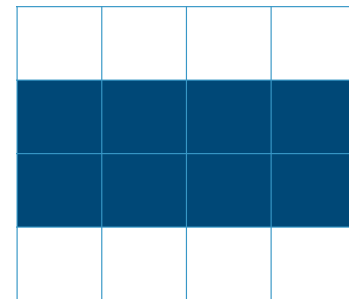
Amber



Green



Red



Blue

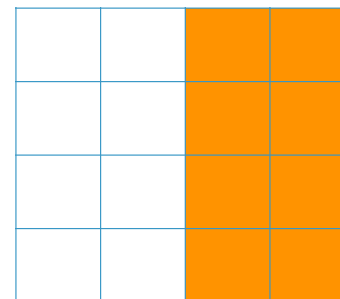
Karnaugh Maps

		AG			
		00	01	11	10
RB	00	0	0	0	0
	01	0	1	1	0
	11	0	1	1	0
	10	0	0	0	0

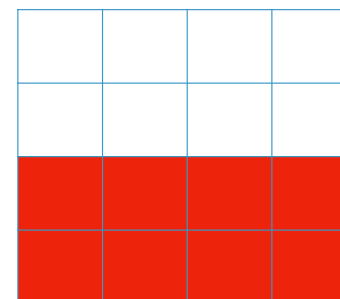
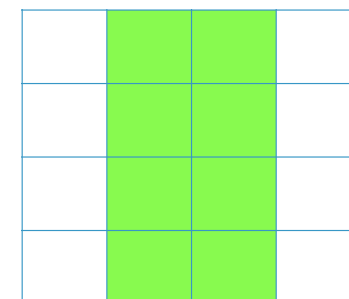
$$G \wedge B$$

4 atoms: 16 states: 64K subsets

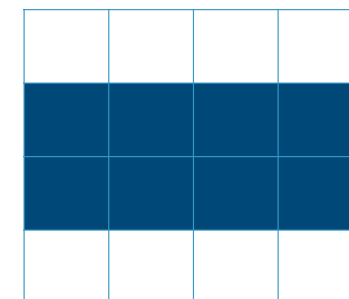
Amber



Green

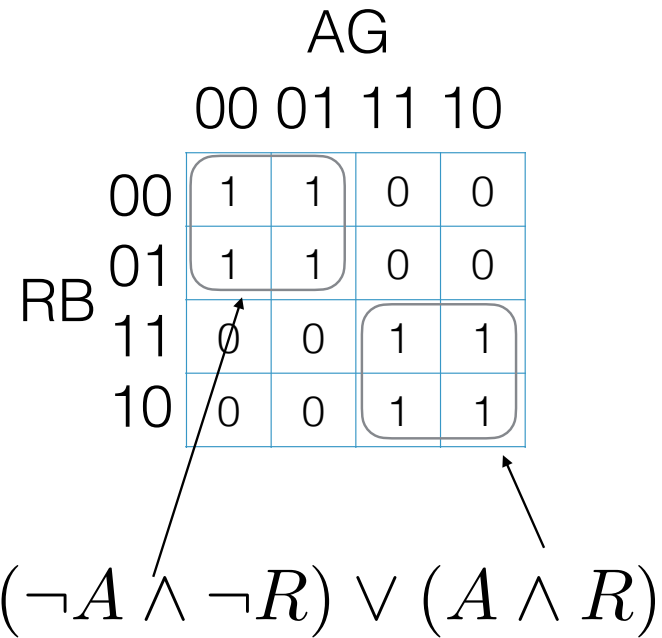


Red

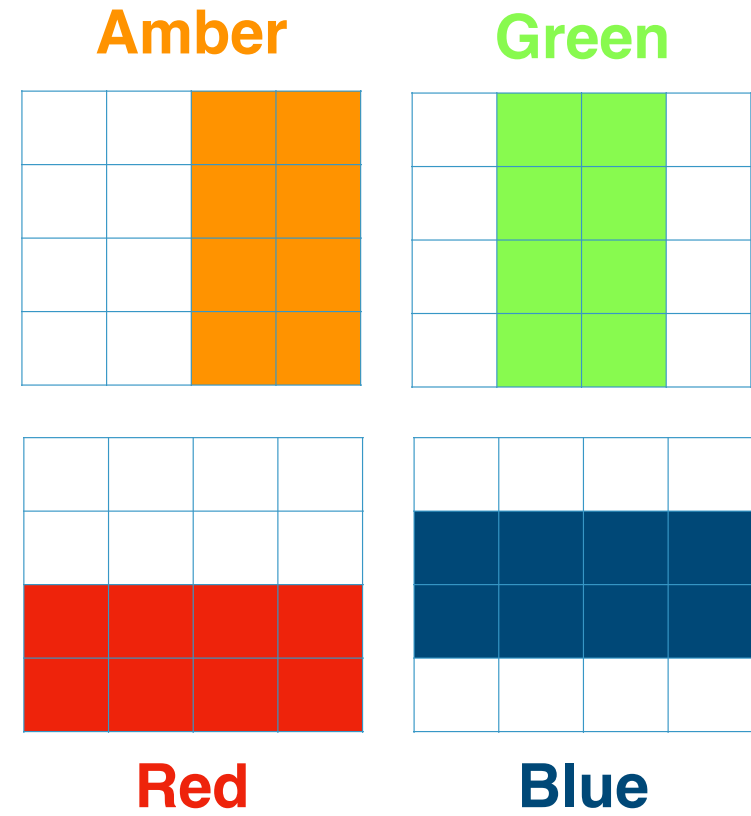


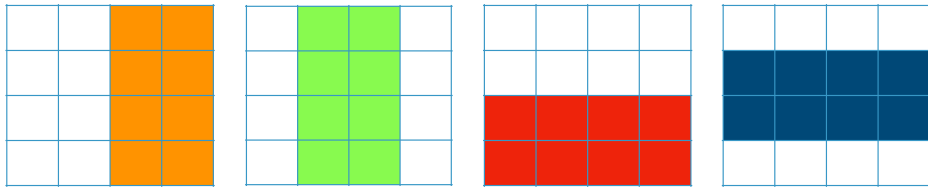
Blue

Karnaugh Maps



4 atoms: 16 states: 64Ki subsets





Amber

Green

Red

Blue

		AG			
		00	01	11	10
RB	00	1	0	0	0
	01	1	1	0	0
	11	0	1	1	1
	10	0	0	1	1

DNF disjunctive normal form

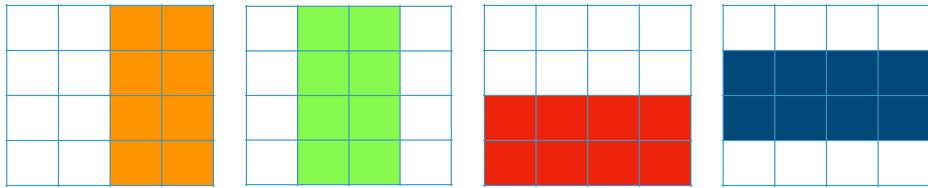
disjunction of conjunctions of literals

to produce a DNF / SOP

identify blocks of 1s

and write a product for each

Karnaugh Maps



Amber

Green

Red

Blue

Karnaugh Maps

		AG			
		00	01	11	10
RB	00	1	0	0	0
	01	1	1	0	0
	11	0	1	1	1
	10	0	0	1	1

to produce a DNF / SOP
 identify blocks of 1s
 and write a product for each

$$RA + BA'G + R'A'G'$$

$$(R \wedge A) \vee (B \wedge \neg A \wedge G) \vee (\neg R \wedge \neg A \wedge \neg G)$$

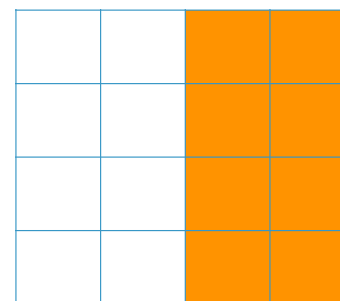
Karnaugh Maps

		AG			
		00	01	11	10
RB	00	0	0	0	0
	01	0	1	1	0
	11	0	1	1	0
	10	0	0	0	0

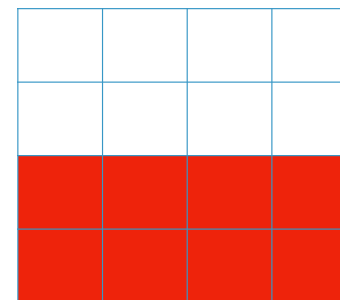
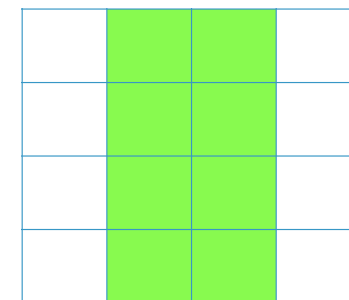
$$G \wedge B$$

4 atoms: 16 states: 64Ki subsets

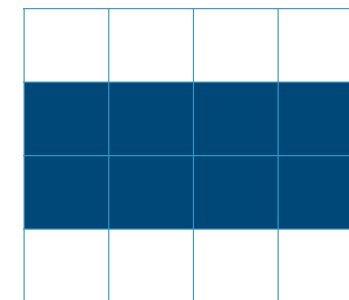
Amber



Green



Red



Blue

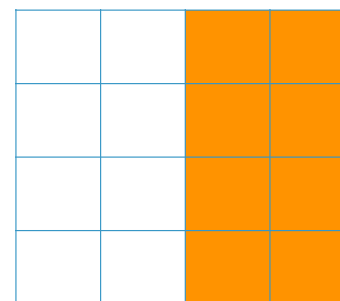
Karnaugh Maps

		AG			
		00	01	11	10
RB	00	1	1	1	1
	01	1	0	0	1
	11	1	0	0	1
	10	1	1	1	1

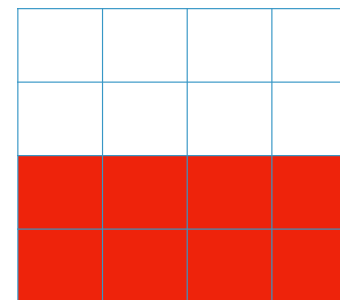
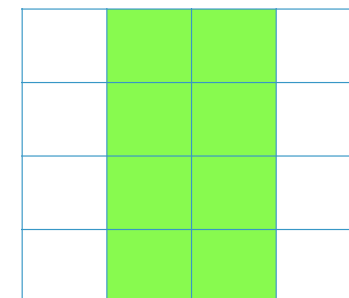
$$\neg B \vee \neg G$$

4 atoms: 16 states: 64Ki subsets

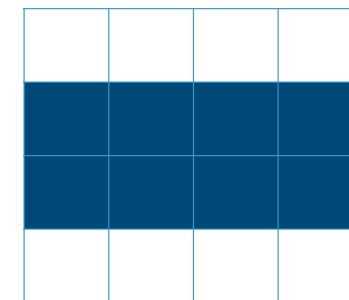
Amber



Green

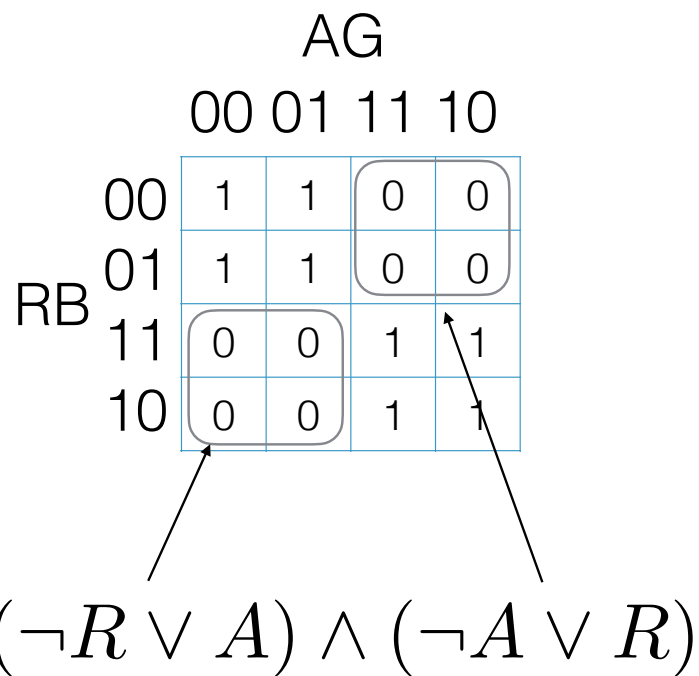


Red

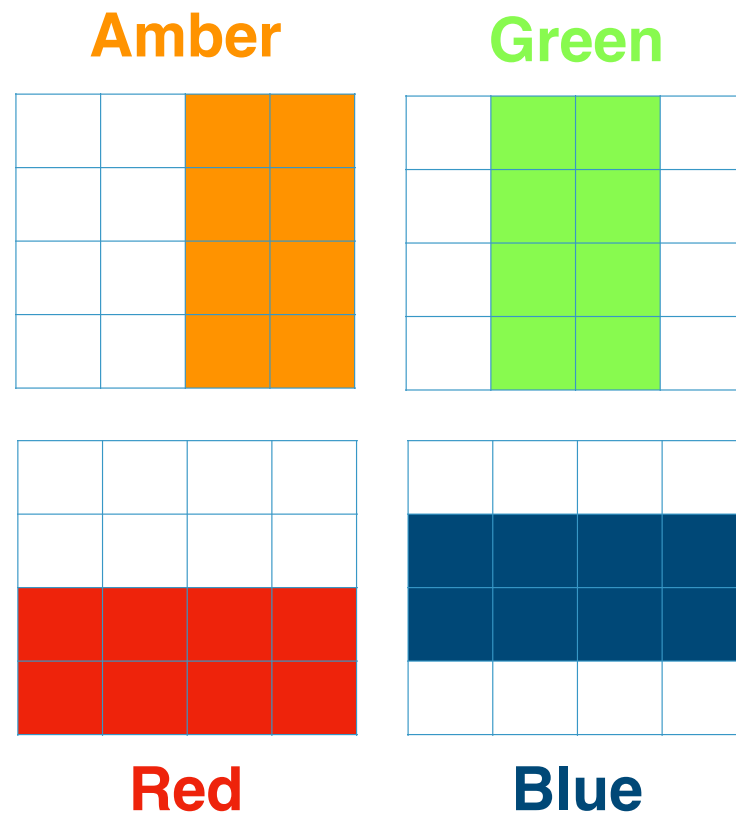


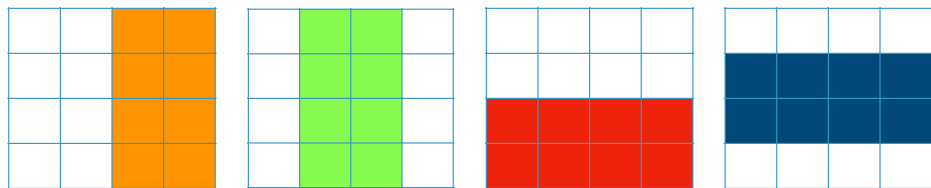
Blue

Karnaugh Maps



4 atoms: 16 states: 64Ki subsets





Amber

Green

Red

Blue

Karnaugh Maps

		AG			
		00	01	11	10
RB	00	1	0	0	0
	01	1	1	0	0
	11	0	1	1	1
	10	0	0	1	1

CNF conjunctive normal form

conjunction of disjunctions of literals

to produce a CNF / POS

identify blocks of 0s

and write a sum for each

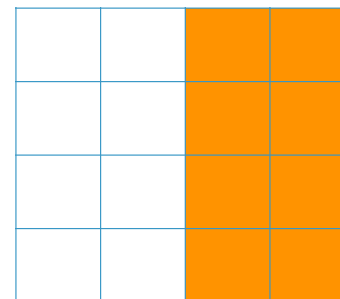
Karnaugh Maps

		AG			
		00	01	11	10
RB	00	0	0	0	0
	01	0	1	1	0
	11	0	1	1	0
	10	0	0	0	0

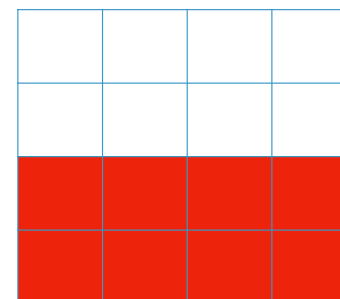
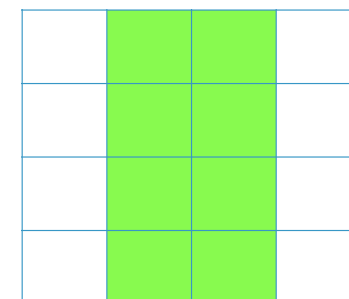
$$G \wedge B$$

4 atoms: 16 states: 64Ki subsets

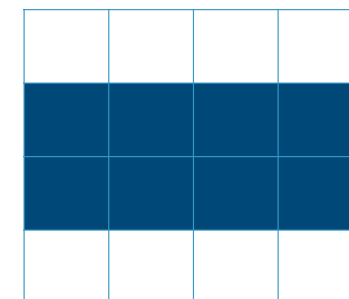
Amber



Green



Red



Blue

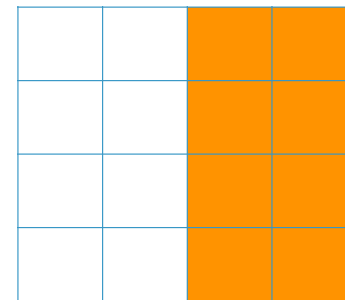
Karnaugh Maps

		AG			
		00	01	11	10
RB	00	1	1	1	1
	01	1	0	0	1
	11	1	0	0	1
	10	1	1	1	1

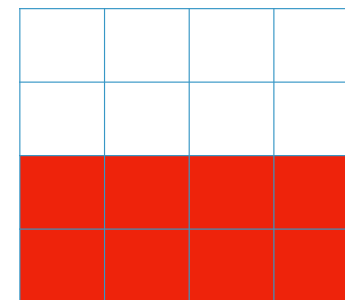
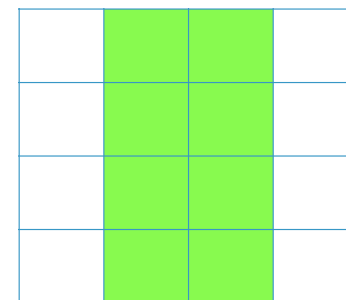
$$\neg B \vee \neg G$$

4 atoms: 16 states: 64Ki subsets

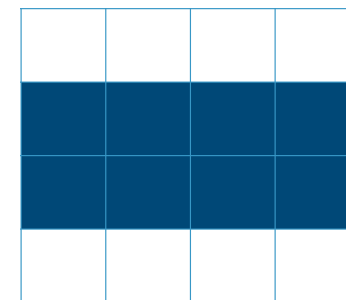
Amber



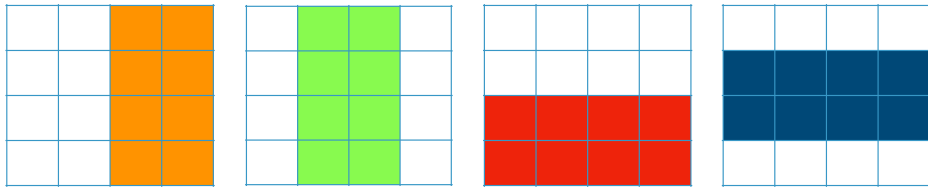
Green



Red



Blue



Amber

Green

Red

Blue

Karnaugh Maps

		AG			
		00	01	11	10
RB	00	1	0	0	0
	01	1	1	0	0
	11	0	1	1	1
	10	0	0	1	1

$$(R' + A + G)(B + A + G')(R + A')$$

$$(\neg R \vee A \vee G) \wedge (B \vee A \vee \neg G) \wedge (R \vee \neg A)$$

to produce a CNF / POS
 identify blocks of 0s
 and write a sum for each